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CoCos, Bail-In, and Tail Risk

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Abstract

Contingent convertibles (CoCos) and bail-in debt for banks have been proposed as potential mechanisms to enhance financial stability. They function by converting to equity when a bank approaches insolvency. We develop a capital structure model to analyze the incentives created by these forms of contingent capital. Our formulation includes firm-specific and market-wide tail risk in the form of two types of jumps and leads to a tractable jump-diffusion model of the firm's income and asset value. The firm's liabilities include insured deposits and senior and subordinated debt, as well as convertible debt. Our model combines endogenous default, debt rollover, and jumps; these features are essential in examining how changes in capital structure to include CoCos or bail-in debt change incentives for equity holders. We derive closed-form expressions to value the firm and its liabilities, and we use these to investigate how CoCos affect debt overhang, asset substitution, the firm's ability to absorb losses, the sensitivity of equity holders to various types of risk, and how these properties interact with the firm's debt maturity profile, the tax treatment of CoCo coupons, and the pricing of deposit insurance. We examine the effects of varying the two main design features of CoCos, the conversion trigger and the conversion ratio, and we compare the effects of CoCos with the effects of reduced bankruptcy costs through orderly resolution. Across a wide set of considerations, we find that CoCos generally have positive incentive effects when the conversion trigger is not set too low. The need to roll over debt, the debt tax shield, and tail risk in the firm's income and asset value have particular impact on the effects of CoCos. We also identify a phenomenon of debt-induced collapse that occurs when a firm issues CoCos and then takes on excessive additional debt: the added debt burden can induce equity holders to raise their default barrier above the conversion trigger, effectively changing CoCos to junior straight debt; equity value experiences a sudden drop at the point at which this occurs. Finally, we calibrate the model to past data on the largest U.S. bank holding companies to see what impact CoCos might have had on the financial crisis. We use the calibration to gauge the increase in loss absorbing capacity and the reduction in debt overhang costs resulting from CoCos. We also time approximate conversion dates for high and low conversion triggers.

1 Introduction

Contingent convertibles (CoCos) and bail-in debt for banks have been proposed as potential mechanisms to enhance financial stability. They function by converting to equity when a bank

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approaches insolvency and would otherwise have difficulty raising new equity. These contingent capital instruments have the potential to offer the advantages of debt in good times and equity in a crisis.

This paper studies the incentive effects of contingent convertible debt (CoCos) and bail-in debt in a structural model of a financial firm. CoCos and bail-in are forms of debt that convert to equity as a firm's assets lose value. They are points on a continuum of such securities differing primarily along two dimensions. One dimension is the level of the trigger for conversion from debt to equity, with CoCos (often classified as *going-concern* contingent capital) converting as a firm nears, but has not yet reached, financial distress, and bail-in (often classified as *gone-concern* contingent capital) converting at the point of non-viability. The second dimension along which these types of securities vary is their conversion ratio and its impact on the original shareholders: the conversion of CoCos dilutes the original shareholders, but a bail-in is accompanied by a reorganization that wipes out the original shareholders – an infinite dilution. Thus, these forms of contingent capital offer firms and regulators two levers in their design, the conversion trigger and the dilution ratio. We study the incentives created by varying these levers and other features of firm's assets and liabilities.

We develop a model of the capital structure of a financial firm that includes CoCos or bail-in debt along with insured deposits, senior debt, and subordinated debt. Importantly, bankruptcy in our model is endogenous, as in Leland [39] and Leland and Toft [40], meaning that it results from the optimal decision of shareholders to exercise their option to surrender the firm's assets to the creditors. We are thus interested in how the two levers in the design of contingent capital affect the incentives for shareholders to invest additional capital in the firm, and how the levers affect the shareholders' incentives to take on different types of risk in investing the firm's assets. Our model incorporates debt rollover, and a central theme of our analysis is that the shareholders' incentives are strongly influenced by this feature — the cost of debt rollover can motivate shareholders to reduce the firm's leverage and the riskiness of its assets. Also crucial to our analysis is the inclusion of both jumps and diffusion in asset value. We interpret the diffusive risk as the ordinary level of volatility in the firm's business, which is readily observable by a regulator. In contrast, the jumps capture the firm's ability to take on high-yielding tail risk that is much harder to measure if jumps are rare. Among the questions we examine is how replacing straight debt with convertible debt affects the attractiveness of the two types of risk to equity holders.

We obtain explicit expressions to value all pieces of the capital structure by building on results of Chen and Kou [14] and Cai, Chen, and Wan [9]. Using the valuation formulas, we investigate the effect of varying key model parameters including the trigger level for conversion of debt to equity, the dilution ratio at conversion, the mix and average maturity of different types of debt, bankruptcy costs, deposit insurance premiums, the tax-deductibility of interest payments on CoCos, and the riskiness of the firm's investments. We investigate how these parameters affect debt and equity values, the timing of bankruptcy, the risk-sensitivity of equity, the propensity for asset substitution, and the extent of debt overhang as an obstacle to raising capital. We can draw some conclusions theoretically, whereas others are illustrated through comparative statics. Details are provided in later sections, but we highlight some key observations as follows:

(i). So long as the trigger level and the conversion ratio are designed to ensure that conversion occurs prior to endogenous bankruptcy, the precise values of the trigger and the conversion ratio have no effect on the timing of bankruptcy or the asset level at which it occurs. This simple but important observation will underlie several other implications of the model. Conversion in our model is triggered when asset value falls below a specified level. The asset level at which shareholders optimally surrender the firm is insensitive to the conversion level and conversion ratio, so long as the conversion level is higher than the resulting bankruptcy level. If conversion precedes bankruptcy, the optimal bankruptcy level is the level for the post-conversion firm, which does not depend on the conversion trigger or ratio.

(ii). CoCos can reduce default risk, as we explain below. In so doing, they reduce the cost of rolling over straight debt as it matures, and this increases dividends available to equity holders. This effect, together with a desire to avoid unfavorable conversion, can lead equity holders to prefer less risky assets.

(iii). Replacing some straight debt with CoCos has several effects.

- This replacement reduces the value of the debt tax shield if CoCo coupons are not tax deductible. Even assuming tax-deductibility of CoCo coupons, this replacement will reduce the tax shield through the eventual conversion of CoCos to equity, unless the CoCo coupon is so high as to offset this effect.
- This replacement lowers the endogenous default barrier and thus increases the firm's ability to sustain a loss in asset value. It thus reduces bankruptcy costs. The reduction in bankruptcy costs and the reduction in the tax shield have opposite

effects on total firm value, but we find that the reduction in bankruptcy cost is greater in our numerical examples.

- Reducing bankruptcy costs lowers the cost of rolling over the remaining straight debt; thus, replacing some straight debt with convertible debt *can increase the value of equity*, which we interpret as a reduction in the firm's cost of capital. The benefit to shareholders of replacing straight debt with convertible debt increases as asset value decreases.

(iv). We also consider the effects of increasing firm size by issuing CoCos while keeping other forms of debt fixed.

- If the size of the additional CoCo issue is sufficiently large, the increased coupon payments may make it optimal for shareholders to default prior to conversion resulting in greater value destruction at bankruptcy through the increase in the firm's assets and a phenomenon of debt-induced collapse discussed below. We mainly focus on the case (which should be more typical) that the optimal default barrier is lower than the conversion trigger.
- So long as this holds, the default barrier is unchanged and the default risk decreases because the distance to default increases with the value of additional assets.
- The reduced default risk lowers the cost of rolling straight debt, which increases the value of equity. If CoCo coupons are tax-deductible, this further increases equity value, lowering the cost of equity capital.

(v). For completeness, we also consider the effect of replacing some equity with CoCos, though this case is of less interest in practice. If CoCo coupons are tax-deductible, and if the substitution is not so large as to drive the default barrier above the conversion level, then equity holders capture all the value of the increased tax shield with no change in the firm's default risk. However, this replacement can also induce the equity holders to prefer less risky assets in order to preserve the funding advantage provided by unconverted CoCos through the tax shield.

(vi). CoCos can mitigate the debt overhang problem, creating two incentives for new equity investment as the firm's asset value approaches the conversion trigger. If the CoCo coupons are tax deductible, it is optimal for the shareholders to invest in the firm to prevent conversion and preserve the tax shield. Also, assuming the number of shares issued to CoCo investors at conversion is fixed, the value of the equity issued to CoCo investors is largest at the conversion trigger, so the incentive for the shareholders to stave off conversion through additional investment is greatest just above this point.

(vii). CoCos affect asset substitution — the tendency of equity holders to prefer riskier assets after issuing debt — in several ways. As already noted, lowering the cost of rolling over straight debt provides an incentive for equity holders to take on less risk, and this incentive can be increased by the presence of CoCos, particularly in the presence of a tax shield. However, CoCos can also create incentives for equity holders to increase exposure to tail risk (i.e., downward jumps in asset value) because the cost (to shareholders) of conversion is lower if it occurs at a lower asset value.

(viii). As bond investors, holders of CoCos may be unwilling or unable to hold equity following conversion and may therefore receive less than full market value in a forced sale of shares. Anticipating this outcome, they would demand a lower price at the time of their initial investment in CoCos. This effect reduces but, in our examples, does not eliminate the attractiveness to shareholders of replacing some straight debt with CoCos.

(ix). In the pure bail-in case, conversion of debt to equity occurs just as the firm would otherwise declare bankruptcy and the original shareholders are wiped out. We assume that the bail-in avoids the deadweight costs of bankruptcy. Although they are wiped out at bail-in, the original shareholders benefit from replacing straight debt with bail-in debt because the reduction in bankruptcy costs lowers the cost of debt service. The benefit to shareholders of such a replacement increases as asset value decreases.

(x). Our model identifies a phenomenon of "debt-induced collapse" specific to a setting with convertible debt and endogenous default. The phenomenon occurs when a firm issues CoCos and then takes on excessive additional debt. If sufficiently extreme, the additional debt will induce equity holders to default prior to conversion, effectively changing CoCos to junior straight debt. At the point at which this occurs, equity value experiences a sudden drop as the value of the conversion feature to equity holders is eliminated. Avoiding this phenomenon requires setting the conversion trigger to be unambiguous about whether conversion will occur prior to bankruptcy (as with CoCos) or only at bankruptcy (the bail-in case).

(xi). Considering a regulator's perspective, we have already noted that the level of the conversion trigger has no direct effect on the timing of bankruptcy, so long as the conversion trigger remains above the endogenous default barrier. Nevertheless, the regulator can have indirect influence through CoCos. A higher trigger creates a greater incentive for equity holders to invest additional capital in the firm earlier and can reduce incentives to increase the riskiness of the assets; but a lower trigger creates a greater incentive for equity holders to voluntarily replace some straight debt with convertible debt.

(xii). Charging deposit insurance in proportion to all of the firm's debt, including CoCos, reduces some of the positive incentives resulting from CoCos, just as the tax-deductibility of CoCo coupons increases some of these positive incentives.

(xiii). We have calibrated our model to public data on bank balance sheets and stock prices during 2004Q1–2011Q3 for 17 of the 19 largest U.S. bank holding companies. (We exclude Ally because it is privately held and MetLife because it is predominantly an insurance company.) We use the calibration to gauge how much CoCos would have increased banks' ability to sustain losses during the crisis. We also use the calibration to measure debt overhang costs and find that CoCos with a high trigger would have created positive incentives for additional investment in 2008–2009 for most of the banks. Based on the calibration, we time the conversion of CoCos with high and low triggers for each bank and and identify which banks would not have triggered conversion.

Flannery [24] proposed reverse convertible debentures (called contingent capital certificates in Flannery [25]) that would convert from debt to equity based on a bank's stock price. Other designs have been proposed in Bolton and Samama [6], Duffie [17], McDonald [42], Pennacchi, Vermaelen, and Wolf [49], Squam Lake Working Group [52], and Sundaresan and Wang [53]; see Calomiris and Herring [12] or Pazarbasioglu et al. [47], for an overview and comparison. Calello and Ervin [11] outline a bail-in proposal; see Basel Committee on Banking Supervision [3] and Financial Stability Oversight Council [22] for recent regulatory updates.

Structural models for valuation of contingent capital include Albul et al. [1], Glasserman and Nouri [26], Hilscher and Raviv [32], Koziol and Lawrenz [35], Madan and Schoutens [41], and Pennacchi [48]. Glasserman and Nouri [26] distinguish a book-value capital ratio from a market-value ratio and value contingent capital that converts progressively, rather than all at once, as a book-value ratio deteriorates; here we model market values only. Madan and Schoutens [41] incorporate bid-ask spreads in a firm's liabilities. Sundaresan and Wang [53] caution against using a stock price trigger for conversion, observing that the stock price itself depends on the timing of conversion; related points are made in a more general context in Bond, Goldstein, and Prescott [7].

This paper is the first to integrate endogenous default, debt rollover, and jumps into the valuation of contingent capital. We see these features as essential to examining the incentive effects of including CoCos: to understand, for example, if CoCos help overcome debt overhang and motivate equity holders to invest in the firm, we need a model in which equity holders exercise their option to abandon the firm optimally; otherwise, apparent incentive effects from CoCos may simply reflect inconsistent modeling of the decisions of equity holders. Moreover, these incentives are strongly influenced by the need to roll debt, and many of our conclusions would be missing in a model with a single maturity date for debt. Our analysis combines these features with a rich capital structure that includes several types of liabilities with a variety of maturity profiles. On the asset side we capture tail risk through two types of jumps, and we combine these features while achieving tractability in our valuations.

Pennacchi [48] highlights the importance of jumps in valuing contingent convertibles and uses a jump-diffusion model of assets, as we do, but his work differs from ours in several respects. He foregoes tractability, instead using simulation for valuation, and incorporates stochastic interest rates in his valuations. All debt in his model shares a fixed maturity, and default is determined exogenously through a mechanism similar to that of Black and Cox [5]. As already noted, endogenous default and the rollover of maturing debt are key features of our analysis. We combine deposits, straight debt, and CoCos, each with its own maturity profile. Pennacchi's [48] simulation model does not include a debt tax shield or bankruptcy costs; these features are important to our conclusions. Our model also adds alternative assumptions about deposit insurance premiums and allows a potential loss at conversion as CoCo investors with a preference for debt are forced to sell unwanted equity shares at a discount. Moreover, our model identifies the phenomenon of "debt-induced collapse" discussed above, which can be observed only in a model with both contingent convertibles and endogenous default. Also, our model distinguishes firm-specific and market-wide jumps; we capture fire-sale effects by imposing a lower recovery rate on assets when default occurs at a market-wide jump.

This paper, like most of those cited above, takes a structural approach to modeling and valuation. Reduced-form credit risk models of the type in Duffie and Singleton [19] and Jarrow and Turnbull [33] could potentially be used for pricing and hedging CoCos, but they are less well-suited to capturing incentive effects. A limitation of many structural models, including ours, is that they do not incorporate asymmetric information between shareholders and creditors. This is partly mitigated by the inclusion of jumps in asset value, which could reflect a sudden release of information, as in Duffie and Lando [18].

We formulate the model in Section 2 and develop our valuation method in Section 3. Section 4 explores the effects of replacing debt or equity with CoCos, with particular focus on the change in the cost of equity capital. Section 5 examines the effect of CoCos on the problem of debt overhang, and Section 6 examines their effect on asset substitution and risk sensitivity. Section 7 explains debt-induced collapse. Section 8 contrasts resolution authority with contingent capital and differentiates recovery rates for defaults at firm-specific jumps and market-wide jumps. Section 9 calibrates our model to data from individual banks through the financial crisis. Technical results are collected in an appendix.

2 The Model

2.1 Firm Asset Value

Much as in Merton [43], Black and Cox [5], Leland [39], Leland and Toft [40], and Goldstein, Ju, and Leland [27], consider a firm generating cash through its investments and operations continuously at rate $\{\delta_t, t \geq 0\}$. The income flow $\{\delta_t\}$ is exposed to firm-specific and market jump risk, with dynamics given by

$$\frac{d\delta_t}{\delta_{t-}} = \tilde{\mu}dt + \tilde{\sigma}d\tilde{W}_t + d\left(\sum_{i=1}^{\tilde{N}_t^f}(\tilde{Y}_i^f - 1)\right) + d\left(\sum_{i=1}^{\tilde{N}_t^m}(\tilde{Y}_i^m - 1)\right). \tag{1}$$

Here, $\tilde{\mu}$ and $\tilde{\sigma}$ are constants, $\{\tilde{W}_t, t \geq 0\}$ is a standard Brownian motion, and we write δ_{t-} to indicate the value just prior to a possible jump at time t. The last two terms reflect two types of jumps, the first we interpret as firm-specific and the second as market-wide. These are driven by Poisson processes $\{\tilde{N}_t^f, t \geq 0\}$ and $\{\tilde{N}_t^m, t \geq 0\}$ with intensities $\tilde{\lambda}_f$ and $\tilde{\lambda}_m$. The jump sizes $\{\tilde{Y}_i^f, i = 1, 2, \ldots\}$ and $\{\tilde{Y}_i^m, j = 1, 2, \ldots\}$, and $\tilde{N}^f$, $\tilde{N}^m$, and $\tilde{W}$ are all independent of each other. Since we are mainly concerned with the impact of downside shocks to the firm's business, we assume that the $\tilde{Y}_i^f$ and $\tilde{Y}_i^m$ are are all less than 1. We can represent the common distribution of the $\tilde{Y}_i^f$ and the common distribution of the $\tilde{Y}_i^m$ by setting $\tilde{Z}_f := -\log(\tilde{Y}^f)$ and $\tilde{Z}_m := -\log(\tilde{Y}^m)$ and positing, for tractability, that these have exponential distributions,

$$f_{\tilde{Z}_f}(z) = \tilde{\eta}_f e^{-\tilde{\eta}_f z} \quad \text{and} \quad f_{\tilde{Z}_m}(z) = \tilde{\eta}_m e^{-\tilde{\eta}_m z}, \quad z \geq 0, \tag{2}$$

for some $\tilde{\eta}_f, \tilde{\eta}_m > 0$. In addition, we assume a constant risk-free interest rate r.

In a rational expectations framework with a representative agent having HARA utility, the equilibrium price of any claim on the future income of the firm can be shown to be the expectation of the discounted payoff of the claim under a "risk-neutral" probability measure $\mathbb{Q}$; see Naik and Lee [46] and Kou [34] for a detailed justification of this assertion. The value of the firm's assets is the present value of the future cash flows they generate,

$$V_t = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(u-t)}\delta_u du \Big| \delta_t\right],$$

for all $t \geq 0$. Following Naik and Lee [46] and Kou [34], we can easily show that $\delta := V_t/\delta_t$ is a constant and V_t evolves as a jump-diffusion process

$$\frac{dV_t}{V_{t-}} = (r-\delta)\,dt + \sigma\,dW_t + d\left(\sum_{i=1}^{N_t^f}(Y_i^f - 1)\right) + d\left(\sum_{j=1}^{N_t^m}(Y_j^m - 1)\right) - (\lambda_m + \lambda_f)\xi\,dt \quad (3)$$

with the parameter $\xi < 0$ given by

$$\xi = \frac{\lambda_f}{\lambda_m + \lambda_f}\cdot\frac{\eta_f}{\eta_f + 1} + \frac{\lambda_m}{\lambda_m + \lambda_f}\cdot\frac{\eta_m}{\eta_m + 1} - 1.$$

Under $\mathbb{Q}$, $\{W_t\}$ in (3) is a standard Brownian motion and $\{N_t^f\}$ and $\{N_t^m\}$ are two Poisson processes with intensities λ_f and λ_m. The distributions of the jump sizes Y_i^f and Y_i^m have the same form as before, but now with parameters η_f and η_m. Kou [34] gives explicit expressions for the parameters in (3) in terms of the parameters in (1). We will value pieces of the firm's capital structure as contingent claims on the asset value process V, taking expectations under $\mathbb{Q}$ and using the dynamics in (3).

2.2 The Capital Structure

The firm finances its assets by issuing four kinds of liabilities: insured deposits, senior and junior debt, contingent capital, and equity. We detail these in order of seniority.

A. Insured Deposits

Insured deposits have no contractual maturity and are subject to withdrawal at any time. We model this by assigning to each deposit a randomly distributed lifetime; for tractability, we take this lifetime to be exponentially distributed (as in Leland and Toft [40]) with a mean of $1/m_1$. More explicitly, the firm issues new accounts with a par value of $p_1 dt$ at every moment $(t, t+dt)$ for all $t \geq 0$. In each subsequent interval $(t+s, t+s+ds)$, $s \geq 0$, a fraction

$$\phi_1(s) = m_1 e^{-m_1 s} ds$$

of the initial deposit $p_1 dt$ is withdrawn.

This specification generates a stationary profile for the firm's insured deposits. At any moment $t > 0$, the total par value of outstanding deposits is given by

$$\int_t^{+\infty}\left(\int_{-\infty}^{t} p_1\phi_1(s-u)du\right) ds = \frac{p_1}{m_1} =: P_1,$$

and this remains constant until the default of the firm. Deposits earn interest at rate c_1, so the total interest paid on deposits in each interval $(t, t+dt)$ is $c_1 P_1 dt$.

At bankruptcy, depositors have the most senior claim on the firm's assets. If these assets are insufficient to repay depositors, government insurance makes up the difference, so depositors are guaranteed repayment at par. Prior to bankruptcy, the firm pays premiums for deposit insurance at rate φ. The insurance premium may be fairly priced — exactly offsetting the expected payout from the insurance fund — but need not be. We also examine the implications of making insurance premiums proportional to all the firm's debt (consistent with rules adopted by the FDIC in 2011), and not just deposits.

B. Senior and Subordinated Debt

In addition to deposits, the firm issues unsecured senior and subordinated debt. The costs and consequences of debt rollover are important to our analysis,[1] so we use the exponential maturity framework for these instruments as well. The firm continuously issues two classes of straight bonds, senior and subordinated, with respective par values $p_2 dt$ and $p_3 dt$ in $(t, t+dt)$ for all $t \geq 0$. The maturities of the newly issued bonds are exponentially distributed; that is, a portion

$$\phi_i(s) = m_i e^{-m_i s} ds,$$

of the total amount $p_i dt$, $i = 2, 3$, matures during the time interval $(t+s, t+s+ds)$, for all $s \geq 0$. As long as the firm is not in default, the par values of the outstanding senior and subordinated debt remains constant at levels $P_i := p_i/m_i$, $i = 2, 3$. In effect, we assume the firm manages its debt issuance to target a fixed maturity profile and fixed levels of various types of debt.

Senior and subordinated debt pay coupons at rates c_i, $i = 2, 3$, respectively. The total coupon payment on these bonds is then $(c_2 P_2 + c_3 P_3)dt$ in each interval $(t, t+dt)$, up to the default of the firm.

Upon default, we assume that a fraction $(1-\alpha)$, $0 \leq \alpha \leq 1$, of the firm's asset value is lost to bankruptcy and liquidation costs. If we let V denote asset value at the moment of bankruptcy and thus αV the value just after bankruptcy, then repaying depositors leaves $(\alpha V - P_1)^+$. Senior bond holders are repaid to the extent that the remaining funds suffice, so they get get $P_2 \wedge (\alpha V - P_1)^+$, and the junior bond holders similarly get $P_3 \wedge (\alpha V - P_1 - P_2)^+$.

[1] Short-term debt addresses problems of asymmetric information and monitoring, as discussed in Calomiris and Kahn [13], Diamond and Rajan [16], and Gorton and Pennacchi [29]. But discussions of the financial crisis, including Brunnermeier [8], Krishnamurthy [36], and Shin [51], have highlighted the role of short-term financing and the resulting rollover risk (e.g., He and Xiong [31]).

This discussion presupposes recovery at par value, in the sense that the bond holders have a claim of P_i, $i = 2, 3$, in bankruptcy, as is the case in practice. Alternative modeling assumptions used in the literature include recovery at market value or at a fraction of an otherwise equivalent Treasury note; see Duffie and Singleton [19], Jarrow and Turnbull [33], and Lando [38]. The differences in these conventions are relatively minor and would not change our conclusions qualitatively.

C. Contingent Convertibles

We use the same basic framework to model the issuance and maturity of CoCos as we use for other types of debt. In all cases, we would retain tractability if we replaced the assumption of exponential maturity profile with consols, but, as already noted, debt rollover is an important part of our analysis, so we use finite-maturity debt. We denote by P_4 the par value of CoCos outstanding, which remains constant prior to conversion or default and pays a continuous coupon at rate c_4. The mean maturity is $1/m_4$, and new debt is issued at rate p_4.

Conversion of CoCos from debt to equity is triggered when the value of the firm's assets fall below an exogenously specified threshold V_c. Thus, conversion occurs at

$$\tau_c = \inf\{t \geq 0 : V_t \leq V_c\},$$

and we assume the trigger V_c is lower than the initial asset level V_0. (Because earnings δV_t are proportional to asset value, the trigger could equivalently be based on earnings, as posited in Koziol and Lawrenz [35].) At the instant of conversion, the CoCo liability is erased and CoCo investors receive Δ shares of the firm's equity for every dollar of principal, for a total of ΔP_4 shares. We normalize the number of shares to 1 prior to conversion. Thus, following conversion, the CoCo investors own a fraction $\Delta P_4/(1 + \Delta P_4)$ of the firm. In the bail-in case, $\Delta = \infty$, so the original shareholders are wiped out and the converted investors take control of the firm. We think of the parameters (V_c, Δ) as part of the contractual terms of the convertible debt and examine the consequences of varying these parameters.[2]

2.3 Endogenous Default

The firm has two types of cash inflows and three types of cash outflows. The inflows are the income stream $\delta_t dt = \delta V_t dt$ and the proceeds from new bond issuance $b_t dt$, where b_t is the total market value of bonds issued at time t. The cash outflows are the after-tax coupon payments,

[2]We do not distinguish between contractual and statutory conversion. Under the former, conversion is an explicit contractual feature of the debt. The statutory case refers to conversion imposed on otherwise standard debt at the discretion of a regulator granted explicit legal authority to force such a conversion.

the principal due $(p_1 + p_2 + p_3 + p_4)dt$ on maturing debt, and insurance premiums $\varphi P_1 dt$ or, more generally, $\varphi P dt$ for some assessment base P. The firm has a marginal tax rate of κ, and we assume that interest payments on deposits and straight debt are tax-deductible. Thus, the after-tax coupon payment rate is given by $A_t = (1 - \kappa)(c_1 P_1 + c_2 P_2 + c_3 P_3 + c_4 P_4)$ or $A_t = (1 - \kappa)(c_1 P_1 + c_2 P_2 + c_3 P_3) + c_4 P_4$, depending on whether or nor coupon payments on CoCos are also tax deductible.

Let $\bar{p}$ denote the total rate of issuance (and retirement) of par value of debt, just as b_t denotes the total rate of issuance measured at market value. We have $\bar{p} = p_1 + p_2 + p_3 + p_4$ prior to conversion of any CoCos and $\bar{p} = p_1 + p_2 + p_3$ after conversion. Whenever

$$b_t + \delta V_t > A_t + \bar{p} + \varphi P_1 \tag{4}$$

the firm has a net inflow of cash, which is distributed to equity holders as a dividend flow. When the inequality is reversed, the firm faces a cash shortfall. The equity holders then face a choice between making further investments in the firm — in which case they invest just enough to make up the shortfall — or abandoning the firm and declaring bankruptcy. Bankruptcy then occurs at

$$\tau_b = \inf\{t \geq 0 : V_t \leq V_b^*\},$$

the first time the asset level is at or below V_b^*, with V_b^* chosen optimally by the equity holders. In fact, it would be more accurate to say that V_b^* is determined simultaneously with b_t, because the market value of debt depends on the timing of default, just as the firm's ability to raise cash through new debt influences the timing of default.

In Section 3 and the appendix, we derive explicit expressions for all the firm's liabilities, including the (before-conversion) equity value $E^{\mathsf{BC}}(V; V_b)$ as a function of the current asset value V and the default barrier V_b. In determining the optimal threshold at which to default, the equity holders seek to maximize the value of equity. They solve

$$\max_{V_b} E^{\mathsf{BC}}(V; V_b)$$

subject to the limited liability constraint

$$E^{\mathsf{BC}}(V'; V_b) \geq 0, \quad \text{for all } V' \geq V_b.$$

If the solution V_b^* to this problem is below the conversion trigger V_c, then the same default barrier solves the corresponding problem for the post-conversion equity value $E^{\mathsf{PC}}(V; V_b)$:

$$\max_{V_b} E^{\mathsf{PC}}(V; V_b)$$

subject to the limited liability constraint

$$E^{\mathsf{PC}}(V'; V_b) \geq 0, \quad \text{for all } V' \geq V_b.$$

3 Valuing the Firm's Liabilities

We value the firm's liabilities by discounting their cashflows and taking expectations under the risk neutral probability $\mathbb{Q}$.

A. Insured Deposits

In the presence of deposit insurance, repayment is guaranteed, but the timing of repayment may be accelerated by the default of the firm. To value a unit of deposit at time t, to be held on deposit until time $t+T$, we discount the interest earned over the interval $[t, (t+T) \wedge \tau_b]$ and the principal received at $(t+T) \wedge \tau_b$ to get a market value of

$$\begin{aligned} b_1(V_t; T) &= \mathbb{E}^{\mathbb{Q}} \left[\int_t^{(t+T)\wedge\tau_b} c_1 e^{-r(u-t)} du + e^{-r(T\wedge(\tau_b - t))} \middle| V_t \right] \\ &= \frac{c_1}{r} + \left(1 - \frac{c_1}{r}\right) \mathbb{E}^{\mathbb{Q}} \left[e^{-r(T\wedge(\tau_b-t))} \mid V_t \right]. \end{aligned}$$

To simplify notation, we will henceforth take $t = 0$ and omit the conditional expectation given V_t, though it should be understood that the value of each liability is a function of the current value V of the firm's assets.

Recall that we take the deposit lifetimes T to be exponentially distributed with density $m_1 \exp(-m_1 T)$, and the total amount in deposits is P_1; the total market value of deposits then evaluates to

$$\begin{aligned} B_1(V) &= P_1 \int_0^\infty b_1(V; T) m_1 e^{-m_1 T} dT \\ &= \frac{c_1}{r} P_1 + m_1 P_1 \left(1 - \frac{c_1}{r}\right) \left(\frac{1}{m_1 + r} + \left(\frac{1}{m_1} - \frac{1}{m_1 + r} \right) \mathbb{E}^{\mathbb{Q}} \left[e^{-(m_1+r)\tau_b} \right] \right). \end{aligned}$$

The key to the valuation is thus the transform of the default time τ_b, which is given explicitly by Cai and Kou [10] and Cai, Chen, and Wan [9].

The asset value remaining just after bankruptcy is αV_{τ_b}, and if this amount is less than the total deposits P_1, the difference is covered by deposit insurance. The market value of this guarantee is therefore given by

$$\mathbb{E}^{\mathbb{Q}} \left[e^{-r\tau_b} (P_1 - \alpha V_{\tau_b}) \mathbf{1}_{\{\alpha V_{\tau_b} < P_1\}} \right].$$

The firm pays a premium for deposit insurance at rate φ. If this rate is charged on the deposit base P_1, the cost of insurance is

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_b} \varphi P_1 e^{-rs} ds\right].$$

We also consider an alternative in which the premium is applied to all of the firm's debt, for which we replace P_1 with $P_1 + P_2 + P_3 + P_4$.

B. Senior and Subordinated Debt

We follow a similar approach to valuing straight debt, first considering a bond with a face value of 1 and a time-to-maturity T. The value of a senior bond with these terms is as follows:

$$\begin{aligned}
b_2(V;T) \;=\; & \mathbb{E}^{\mathbb{Q}}\left[e^{-rT}\mathbf{1}_{\{\tau_b > T\}}\right] \quad \text{(principal payment if no default)} \\
& + \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\mathbf{1}_{\{\tau_b \le T\}}\mathbf{1}_{\{\alpha V_{\tau_b} \ge P_1+P_2\}}\right] \quad \text{(payment at default, full recovery)} \\
& + \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\mathbf{1}_{\{\tau_b \le T\}}\frac{\alpha V_{\tau_b} - P_1}{P_2}\mathbf{1}_{\{P_1 \le \alpha V_{\tau_b} < P_1+P_2\}}\right] \quad \text{(partial recovery)} \\
& + \mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_b \wedge T} c_2 e^{-r(u-t)} du\right] \quad \text{(coupon payments)}
\end{aligned} \tag{5}$$

In this expression, $\{\tau_b \le T\}$ is the event that default occurs before the bond matures, and αV_{τ_b} gives the value of the firm's assets just after default. If αV_{τ_b} exceeds $P_1 + P_2$, the senior bonds are repaid in full; if $P_1 \le \alpha V_{\tau_b} < P_1 + P_2$, then each dollar of face value of senior debt recovers $(V_{\tau_b} - P_1)/P_2$.

Proceeding as we did for the value of the deposits, we calculate the total market value of senior debt to be

$$\begin{aligned}
B_2(V) &= P_2 \int_0^{\infty} b_2(V;T) e^{-m_2 T} dT \\
&= P_2 \left[\left(1 - \frac{c_2}{r}\right) \cdot \frac{m_2}{m_2 + r} + \frac{c_2}{r}\right] \cdot \mathbb{E}^{\mathbb{Q}}\left[1 - e^{-(m_2+r)(\tau_b - t)}\right] \\
&+ P_2 \mathbb{E}^{\mathbb{Q}}\left[e^{-(m_2+r)(\tau_b - t)}\left(\mathbf{1}_{\{\alpha V_{\tau_b} \ge P_1+P_2\}} + \frac{\alpha V_{\tau_b} - P_1}{P_2}\mathbf{1}_{\{P_1 \le \alpha V_{\tau_b} < P_1+P_2\}}\right)\right].
\end{aligned} \tag{6}$$

The expectation in (6) is evaluated in the appendix.

By exactly the same argument, the total market value of the subordinated debt is

$$\begin{aligned}
B_3(V) &= P_3 \left[\left(1 - \frac{c_3}{r}\right) \cdot \frac{m_3}{m_3 + r} + \frac{c_3}{r}\right] \cdot \mathbb{E}^{\mathbb{Q}}\left[1 - e^{-(m_3+r)\tau_b}\right] \\
&+ P_3 \mathbb{E}^{\mathbb{Q}}\left[e^{-(m_3+r)\tau_b}\left(\mathbf{1}_{\{\alpha V_{\tau_b} \ge P_1+P_2+P_3\}} + \frac{\alpha V_{\tau_b} - P_1 - P_2}{P_3}\mathbf{1}_{\{P_1+P_2 \le \alpha V_{\tau_b} < P_1+P_2+P_3\}}\right)\right].
\end{aligned}$$

C. Contingent Convertibles

The market value of a CoCo combines the value of its coupons, its principal, and its potential conversion to equity.[3] Fix a default barrier V_b, and suppose that $V_c > V_b$, so that bankruptcy cannot occur prior to conversion. A CoCo with maturity T and unit face value has market value

$$b_4(V;T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-rT}\mathbf{1}_{\{\tau_c>T\}}\right] + \mathbb{E}^{\mathbb{Q}}\left[\int_0^{T\wedge\tau_c} c_4 e^{-rs}ds\right] + \frac{\Delta}{1+\Delta P_4}\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_c}E^{\mathsf{PC}}(V_{\tau_c})\mathbf{1}_{\{\tau_c<T\}}\right],$$

where $E^{\mathsf{PC}}(V)$ is the post-conversion value of equity at an asset value of V. At conversion, the CoCo investors collectively receive ΔP_4 shares of equity for each share outstanding, giving them a fraction $\Delta P_4/(1+\Delta P_4)$ of the firm, and dividing this by P_4 yields the amount that goes to a CoCo with a face value of 1. The total market value of CoCos outstanding is then

$$\begin{aligned} B_4(V) &= P_4\int_0^{+\infty} b_4(V;T)\, m_4 e^{-m_4 T}dT \\ &= P_4\left[\left(1-\frac{c_4}{r}\right)\frac{m_4}{m_4+r}+\frac{c_4}{r}\right]\left(1-\mathbb{E}\left[e^{-(r+m_4)\tau_c}\right]\right)+\frac{\Delta P_4}{1+\Delta P_4}\mathbb{E}\left[e^{-(r+m_4)\tau_c}E^{\mathsf{PC}}(V_{\tau_c})\right]. \end{aligned}$$

If $V_c \le V_b$, so that conversion does not occur prior to bankruptcy, a similar calculation yields

$$\begin{aligned} B_4(V) = {} & P_4\left[\left(1-\frac{c_4}{r}\right)\frac{m_4}{m_4+r}+\frac{c_4}{r}\right]\left(1-\mathbb{E}\left[e^{-(r+m_4)\tau_b}\right]\right)+P_4\mathbb{E}\left[e^{-(r+m_4)\tau_b}\right] \\ & +\mathbb{E}^{\mathbb{Q}}\left[e^{-(r+m_4)\tau_b}\left((\alpha V_{\tau_b}-P_1-P_2-P_3)\,\mathbf{1}_{\{P_1+P_2+P_3\le\alpha V_{\tau_b}<P_1+P_2+P_3+P_4\}}\right.\right. \\ & \left.\left.+P_4\mathbf{1}_{\{\alpha V_{\tau_b}\ge P_1+P_2+P_3+P_4\}}\right)\right] \end{aligned}$$

It remains to calculate $E^{\mathsf{PC}}(V)$, the post-conversion equity value with a default barrier of V_b. We derive this value by calculating total firm value and subtracting the value of debt. After conversion, total firm value is given by

$$\begin{aligned} F^{\mathsf{PC}}(V) = {} & V + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_b}\kappa(c_1P_1+c_2P_2+c_3P_3)e^{-rs}ds\right]}_{\text{tax benefits}} \qquad (7) \\ & + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(P_1-\alpha V_{\tau_b})\mathbf{1}_{\{\alpha V_{\tau_b}\le P_1\}}\right]}_{\text{deposit insurance}} - \underbrace{\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right]}_{\text{bankruptcy costs}} - \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_b}\varphi P_1 e^{-rs}ds\right]}_{\text{insurance premiums}} \\ = {} & V + \left[(c_1P_1+c_2P_2+c_3P_3)\frac{\kappa}{r}-P_1\frac{\varphi}{r}\right]\left(1-\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right) \\ & +\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(P_1-\alpha V_{\tau_b})\mathbf{1}_{\{\alpha V_{\tau_b}\le P_1\}}\right]-\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right] \qquad (8) \end{aligned}$$

[3] In applying the same pricing measure with and without CoCos, we are implicitly assuming that the impact of CoCos is not so great as to change the market's stochastic discount factor.

The post-conversion equity value (for a given barrier V_b) is then given by

$$E^{\mathsf{PC}}(V) \equiv E^{\mathsf{PC}}(V; V_b) = F^{\mathsf{PC}}(V) - B_1(V) - B_2(V) - B_3(V),$$

the value that remains after subtracting deposits and senior and subordinated debt from total firm value. The expression we need to value the CoCos,

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_c} E^{\mathsf{PC}}(V_{\tau_c})\mathbf{1}_{\{\tau_c<T\}}\right], \tag{9}$$

is evaluated using the method in the appendix.

D. Equity Value Before Conversion

Just as we did for the post-conversion value of equity, we calculate the value before conversion by first calculating total firm value before conversion, $F^{\mathsf{BC}}(V)$, given a default barrier V_b. To the expression above for the post-conversion value, we need to add the tax shield on CoCos,

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_c\wedge\tau_b} \kappa c_4 e^{-ru} du\right],$$

if the CoCo coupons are tax-deductible. Thus,

$$F^{\mathrm{BC}}(V) = F^{\mathrm{PC}}(V) + \begin{cases} c_4 P_4 \dfrac{\kappa}{r}\left(1 - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_b}\right]\right), & \text{if } V_c \le V_b; \\ c_4 P_4 \dfrac{\kappa}{r}\left(1 - \mathbb{E}^{\mathbb{Q}}\left[e^{-r\tau_c}\right]\right), & \text{if } V_c > V_b. \end{cases}$$

If insurance premiums are charged in proportion to all debt, and not just deposits, then conversion of CoCos also reduces premium payments, and we need to add

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_c\wedge\tau_b} \varphi P_4 e^{-rs} ds\right]$$

to $F^{\mathsf{PC}}(V)$ to get $F^{\mathsf{BC}}(V)$. The market value of the firm's equity before conversion is then given by

$$E^{\mathsf{BC}}(V) = F^{\mathsf{BC}}(V) - B_1(V) - B_2(V) - B_3(V) - B_4(V). \tag{10}$$

Both $F^{\mathsf{BC}}(V)$ and $E^{\mathsf{BC}}(V)$ admit closed-form expressions.

We thus have explicit expressions for the market values of all pieces of the firm's capital structure, for a given default barrier V_b. To complete the valuation, we endogenize default, letting equity holders choose V_b optimally, as discussed in Section 2.3.

To solve this type of problem in a pure-diffusion model without CoCos, Leland [39] and Leland and Toft [40] use a smooth-pasting principle. In our setting, this becomes

$$\frac{\partial E^{\mathsf{BC}}(V)}{\partial V}\Big|_{V=V_b^*} = 0, \tag{11}$$

with the understanding that $E^{\mathsf{BC}}(V) = E^{\mathsf{PC}}(V)$ if $V \le V_c$. We take the solution to (11) to be the optimal barrier V_b^*. Chen and Kou [14] justify this approach in a jump-diffusion model without CoCos (see also Kyprianou and Surya [37]), and their argument carries over to our setting as well.[4] This calculation has to be done numerically, so we do not have explicit expressions for V_b^* or for the value of equity using V_b^*.

4 Changes in Capital Structure

We can use the valuation results of the previous section to investigate the effects of changes in the firm's liability structure. We focus in particular on the perspective of the equity holders and the impact of issuing CoCos.

For our base case, we use the parameters in Table 1. The firm initially funds 100 in assets with par values of 40 in deposits, 30 in senior debt, 15 in subordinated debt, and 15 in equity or a combination of equity and CoCos. The proceeds of issuing new debt may be used to scale up the firm's assets, to pay down another form of debt, or to buy back equity. We consider all three cases. Under any change in capital structure, we recompute the optimal default barrier and recompute the value of the firm and its liabilities.

The process of rolling debt is important to our analysis, so we briefly describe how this works. Under our exponential maturity assumption, old debt is continuously maturing and new debt is continuously issued. Within each debt category, the coupon and the total par value outstanding remain constant; but while debt matures at par value, it is issued at *market* value. If the par value is greater, the difference is a cash shortfall that needs to be paid out by the firm; if the market value is greater, the difference generates additional cash for the firm. We refer to these as rollover costs — a negative cost in the first case, a positive cost in the second — and treat them the same way we treat coupon payments. Rollover costs will change as the firm's asset value changes, becoming more negative as asset value declines, the firm gets closer to default, and the market value of its debt decreases. Rollover costs thus capture the increased yield demanded of riskier firms.

[4] Décamps and Villeneuve [15] show that the situation is more complex, and the resulting equilibrium unknown in general, when equity holders may deviate from a stated default rule and creditors anticipate this possibility. The added generality would be needed to incorporate strategic behavior on the part of creditors and the possibility that creditors might force the firm into bankruptcy.

Parameter		Value
initial asset value	V_0	100
debt principal	(P_1, P_2, P_3)	$(40, 30, 15)$
risk free rate	r	6%
volatility	σ	8%
payout rate	δ	1%
tax rate	κ	35%
firm specific jump intensity	λ_f	.2
market jump intensity	λ_m	.05
firm specific jump exponent	η_f	4
market jump exponent	η_m	3
coupon rates	(c_1, c_2, c_3, c_4)	$(r, r + 3\%, r + 3\%, r)$
deposits insurance premium rate	φ	1%
contingent capital principal	P_4	1 or 5
maturity profile exponent – base case	(m_1, m_2, m_3, m_4)	$(1, 1/4, 1/4, 1/4)$
maturity profile exponent – long maturity	(m_1, m_2, m_3, m_4)	$(1, 1/16, 1/16, 1/16)$
		or $(1, 1/25, 1/25, 1/25)$
conversion trigger	V_c	75 (in most cases)
conversion loss (if applied)		20% of value of shares

Table 1: Base case parameters. Asset returns have a total volatility (combining jumps and diffusion) of 20.6 percent and overall drift rate of 3.3 percent. In the base case, the number of shares Δ issued at conversion is set such that if conversion happens at exactly V_c, the market value of shares delivered is the same as the face value of the converted debt.

4.1 Replacing Straight Debt with CoCos

We begin by replacing some straight debt — either senior or subordinated — with CoCos. We assume that the conversion trigger V_c is higher than the bankruptcy barrier V_b^*, so bankruptcy cannot precede conversion. This holds in our base case. As noted in Section 1, so long as the trigger is above the barrier, it has no effect on the barrier; in other words, $\partial V_b^* / \partial V_c = 0$ whenever $V_b^* > V_c$.

We can walk through the consequences of the substitution as follows.

- If coupon payments on CoCos are not tax deductible, then replacing straight debt with CoCos has the immediate effect of reducing firm value by reducing the value of the tax shield. Even if CoCo coupons are tax deductible, this benefit ends at conversion, so, other things being equal, the substitution still has the immediate impact of reducing firm value; see (7)–(8). The reduction in firm value has the direct effect of lowering the value of equity.

- However after conversion the firm will have less debt outstanding and lower debt service payments (coupons and rollover costs) than it would without the substitution of CoCos for straight debt. With lower debt service, more of the cash $\delta V_t\, dt$ generated by the firm's assets flows to equity holders in dividends. This reduces the default barrier V_b^*, which extends the life of the firm, reduces the bankruptcy cost $\mathbb{E}\left[e^{-r\tau_b}(1-\alpha)V_{\tau_b}\right]$, and thus increases firm value in (7)–(8).

- We thus have two opposite effects on firm value: the reduced tax shield from CoCos reduces firm value, but the reduced default probability and bankruptcy cost increases firm value. In our numerical examples, we find that the second effect dominates over a wide range of parameter values, so that the net effect of replacing straight debt with CoCos is to increase firm value.

- Part of this increase in firm value is captured by senior and subordinated debt holders because the reduced bankruptcy risk increases the value of the debt; see (6), for example. Part of the increase is also captured by equity holders: the increased debt value reduces rollover costs which increases the flow of dividends. Thus, *equity holders have a positive incentive to issue CoCos.*

This conclusion contrasts with that of Albul et al. [1], who find that equity holders would never voluntarily replace straight debt with contingent convertibles. In their model, straight debt has infinite maturity and is never rolled. As a result, all of the benefit of reduced

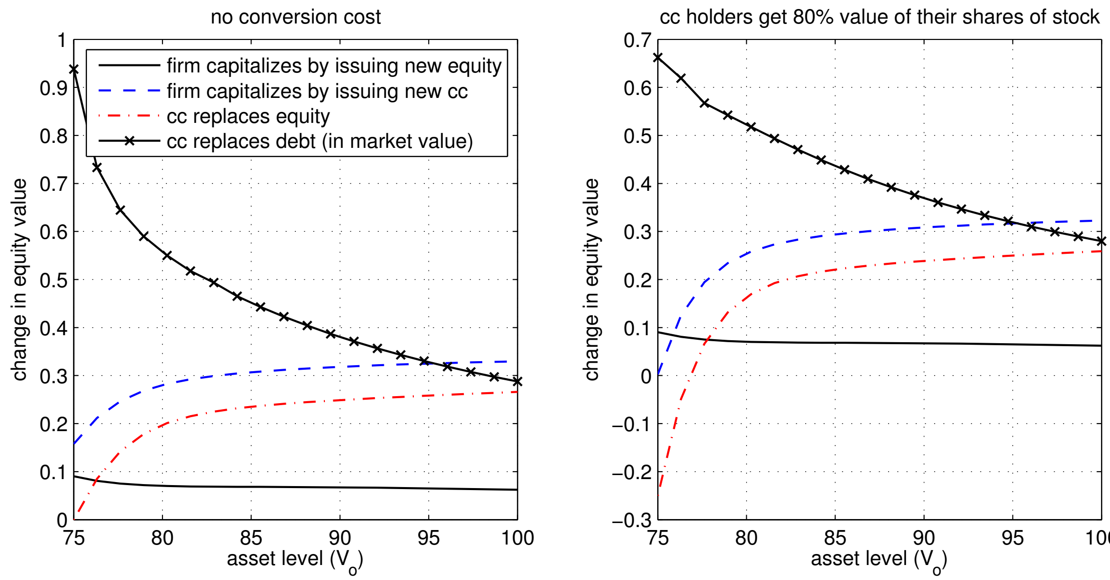

Figure 1: Change in equity value resulting from various changes in capital structure. In the right hand figure the CoCo holders dump their shares in the market following the conversion and as a result lose 20 percent of the value of their shares due to price impact and transaction fees.

bankruptcy costs from CoCos is captured by debt holders, indeed leaving no incentive for equity holders. This difference highlights the importance of debt rollover in influencing incentives for equity holders, an effect we return to at several points.

The line marked with crosses in the left panel of Figure 1 shows the increase in equity value resulting from a substitution of one unit (market value) of CoCos for one unit (market value) of straight debt, plotted against the value of the firm's asset value. The conversion level V_c is 75. Despite the dilutive effect of conversion, the benefit to equity holders of the substitution is greatest just above the conversion level and decreases as asset level increases. This follows from the fact that the benefit to equity holders derives from the reduction in bankruptcy costs, which is greater at lower asset values. We will discuss the other curves in the left panel shortly.

The right panel of Figure 1 incorporates a friction in the conversion of debt to equity. To this point, we have valued each security as the expected present value of its cash flows. In practice, the markets for debt and equity are segmented, and some bond investors may be unwilling (or unable under an investment mandate) to own equity. Such investors would value CoCos at less than their present value, and this effect could well move the price at which the market clears, given the comparatively small pool of investors focused on hybrid securities.

To capture this effect, we suppose that the equity received by CoCo investors at conversion is valued at 80 percent of market value. For example, we can think of CoCo investors as dumping their shares at a discount, with the discount reflecting a market impact that is only temporary and therefore does not affect the original equity holders. CoCo investors anticipate that they will not receive the full value of equity at conversion and thus discount the price of CoCos up front. This makes CoCos more expensive for the firm as a source of funding. The line marked with crosses in the right panel shows the benefit to equity holders of the same substitution examined in the left panel. As one would expect, the benefit is substantially reduced near the conversion trigger of 75 (comparing the two panels); at higher asset values, the difference between the cases vanishes, with the crossed lines in both panels near 0.3 at an asset level of 100. This discussion can be summarized as follows: *Segmentation between debt and equity investors that creates a friction in conversion reduces the benefit of issuing CoCos; this effect is partially offset by lowering the conversion trigger.*

4.2 Increasing the Balance Sheet with CoCos

We now consider the effects of issuing CoCos without an offsetting reduction in any other liabilities. The proceeds from issuing CoCos are used to scale up the firm's investments. The consequences of this change are as follows:

- Because the post-conversion debt outstanding is unchanged, the endogenous default barrier V_b^* is unchanged, so long as the optimal barrier is below the conversion trigger V_c.
- In this case, the risk of default decreases because an increase in assets moves the firm farther from the default barrier. The reduction in bankruptcy costs increases firm value and the value of straight debt. The additional tax shield from issuing CoCos (assuming their coupons are tax-deductible) further increases firm value.
- Shareholders benefit from the increase in firm value combined with the decrease in rollover costs for straight debt and the increase in cash generated from the larger asset base. These benefits work in the opposite direction of the increase in coupon payments required for the new CoCos.
- With a sufficiently large CoCo issue, it becomes optimal for equity holders to default prior to conversion. At this point, the firm moves, in effect, from one equilibrium to another, and the increased default risk works against the increase in firm, debt, and equity value. We return to this in Section 7.

The dashed line in each panel of Figure 1 shows the benefit to shareholders of issuing a unit of new CoCos with $V_b^* < V_c$. The benefit is lower on the right in the presence of a conversion friction. Whereas the incentive for debt substitution decreases with asset value, the incentive for issuing new CoCos increases with asset value.

4.3 Replacing Equity with CoCos

We include this case for completeness. The firm issues CoCos and the proceeds are used to buy back equity; the cash from the buy-back is part of the benefit received by equity holders. If the conversion trigger is above the default barrier, the post-conversion firm is unaffected because the CoCos that replaced the equity convert to equity; thus, the timing of default is unchanged. The value of straight debt is therefore unaffected by the replacement, and there are no changes in rollover costs of straight debt. If the coupon on CoCos is tax deductible, this benefit from the newly issued CoCos is entirely captured by the equity holders. The total benefit to equity holders (the buy-back cash and the tax shield) is illustrated by the dash-dot line in each panel of Figure 1. In the right panel, the benefit is negative at asset levels near the conversion trigger because the conversion friction increases the cost of issuing CoCos and decreases the cash received in the buy back.

Finally, the solid line in the figures shows the net benefit to shareholders of a unit increase in their equity investment. The benefit is positive because the additional equity reduces default risk and thus lowers rollover costs, a phenomenon not observed in a static model of capital structure. Thus, the negative effect of debt overhang does not overwhelm the potential value of additional investment. We further develop this point in Section 5.

4.4 The Bail-In Case

We model bail-in debt by taking $\Delta = \infty$, meaning that the original shareholders are infinitely diluted and thus wiped out. Also, there is no exogenous trigger level V_c in the bail-in case; instead, conversion occurs at the moment that the original equity holders declare bankruptcy. The convertible debt converts to equity, the converted investors become the sole shareholders, and they then determine a new default barrier endogenously. We assume that the bail-in avoids all bankruptcy costs, but the key assumption is that the asset recovery rate at bail-in is greater than the recovery rate α at default.

Figure 2 illustrates the same comparisons made in the left panel of Figure 1. The main observation is that the incentive to issue convertible debt is greater in Figure 2 than in Figure 1. This is primarily due to the lowering of the conversion threshold — the trigger is 75 in Figure 1 whereas the bail-in point is a bit below 70 in Figure 2. As long as conversion occurs before bankruptcy, the level of the conversion threshold has no effect on firm value or the value of straight debt. It does affect how value is apportioned between equity holders and investors in the convertible debt.

5 Debt Overhang and Investment Incentives

In most capital structure models, equity holders are least motivated to invest in a firm precisely when the firm most needs additional equity. For a firm near bankruptcy, much of the value of an additional equity investment is captured by debt holders as the additional equity increases the market value of the debt by reducing the chances of bankruptcy. This is a problem of debt overhang (Myers [45]), and it presents a significant obstacle to recapitalizing ailing banks. Duffie [17] has proposed mandatory rights offerings as a mechanism to compel investment. Here we examine the effect of CoCos on investment incentives.

The phenomenon of debt overhang is easiest to see in a static model, viewing equity as a call option on the assets of a firm with a strike price equal to the face value of debt, as in Merton [43]. At a low asset value, where the option is deep out-of-the-money, the option delta is close to zero: a unit increase in asset value produces much less than a unit increase in option value,

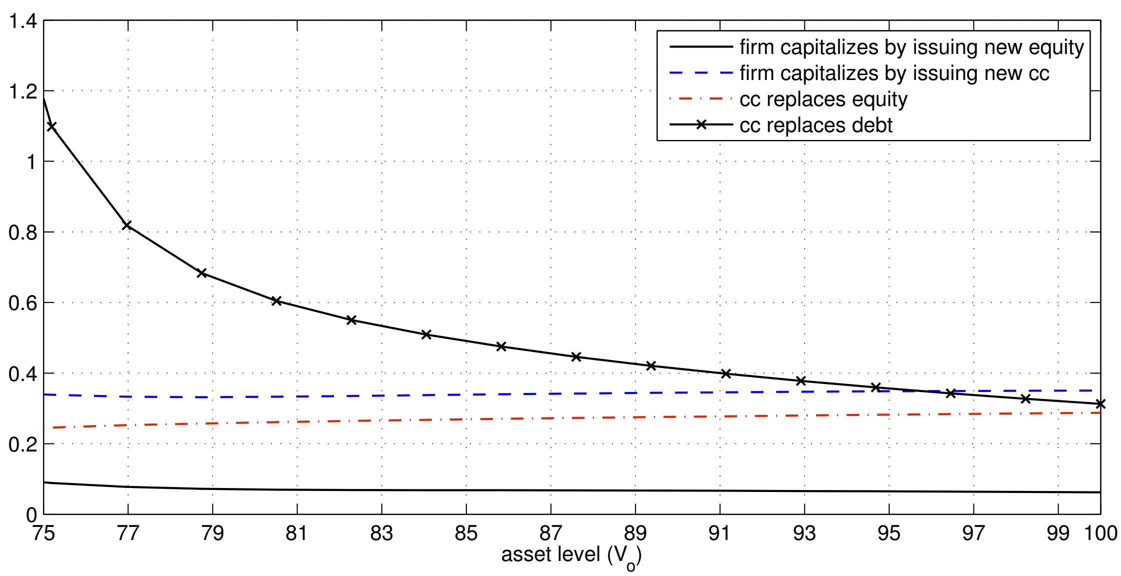

Figure 2: Change in equity value resulting from various changes in capital structure with bail-in debt.

so equity holders have no incentive to invest. Indeed, in this static model, the net benefit of investment is always negative.

At least three features distinguish our setting from the simple static model. First, the reduction in rollover costs that follows from safer debt means that equity holders have the potential to derive some benefit from an increase in their investment. Second, the dilutive effects of CoCo conversion creates an incentive for shareholders to invest to prevent conversion. Third, if CoCo coupons are tax deductible, shareholders have an added incentive to invest in the firm near the conversion trigger to avoid the loss of this tax benefit.

Figure 3 shows the cost to equity holders of an additional investment of 1 in various scenarios. Negative costs are benefits. For this example, we use the longer maturities for debt in Table 1, as the overhang problem is more acute in this case. This is illustrated by the solid black line in the left panel, which shows the overhang cost is positive throughout the range of asset values displayed.

The solid blue line and the dashed line show the overhang cost after the firm has issued CoCos. The blue line corresponds to replacing equity with CoCos, and the dashed line corresponds to replacing straight debt with CoCos. As we move from right to left, tracing a decline in asset value toward the conversion threshold $V_c = 75$, we see a dramatic increase in the benefit (negative cost) to equity holders of an additional investment. In other words, *the presence of CoCos creates a strong incentive for equity holders to invest in the firm to avoid conversion.* After conversion (below an asset level of 75), the overhang cost reverts to its level in a firm without CoCos.

The right panel of Figure 3 provides further insight into the investment incentive illustrated in the left panel. If we lower the conversion trigger from 75 to 70, we see from the solid black line that the investment incentive becomes greatest at 70, as expected, where it is a bit greater than the greatest value in the left figure. Removing the tax-deductibility of CoCo coupons yields the dashed black line, which shows that the investment incentive is reduced but not eliminated. In the solid red line, we have returned the conversion trigger to 75 but removed the jumps from the asset process. This eliminates close to half the incentive for investment, compared to the left panel. Removing both the tax shield on CoCos and jumps in asset value eliminates almost all the investment incentive, as indicated by the dashed red line.

The tax effect is immediate: the tax shield increases the value to shareholders of avoiding the conversion of CoCos and thus creates a greater incentive for investment. The jump effect requires some explanation. Recall that the conversion ratio Δ is set so that the market value of the shares into which the CoCos convert equals the face value of the converted debt *if conversion*

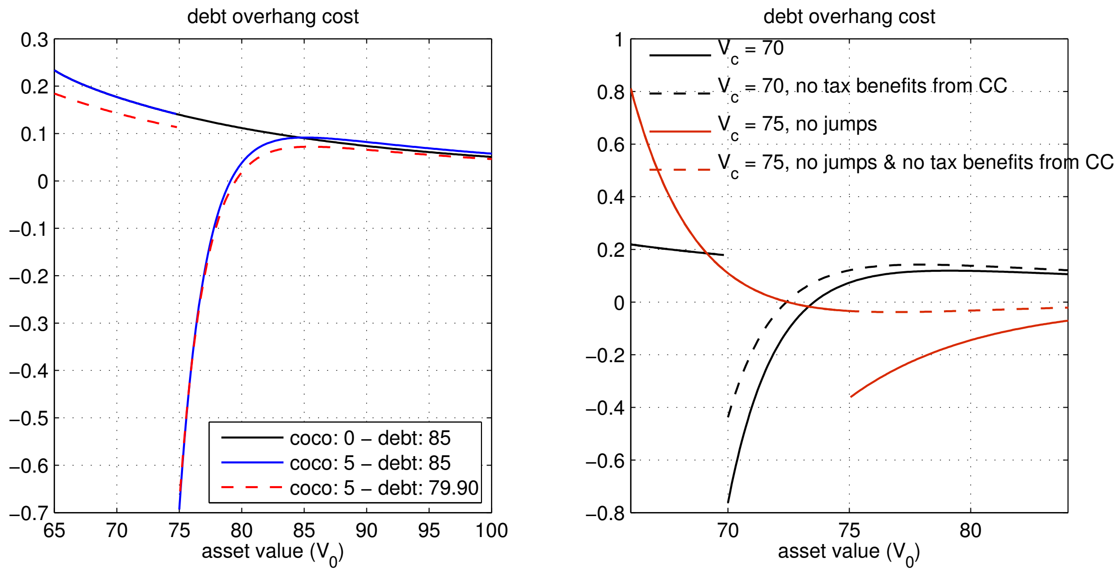

Figure 3: Net cost to shareholders of increasing the firm's asset by 1. Negative costs are gains. The figures show that CoCos and tail risk create a strong incentive for additional investment by equity holders near the conversion trigger.

occurs at an asset level of V_c. If a downward jump takes V_t from a level above the trigger V_c to a level below it, then conversion occurs at an asset level lower than V_c, and the market value of the equity granted to CoCo investors is less than the face value of the debt. Equity holders thus prefer conversion following a jump to conversion at the trigger; indeed, conversion right at the trigger is the worst conversion outcome for equity holders, and this creates an incentive for investment as asset value approaches the trigger. The equity holders would prefer to delay conversion and, in effect, bet on converting at a jump rather than right at the trigger. This suggests that CoCos may create an incentive for equity investors to take on further tail risk, an issue we investigate in the next section.

We close this section by examining the value of equity and CoCos across the conversion trigger. In Figure 4, we fix the conversion ratio Δ at the "fair" value for a CoCo size of 5 and a conversion trigger of $V_c = 80$; with $\Delta = .108$, the market value of the 5Δ shares issued to the CoCo investors equals the par value of 5 if the asset value V equals V_c. We keep Δ fixed at this level and vary V_c so that the value received at the trigger is either greater ($V_c = 85$) or smaller ($V_c = 75$).

The figures indicate that there is no instantaneous value transfer across the conversion trigger, a property introduced by Sundaresan and Wang [53]. In other words, the values are

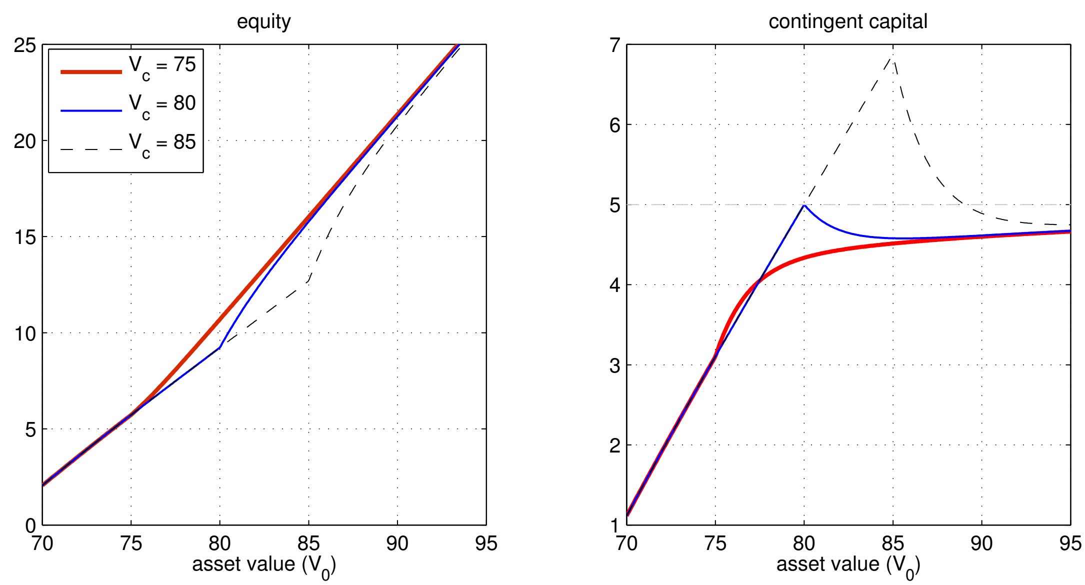

Figure 4: Equity and CoCo values are continuous functions of asset level — there is no abrupt value transfer at conversion. The three curves use the same conversion ratio Δ, set here so that the value of the equity held by CoCo investors just after conversion equals the par value of the CoCos if conversion occurs at an asset level of 80. With conversion at 85, CoCo investors get more than the par value in equity; with conversion at 75, they get less the par value.

continuous across the trigger. There does appear to be a kink at conversion — a discontinuity in the derivative. The higher sensitivity of equity to asset value above V_c is consistent with higher leverage. Conversion that is less attractive to CoCo investors (a lower V_c with the same Δ) produces a small equity sensitivity above the trigger, suggesting that this may also reduce risk-taking incentives; we examine this in greater detail in the next section.

The right panel shows an interesting pattern for CoCo value near the conversion trigger. When conversion grants shares at a discount price advantageous to CoCo investors (the dashed line), CoCo value increases as asset value decreases toward the trigger. We see a similar but less pronounced increase in the blue line, in which conversion is at par. The red line reflects conversion at a premium price per share, and in this case CoCo value declines steadily.

A parallel pattern has been observed for contingent capital with a stock price trigger, and the possibility that the CoCo value would increase as the stock price decreases has raised concerns about potential market manipulation and a downward spiral as CoCo investors short the stock to try to trigger conversion. This concern is not directly applicable to our setting, as investors have little chance of moving asset value. Nevertheless, the two panels of Figure 4 suggest that conversion at a small premium (the solid red line) yields lower volatility in CoCo and equity value around the conversion trigger.

Calomiris and Herring [12] argue that CoCos should be designed so that they convert at a ratio punitive to shareholders and also so that their yield spreads widen as they approach conversion, providing a signal to the market of the firm's condition (as has been discussed for subordinated debt — see Evanoff and Wall [20], Flannery [23], and Hancock and Kwast [30]). The right panel of Figure 4 shows that these objectives are incompatible: for spreads to widen near conversion, the conversion should be at small premium over the market value of the shares earned.

6 Asset Substitution and Risk Sensitivity

We reviewed the problem of debt overhang in the previous section in Merton's [43] model, which views equity as a call option on the firm's assets. The same model predicts that equity value increases with the volatility of the firm's assets, giving equity holders an incentive to increase the riskiness of the firm's investments after they have secured funding from creditors. In this section, we examine this phenomenon in our dynamic model, focusing on how CoCos change the incentives. Related questions of risk-shifting incentives are studied in Albul et al. [1], Hilscher and Raviv [32], Koziol and Lawrenz [35], and Pennacchi [48] with contingent capital and in Bhanot and Mello [4] for debt with rating triggers. Morellec [44] studies the impact of asset liquidity on debt capacity.

We can summarize our main observations as follows. Because of the need to roll maturing debt, equity holders do not necessarily prefer more volatile assets in a dynamic model; longer debt maturity makes riskier assets more attractive to equity holders. Even when equity value does increase with asset volatility, CoCos can mitigate or entirely offset this effect, in part because equity holders are motivated to avoid conversion. In some cases, CoCos can make tail risk more attractive to equity holders even while making diffusive risk less attractive.

To illustrate these points, we start with the lower panel of Figure 5, which shows the sensitivity of equity to diffusive volatility as a function of asset value. The solid black line corresponds to a firm with no contingent capital — the sensitivity of equity to σ is positive throughout the range and peaks just above the default barrier. As the firm nears bankruptcy, the equity holders are motivated to take on extra risk in a last-ditch effort at recovery.

We see a very different pattern in the two blue lines, corresponding to a firm in which some straight debt has been replaced with CoCos, and the two red lines, based on replacing some equity with CoCos. In both cases, the solid line is based on a conversion trigger of 85, and the dashed line uses a trigger of 70. This gives us four combinations of capital structure and trigger level. In all four, the sensitivity is *negative* at high asset values and turns sharply negative as asset value decreases toward the conversion boundary before becoming slight positive just above the trigger, where equity holders would prefer to gamble to avoid conversion. After conversion, the pattern naturally follows that of a firm without CoCos. The key implication of the figure is that CoCos decrease, and even reverse, the incentive for the shareholders to increase the riskiness of the firm's assets.

The top half illustrates the effect of debt maturity and bankruptcy costs on the risk-shifting incentive. In each pair of lines, the dashed line has the same level of deposits and straight debt as the solid line but it also has CoCos. Considering first the solid lines, we see that with long-maturity debt, the risk-shifting incentive is positive, even at rather high recovery rate of $\alpha = 90\%$. In contrast, with shorter maturity debt, the sensitivity is nearly always negative, even with a recovery rate of 100 percent — i.e., with no bankruptcy costs. Thus, debt maturity and not bankruptcy cost is the main driver of the sign of the risk-sensitivity. CoCos therefore have a greater effect on the risk-shifting incentive when the rest of the firm's debt has longer average maturity. The impact of CoCos is not very sensitive to the recovery rate α.

Figures 6 and 7 illustrate similar comparisons but with the sensitivity at each asset level normalized by the value of equity at that asset level; we interpret this as measuring the risk-shifting incentive per dollar of equity. Also, the figures compare sensitivities to diffusive volatility on the left with sensitivity to tail risk, as measured by $1/\eta_f$, on the right. Figure 7 uses a longer average maturity of debt than Figure 6.

The left panels of Figures 6 and 7 are consistent with what we saw in Figure 5 for the unnormalized sensitivities: with longer maturity debt, CoCos reverse the risk-shifting incentive; with shorter maturity debt, equity holders already have an incentive to reduce risk, particularly at low asset values, and CoCos make the risk sensitivity more negative.

The right panels add new information by showing sensitivity to tail risk. In both Figures 6 and 7, equity holders have a positive incentive to add tail risk, particularly with long maturity debt, but also with short maturity debt at low asset levels. Indeed, the incentive becomes very large in both cases as asset value falls. Increasing the size of the firm's balance sheet by adding CoCos leads to a modest increase in this incentive above the conversion trigger. Replacing some straight debt with CoCos reduces the incentive to take on tail risk but does not reverse it. Related comparisons are examined in Albul et al. [1] and Pennacchi [48]. Pennacchi's [48] conclusions appear to be consistent with ours, though modeling differences make a direct comparison difficult; the conclusions in Albul et al. [1] are quite different, given the absence of jumps and debt rollover in their framework.

The patterns in our results can be understood, at least in part, from the asset dynamics in (3); in particular, whereas the diffusive volatility σ plays no role in the (risk-neutral) drift, increasing the jump parameter $1/\eta_f$ increases the drift. In effect, the firm earns a higher continuous yield on its assets by taking on greater tail risk. This has the potential to generate additional dividends for shareholders, though the additional yield needs to be balanced against increased rollover costs resulting from increased default risk. In addition to generating a higher yield, jump risk is attractive to shareholders because the cost of conversion is lower if it takes place at a lower asset value than at the conversion trigger. Moreover, shareholders are indifferent between a bankruptcy at an asset value below their default barrier or right at their barrier, so they are motivated to earn the higher yield from tail risk without bearing all of the downside consequences.

7 Debt-Induced Collapse

At several points in our discussion we have qualified our remarks with the condition that conversion precedes bankruptcy — in other words, that the the conversion trigger V_c is above the endogenous bankruptcy boundary V_b. We now examine this condition in greater detail, highlighting a phenomenon of debt-induced collapse in equity value: an increase in the firm's debt drives its bankruptcy level V_b higher; if the increase is sufficiently extreme to drive the bankruptcy level above the conversion trigger, then just at the point at which V_b crosses V_c — where the CoCos become junior debt — equity value experiences a sharp decline. No comparable phenomenon can occur in the absence of CoCos.

To explain this phenomenon, we introduce two other firms that are identical to the original firm except that in one the CoCos have already been converted, and in the other the conversion feature has been removed so the CoCos will never convert. Call these the AC and NC firms, respectively. The equity holders of the NC firm (which has no convertible debt) set an optimal default barrier $V_b(NC)$. If $V_b(NC) \geq V_c$, then $V_b(NC)$ is a feasible choice of default barrier for the original firm (and makes equity values for the two firms identical). It is feasible in the sense that if the original firm chose $V_b = V_b(NC)$, then equity value would be positive prior to default and equal to zero at the time of default. These assertions follow from the fact that equity value in the original firm would equal equity value in the NC firm if $V_b = V_b(NC) \geq V_c$, because conversion would never precede default under this condition.

To illustrate, we consider an example. The heavy solid line in Figure 8 shows equity value for the NC firm. The optimal default barrier $V_b(NC)$ is at 86.1, and the NC equity value and its derivative are equal to zero at this point. If the conversion trigger V_c is below 86.1 (two cases are considered in the figure), then this is a feasible default level — and a feasible equity value — for the original firm.

Denote by $V_b(AC)$ the optimal default barrier for the AC firm. We always have $V_b(AC) \leq V_b(NC)$ because the NC firm has all the debt of the AC firm plus additional debt. Suppose $V_b(AC) < V_c$. Below the conversion trigger V_c, the original firm is identical to the AC firm, so for asset values below V_c the only possible choice of default barrier for the original firm is $V_b(AC)$. However, this choice may not be feasible for the original firm because it potentially produces negative equity values at higher asset levels. But equity holders would default rather than accept negative value; the inconsistency in such a case would indicate that $V_b(AC)$ would not be a feasible choice of default barrier for the original firm.

Both cases are illustrated in Figure 8. The dashed line corresponds to a conversion trigger of $V_c = 81.7$, where the kink occurs. The equity holders choose $V_b = V_b(AC) = 66.3$ as their default barrier, conversion occurs prior to default, equity value is always nonnegative, and it is characterized by the smooth pasting condition at the default barrier. However, at a conversion trigger of $V_c = 72.9$, an attempt to choose $V_b = V_b(AC) = 66.3$ as the default barrier would result in negative equity at a higher asset value, which means that equity holders would actually default near 78; but this choice would then change the entire path of equity value, meaning that $V_b = V_b(AC)$ fails to be internally consistent — it is not a feasible choice. The only feasible default barrier for the original firm is then $V_b = V_b(NC)$.

Now consider the implications of having the two candidate solutions $V_b(AC)$ and $V_b(NC)$ for optimal default barrier of the original firm. With a conversion trigger of $V_c = 81.7$, the optimal default barrier is $V_b = V_b(AC) = 66.3$, conversion occurs prior to default, and equity value follows the dashed line. But if we lower V_c, we eventually get to a point (somewhere before $V_c = 72.9$) at which $V_c(AC)$ becomes infeasible and the optimal default barrier jumps to $V_b(NC) = 86.1$. This jump up in default barrier is accompanied by a sudden drop in equity value.

The economic explanation is that lowering the conversion trigger eventually has the effect of changing the CoCos into junior straight debt — debt with no ability to absorb losses. In an equilibrium in which conversion occurs prior to bankruptcy, equity holders derive greater benefit from the presence of the convertible debt and are thus more willing to continue to sustain the firm at times when the inequality in (4) is reversed. In an equilibrium in which bankruptcy precedes conversion, the convertibility feature has no value to equity holders, equity holders have less incentive to sustain the firm, and equity value drops.

Although we have described this phenomenon through a change in V_c, a similar and more significant pattern holds if V_c is fixed but the firm increases its debt, whether straight debt or CoCos. Consider an increase in straight debt. This moves both $V_b(AC)$ and $V_b(NC)$ to the right in the figure, which has the same effect as moving V_c to the left.[5] Eventually, the additional debt service becomes so great that equity holders become unwilling to sustain the firm all the way down to $V_b(AC)$ and instead commit to abandoning the firm at $V_b(NC)$, prior to conversion. The equity holders thus effectively eliminate the conversion feature of the CoCos, and at the point at which this happens, equity experiences a sudden drop given by the vertical distance between the dashed line and the heavy solid line in the figure.

We view this scenario as a real phenomenon, and one that is possible only with convertible debt and endogenous default. The implications are as follows: the conversion trigger for CoCos needs to be sufficiently high to ensure unambiguously that conversion will take place prior to default; the firm's capital structure needs to be managed to ensure that this continues to hold if the firm takes on more debt. A switch from conversion prior to bankruptcy to bankruptcy prior to conversion is accompanied by a sharp drop in equity value as the value of the conversion feature is lost.

8 Orderly Resolution Versus Contingent Capital

Resolution authority and contingent capital can be viewed as complementary tools in avoiding financial crises: whereas the objective of replacing straight debt with CoCos is to reduce the likelihood of a bank failure, the objective of orderly resolution is to reduce the costs and negative externalities of a failure. These tools are also complementary in the sense that orderly resolution includes the option of a bail-in mechanism in which equity holders are wiped out and creditors are forced to take some losses and accept repayment in the form of equity in a reorganized entity.[6]

8.1 Varying the Recovery Rate

In our model, the impact of orderly resolution is reflected, in a reduced-form manner, by the parameter α, which measures the recovery rate on assets in the event of default: an ideal and seamless resolution would have $\alpha = 100\%$. We have used $\alpha = 50\%$ as part of our base case

[5]If the additional debt is convertible, $V_b(AC)$ will not change, but equity will become negative somewhere above the conversion boundary, as in Figure 8.

[6]See, for example, Resolution Strategy Overview, FDIC Office of Complex Financial Institutions, January 25, 2012, http://www.fdic.gov/about/srac/2012/2012-01-25_resolution-strategy.pdf.

parameter set. In this section, we examine the relative impact of CoCos and orderly resolution by exploring how much α would need to be increased to achieve the same effect as a CoCo issue of a given size. We examine this tradeoff for a few different performance measures as α and CoCo size vary.

Before doing so, we comment briefly on the relationship between our reduced-form model and the complexities of unwinding a large financial institution. For this, we draw on an analysis by the FDIC [21] of how the failure of Lehman Brothers would have been managed had Title II of the Dodd-Frank act been in effect at the time. The report highlights (p.6) three elements of the FDIC's resolution authority that are particularly relevant to our reduced-form bankruptcy costs: supporting an orderly liquidation that maintains asset values, the ability to continue key operations, and the ability to transfer contracts to preserve value.[7] According to the report, the Chapter 11 reorganization plan filed on January 25, 2011, estimates a 21.4 percent recovery rate for senior unsecured creditors. The report further concludes that an FDIC resolution would have produced a 97 percent recover rate for senior unsecured creditors. These recovery rates are not directly comparable to our α, because α is a recovery rate on assets, not debt, which must be lower. To arrive at a 97 percent recovery rate on senior debt under an orderly resolution, the report estimates that Lehman's problem assets would have experienced a loss in the range of 60–80 percent, and that its \$210 billion in total assets would have suffered \$40 billion in losses, for a recovery rate of 81 percent, eliminating \$35 billion in equity and subordinated debt. To achieve the 21.4 percent recovery rate on senior unsecured debt in the Chapter 11 filing, a similar calculation shows that the recovery rate on assets would have to be less than 17.8 percent. Although none of these values corresponds directly to our α, they suggest an aspiration for a very substantial increase in the recovery rate, from something in the vicinity of 20 percent to something closer to 80-100 percent.[8]

In Figure 9, we vary the loss given default factor, $1 - \alpha$, to achieve the same result as a CoCo issue. In the left panel, we hold expected bankruptcy costs fixed. In our base case (the heavy solid line) we see, for example, that replacing 10 percent of debt with CoCos achieves the same reduction in expected bankruptcy costs as reducing the loss given default from 50 percent to around 34 percent. The other two curves on the left show the same comparison with either σ or η doubled. Reducing the severity of the jumps (increasing η) makes CoCos relatively more

[7] The report also highlights advance resolution planning and prompt distributions to creditors based on anticipated recoveries.

[8] Valukas [54], pp.202–209, observes that Lehman's assets were in principle reported at fair value but that there was public skepticism about Lehman's marks on its illiquid assets. The loss in asset value at bankruptcy may therefore combine a correction in valuation with costs more directly connected to financial distress; the two effects are difficult to disentangle.

effective as measured by the equivalent reduction in $1 - \alpha$.

The right panel shows a similar comparison holding the discount on senior debt constant. The four lines correspond to replacing senior debt with CoCo issues of varying sizes. Asset value increases as we move from left to right. The largest CoCo size has the same effect on the yield of senior debt as reducing the loss given default to 7.5–12.5 percent from the base case value of 50 percent.

Finally, we consider the impact on equity. Figure 10 shows the reduction in loss given default required to achieve the same increase in equity value as a CoCo issuance of the indicated size. The sharp decline near the left end of the curve is due to deposit insurance: equity value is nearly insensitive to the loss given default until the recovery exceeds the deposits. The figure indicates that replacing straight debt with CoCos in an amount equal to 10 percent of assets has the same effect on equity value as reducing the loss given default from 50 percent to just over 20 percent.

Focusing on the impact on debt and equity, these comparisons suggest that replacing approximately 10 percent of debt with CoCos has a similar effect as increasing the recovery rate α from 50 percent to roughly 80–90 percent, for parameter values similar to our base case. This improvement is substantial, though it falls short of the objective of a seamless resolution with nearly 100 percent recovery.

8.2 Market-Wide Jumps and Systemic Effects

Our model distinguishes firm-specific jumps and market-wide jumps, with the interpretation that market-wide jumps are rarer but more severe. The analysis in the appendix further allows us to differentiate recovery rates for defaults triggered by the two types of jumps. Default at a market-wide jump is likely to have spill-over effects: if many banks suffer losses simultaneously, many may need to liquidate assets simultaneously, further depressing prices. We model this systemic effect through a lower recovery rate for defaults that occur at market-wide jumps.

We revisit the comparison on the right side of Figure 5 from this perspective. To achieve a more pronounced separation between systemic and ordinary defaults, we set the baseline recovery rate to 30 percent at market-wide jumps and 70 percent otherwise. Figure 11 compares the sensitivity of equity value to market-wide jump risk and to firm-level jump risk for three different combinations of straight debt and CoCos. In all cases, the sensitivities are positive at low asset values, reflecting an incentive for equity holders to take on additional jump risk in this setting. The sensitivity to market-wide jump risk is consistently higher than the sensitivity to firm-specific jump risk. Replacing straight debt with CoCos reduces the attractiveness of jump risk to equity holders, as measured by the sensitivities, even making the sensitivity to firm-specific jump risk negative. However, the gap between the two sensitivities is not affected. Indeed, as discussed in Section 5, equity holders prefer CoCo conversion to occur at a low asset level rather than near the trigger level, and this creates an incentive to take on tail risk.

Next, we re-examine the tradeoff between CoCos and resolution authority. We suppose that resolution authority can improve the recovery rate when default is due to diffusive risk or a firm-specific jump, but that it cannot offset the fire-sale effects of a market-wide decline in asset values. The setting is the same as that of Figure 9, except that the recovery rate at market-wide jumps is fixed at 30 percent and we vary only the recovery rate for other types of default, starting at the base case value of 50 percent. The results are shown in Figure 12. As one might expect, CoCos translate to a greater improvement in recovery in this setting because the improvement applies in only a subset of cases. However, the change compared to Figure 9 is minor because defaults due to market-wide jumps are rare for the parameters in our base case.

A simple variant of our model would calculate bankruptcy costs from a regulator's perspective using a lower α than the recovery rate used by shareholders and creditors, with the interpretation that these greater costs reflect negative externalities from the failure of a financial institution. Taking this idea a step further, one could try to develop an extension in which the regulator sets CoCo requirements to get shareholders to internalize these externalities. The model in Van den Heuvel [55] is potentially useful in formulating a regulatory objective.

9 Calibration to Bank Data Through the Crisis

In this section, we calibrate our model to specific banks. We focus on the years leading up to and during the financial crisis, with the objective of gauging what impact CoCos might have had, had they been issued in advance of the crisis. We examine the increase in the banks' ability to absorb losses, relative to the amount of straight debt replaced with CoCos, and we calculate the reduction in debt overhang costs as an indication of whether CoCos would have created greater incentives for equity holders to inject private capital at various points in time.

As candidates for our calibration, we chose the 19 bank holding companies (the largest 19 at the time) that underwent the Supervisory Capital Assessment Program (SCAP) in 2009. From this list, we removed MetLife because banking is a small part of its overall business, and we removed GMAC (now Ally) because it is privately held. The banks are listed in Table 2, in order of asset value in 2009.

We obtain quarterly balance sheet information from each bank holding company's quarterly

10-Q/10-K S.E.C. filings from 2004 through the third quarter of 2011, except in the case of American Express, for which we begin in 2006 because of a large spin-off in 2005. Several of the firms became bank holding companies late in our time window, so Y-9 reports would not be available throughout the period. Also, the Y-9 reports contain less information about debt maturities and interest expenses than the quarterly reports. We group all debt into three categories — deposits, short-term debt, and long-term debt — in this order of seniority. We do not separate subordinated debt from other long-term debt because of difficulties in doing so consistently and reliably. The distinction would not have much effect on our calculations. We calculate average debt maturity within each category using information provided in annual reports. We calculate total dividends and interest payments to get a total payout rate.

We linearly interpolate values within each quarter, using values from the beginning of the quarter and the beginning of the subsequent quarter; this gives us values at a weekly frequency and avoids abrupt changes at the end of each quarter. For debt maturities, we interpolate between annual reports.

Our model is driven by asset value, but asset value is not observable. So, we fit our model using balance sheet and market information and then use the model to infer asset value or a model-defined proxy for asset value. In more detail, at each week we use the linearly interpolated values to determine the bank's debt profile, dividends, and interest. As the risk-free rate, we use the Treasury yield corresponding to the weighted average maturity of each bank's debt.

Jump parameters are difficult to estimate, particularly for rare jumps as contemplated by our model. For the calibrations, we simplify the model to a single type of jump and choose from a finite set of values for the jump rate λ and the mean jump size $1/\eta$. For each (λ, η), we calibrate a value for the diffusive volatility σ iteratively as follows. Given a starting value for σ, we can numerically invert our model's formula for equity at each point in time (using the market value of equity at each point in time) to get an implied market value for the assets. We then calculate the annualized sample standard deviation of the implied asset log returns, excluding returns of magnitude greater than 3.3σ, which we treat as jumps, and compare it with σ. We adjust σ up or down depending on whether the standard deviation is larger or smaller than σ, proceeding iteratively until the values match. At that point, we have found a path of underlying assets that reproduces the market value of equity with an internally consistent level of asset volatility, for a fixed (λ, η).

We repeat this procedure over a grid of (λ, η) values. We limit λ to 0.1 or 0.3; for η, we consider integer values between 5 and 10, but if the best fit occurs at the boundary we extend the range to ensure that does not improve the fit. We choose from the set of (λ, η, σ) values by

	Jan-2006		Jan-2007		Jan-2008		Jan-2009	
Bank of America Corp	1.47	7%	1.43	8%	1.63	5%	1.54	3%
JPMorgan Chase & Co.	1.29	6%	1.29	6%	1.49	5%	1.50	5%
Citigroup Inc.	1.34	7%	1.32	6%	1.42	4%	-	2%
Wells Fargo & Company	1.11	19%	1.06	22%	1.44	9%	1.60	5%
Goldman Sachs Group, Inc.	1.35	4%	1.41	5%	1.52	4%	-	4%
Morgan Stanley	1.43	4%	1.38	4%	1.50	5%	-	5%
PNC Financial Services	1.17	19%	1.11	21%	1.29	14%	-	8%
U.S. Bancorp	0.95	32%	0.98	32%	1.11	24%	1.17	18%
Bank of New York Mellon	1.15	24%	1.06	28%	1.04	28%	0.80	17%
SunTrust Banks, Inc.	0.91	21%	0.87	22%	0.91	16%	-	8%
Capital One Financial Corp.	0.93	29%	0.92	26%	0.97	16%	-	12%
BB&T Corporation	1.03	25%	1.03	23%	0.97	14%	-	9%
Regions Financial Corp.	0.90	24%	0.89	19%	0.87	12%	-	4%
State Street Corporation	1.33	18%	1.25	20%	1.07	24%	-	11%
American Express Company	1.15	38%	1.13	36%	1.26	28%	1.50	18%
Fifth Third Bancorp	0.89	26%	0.77	31%	-	17%	-	6%
KeyCorp	1.11	17%	1.01	20%	-	10%	-	5%
mean	1.15	18.81%	1.11	19.23%	1.23	13.73%	1.35	8.15%
median	1.15	19.32%	1.06	20.52%	1.26	13.80%	1.50	5.81%

Table 3: Under each date the left column shows the ratio of the increase in loss absorption (the change in the default boundary after CoCo issuance) to CoCo size (as measured by market value). The right column is the distance to default (without CoCos) as a percentage of asset level. The dilution ratio is 50 percent.

each of the banks would have triggered conversion of CoCos with a high trigger (50 percent dilution ratio) and a low trigger (75 percent dilution ratio). In each case, the CoCo size is equal to 10 percent of the bank's total debt. The calibrations predict that all the banks except JPMorgan Chase, Wells Fargo, and American Express would have crossed the high conversion trigger sometime between November 2007 and January 2009; seven of the banks would have crossed the lower conversion trigger as well.

Next, we consider debt overhang costs. For each bank in each week, we calculate the size of the equity investment required to increase assets by 1 percent. From this we subtract the net increase in equity value, which we calculate by taking the value of equity just after the investment (as calculated by the model) and subtracting the value of equity just before the investment (as observed in the data). This is our measure of debt overhang cost: if it is positive, it measures how much less equity holders get from their investment than they put in. A negative cost indicates a net benefit to investment.

Table 4 presents more detailed information at three dates prior to key points in the financial crisis: one month before the announcement of JP Morgan's acquisition of Bear Stearns; one month before final approval of the acquisition; and one month before the Lehman bankruptcy. For each date, the table shows the debt overhang cost without CoCos and with high-trigger CoCos; the third column under each date shows the distance to the conversion boundary as a percentage of asset value. Interestingly, several of the largest banks show significantly negative debt overhang costs even without CoCos. Recall from Section 5 that this is possible in a model with debt rollover, though not with a single (finite or infinite) debt maturity. Greater asset value implies greater bankruptcy costs and reducing these costs may partly explain the motivation for shareholders to increase their investments in the largest firms. Also, if the market perceives a too-big-to-fail guarantee for the largest banks that is absent from our model, then the model's shareholders may see the largest banks as overly leveraged relative to the market's perception.

We focus on comparisons between columns of the table — a single firm under different conditions — rather than comparisons across rows. With few exceptions, the effect of the CoCos is to lower the debt overhang cost, and the impact is often substantial. The effect depends on the interaction of several factors, including leverage, debt maturity, and the risk-free rate, which enters into the risk-neutral drift. The largest reductions in debt overhang cost generally coincide with a small distance to conversion, and, in most cases in which a bank draws closer to the conversion boundary over time, the resulting reduction in debt overhang cost becomes greater. The values in the table are for 50 percent dilution. The pattern with 75 percent is similar, but the decrease in the debt overhang cost is smaller in that case because the

	Feb-2008			Apr-2008			Aug-2008		
Bank of America Corp	-29%	-32%	6%	-26%	-30%	5%	-28%	-42%	3%
JPMorgan Chase & Co.	-75%	-51%	5%	-43%	-41%	5%	-93%	-60%	3%
Citigroup Inc.	-42%	-53%	3%	-24%	-45%	2%	-54%	-65%	2%
Wells Fargo & Company	-35%	-23%	8%	-33%	-20%	8%	-33%	-21%	7%
Goldman Sachs Group	-51%	-45%	2%	-33%	-42%	2%	-53%	-54%	2%
Morgan Stanley	21%	-42%	1%	21%	-36%	1%	-20%	-58%	2%
PNC Financial Services	-11%	-16%	7%	-7%	-12%	8%	-10%	-12%	8%
U.S. Bancorp	4%	4%	13%	5%	5%	13%	5%	5%	11%
Bank of New York Mellon	-3%	-2%	17%	-1%	0%	14%	6%	4%	8%
SunTrust Banks, Inc.	-2%	-20%	2%	5%	-	-	9%	-	-
Capital One Financial	-4%	-28%	3%	4%	-34%	2%	6%	-	-
BB&T Corporation	2%	-11%	4%	4%	-12%	4%	6%	-60%	1%
Regions Financial Corp.	-7%	-24%	3%	-8%	-42%	2%	-9%	-	-
State Street Corporation	2%	2%	11%	5%	-1%	6%	0%	-11%	5%
American Express Co.	-12%	-13%	20%	-7%	-10%	20%	-10%	-12%	17%
Fifth Third Bancorp	12%	-79%	0%	17%	-	-	19%	-	-
KeyCorp	-6%	-137%	0%	-1%	-	-	5%	-	-

Table 4: Under each date, the first column is the debt overhang cost as a percentage of the increase in assets with no CoCos. The second column quotes the same value when 10 percent of debt is replaced with CoCos and CoCo investors receive 50 percent of equity at conversion. The third column is the distance to conversion as the percentage of assets. The dates correspond to one month before announcement and final approval of acquisition of Bear Stearns by JPMorgan and one month before the Lehman bankruptcy. A table entry is blank if the corresponding date is later than the CoCo conversion date for the corresponding bank.

distance from the conversion trigger is greater. For illustration, Figure 15 plots the conversion boundaries calibrated for SunTrust at the two dilution ratios, and the lower figure plots debt overhang costs with and without CoCos. The CoCos create a positive incentive for investment as the asset level approaches the conversion boundary.

The magnitudes of the quantities reported in these tables and figures are subject to the many limitations and simplifications of our model and calibration. We see these results as providing a useful additional perspective on the comparative statics of earlier sections of the paper; the directional effects and the comparisons over time should be more informative than the precise numerical values. These calibrations and our exploration of counterfactual scenarios, though hypothetical, shed light on how CoCo issuance in advance of the financial crisis might have affected loss absorption capacity, incentives for additional equity investment, and how the choice of conversion trigger and dilution ratio might have determined the timing of conversion.

10 Concluding Remarks

The key contribution of this paper lies in combining endogenous default, debt rollover, and jumps and diffusion in income and asset value to analyze the incentive effects of contingent convertibles and bail-in debt. Through debt rollover, shareholders capture some of the benefits (in the form of lower bankruptcy costs) from reduced asset riskiness and lower leverage — benefits that would otherwise accrue solely to creditors. These features shape many of the incentives we consider, as do the tax treatment of CoCos, the deposit insurance base, and tail risk. The phenomenon of debt-induced collapse, which is observable only when CoCos are combined with endogenous default, points to the need to set the conversion trigger sufficiently high so that conversion unambiguously precedes bankruptcy, or else to use bail-in debt structured to avoid bankruptcy costs. Our calibrations suggest that CoCos could have had a significant impact on the largest U.S. bank holding companies in the lead up to the financial crisis.

Our analysis does not include asymmetric information, nor does it directly incorporate agency issues; both considerations are potentially relevant to the incentives questions we investigate. Some important practical considerations, such as the size of the investor base for CoCos, the behavior of stock and bond prices near the trigger, and the complexity of these instruments are also outside the model. The analysis provided here should nevertheless help inform the discussion of the merits and potential shortcomings of CoCos and other hybrid capital instruments.

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Technical Appendix

All the valuations used in this paper reduce to expectations of certain functions of the asset value process V_t in (3) and the default time τ_b. This appendix derives the necessary formulas.

Our analysis builds on work on the hyperexponential jump-diffusion process in Cai et al. [9]. There is an extensive body of work on ruin probabilities and random walks that uses related techniques; see Asmussen and Albrecher [2] for a thorough treatment of the topic and extensive references.

Let $X_t = \log(V_t)$ and write

$$X_t = X_0 + \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i. \tag{12}$$

Here, $N_t = N_t^{(m)} + N_t^{(f)}$, and the jump sizes Y_i are i.i.d. with density

$$f_Y(y) = q_f \eta_f e^{\eta_f y} \mathbf{1}_{\{y<0\}} + q_m \eta_m e^{\eta_m y} \mathbf{1}_{\{y<0\}}, \tag{13}$$

where $q_f = \Lambda_f/(\Lambda_f + \Lambda_m)$ and $q_m = 1 - q_f$ are the probabilities of the two types of jumps. This is a Lévy process with Lévy exponent

$$G(x) := \frac{1}{t} \log \mathbb{E}[\exp(xX_t)] = x\mu + \frac{1}{2}x^2\sigma^2 + (\Lambda_f + \Lambda_m)\left(\frac{q_f \eta_f}{\eta_f + x} + \frac{q_m \eta_m}{\eta_m + x} - 1\right).$$

By some elementary calculus, it can be shown that for any given $a > 0$, the equation $G(x) = a$ has four distinct real roots β, $-\gamma_1$, $-\gamma_2$, and $-\gamma_3$, where $\beta, \gamma_j > 0$ for $j = 1, 2, 3$. All these roots are different from η_f and η_m. See Cai et al. [9].

Given a constant b, define $\tau_b \equiv \inf\{t \geq 0 : X_t \leq b\}$. The process X can reach or cross level b in three ways: without a jump at τ_b, with a firm-specific jump at τ_b, or with a market-wide jump at time τ_b. Let J_0, J_1, and J_2 denote these three events. We need to consider the overshoot across level b in these three cases, so we define the events $F_0 := \{X_{\tau_b} = b\} \cap J_0$, $F_1 := \{X_{\tau_b} < b + y\} \cap J_1$, and $F_2 := \{X_{\tau_b} < b + y\} \cap J_2$ for some negative y. The pricing equations in Section 3 all reduce to evaluating quantities of the form

$$u_i(x) = \mathbb{E}\left[e^{-a\tau_b + \theta X_{\tau_b}} \mathbf{1}_{F_i} | X_0 = x\right], \ i = 0, 1, 2,$$

where $a \geq 0$ and θ are constants.

Introduce a matrix

$$M = \begin{bmatrix} e^{-\gamma_1 b} & e^{-\gamma_1 b}\frac{\eta_f}{\eta_f - \gamma_1} & e^{-\gamma_1 b}\frac{\eta_m}{\eta_m - \gamma_1} \\ e^{-\gamma_2 b} & e^{-\gamma_2 b}\frac{\eta_f}{\eta_f - \gamma_2} & e^{-\gamma_2 b}\frac{\eta_m}{\eta_m - \gamma_2} \\ e^{-\gamma_3 b} & e^{-\gamma_3 b}\frac{\eta_f}{\eta_f - \gamma_3} & e^{-\gamma_3 b}\frac{\eta_m}{\eta_m - \gamma_3} \end{bmatrix}.$$

The matrix M is invertible because the roots γ_j are distinct. We can use it to express the functions $u_i(x)$ explicitly:

Theorem 1 *Given $a > 0$ and the negative roots $-\gamma_j$, $j = 1, 2, 3$, of the algebraic equation $G(x) = a$, let $w(x) := (\exp(-\gamma_1 x), \exp(-\gamma_2 x), \exp(-\gamma_3 x))^\top$. Then,*

$$\begin{bmatrix} u_0(x) \\ u_1(x) \\ u_2(x) \end{bmatrix} = DM^{-1}w(x),$$

where

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\theta b}\frac{\eta_f}{\eta_f+\theta}e^{(\theta+\eta_f)y} & 0 \\ 0 & 0 & e^{\theta b}\frac{\eta_f}{\eta_f+\theta}e^{(\theta+\eta_f)y} \end{bmatrix}.$$

Proof. Conditional on the event J_1, the memoryless property of the exponential distribution implies that $b - X_{\tau_b}$ is exponentially distributed with mean $1/\eta_f$, independent of η_f. Therefore,

$$\begin{aligned} \mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b})\mathbf{1}_{F_1}|X_0 = x] &= e^{\theta b}\mathbb{E}[\exp(-a\tau_b + \theta(X_{\tau_b} - b))\mathbf{1}_{F_1}|X_0 = x] \\ &= e^{\theta b}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_1}|X_0 = x]\frac{\eta_f}{\theta + \eta_f}e^{(\theta+\eta_f)y}. \quad (14) \end{aligned}$$

Similarly, we have

$$\mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b})\mathbf{1}_{F_2}|X_0 = x] = e^{lb}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_2}|X_0 = x]\frac{\eta_m}{\theta + \eta_m}e^{(\theta+\eta_m)y}, \quad (15)$$

and

$$\mathbb{E}[\exp(-a\tau_b + \theta X_{\tau_b})\mathbf{1}_{F_0}|X_0 = x] = e^{lb}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_0}|X_0 = x] \quad (16)$$

Thus, we need to find

$$\mathbb{E}\left[e^{-a\tau_b + \theta X_{\tau_b}}\mathbf{1}_{J_i}|X_0 = x\right], \quad i = 1, 2, 3.$$

For any $a > 0$ and any purely imaginary number l (i.e., $l = \sqrt{-1}c$ for some real c),

$$M_t := \exp(-at + lX_t) - \exp(lX_0) - (G(l) - a)\int_0^t \exp(-as + lX_s)ds$$

is a zero-mean martingale. By the optional sampling theorem for martingales, we know that $\mathbb{E}[M_{\tau_b}|X_0 = x] = 0$, i.e.,

$$0 = \mathbb{E}[\exp(-a\tau_b + lX_{\tau_b})|X_0 = x] - e^{lx} - (G(l) - a)\mathbb{E}\left[\int_0^{\tau_b} \exp(-as + lX_s)ds|X_0 = x\right]. \quad (17)$$

On the other hand, we can further decompose the first term on the right as

$$\begin{aligned} \mathbb{E}[\exp(-a\tau_b + lX_{\tau_b})|X_0 = x] &= \sum_{i=1}^{3}\mathbb{E}[\exp(-a\tau_b + lX_{\tau_b})\mathbf{1}_{J_i}|X_0 = x] \\ &= e^{lb}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_0}|X_0 = x] + e^{lb}\frac{\eta_f}{\eta_f + \theta}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_1}|X_0 = x] \\ &\quad + e^{lb}\frac{\eta_m}{\eta_m + \theta}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_2}|X_0 = x] \end{aligned}$$

From (17) and (18), we know that

$$
\begin{aligned}
0 &= \mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_0}|X_0 = x]e^{lb} + e^{lb}\frac{\eta_f}{\eta_f+\theta}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_1}|X_0 = x] \\
&\quad + e^{lb}\frac{\eta_m}{\eta_m+\theta}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_2}|X_0 = x] - e^{lx} - (G(l)-a)\mathbb{E}[\int_0^{\tau_b}\exp(-as+lX_s)ds|X_0 = x].
\end{aligned}
\tag{18}
$$

Denote the right side of (18) by $h(l)$. The equality (18) indicates that $h(l) \equiv 0$ for all imaginary l. Multiply $h(l)$ by $(l+\eta_m)(l+\eta_f)$ to obtain a new function

$$H(l) = h(l)\cdot(l+\eta_m)\cdot(l+\eta_f).$$

Then, $H(l)$ is well-defined and analytic in the whole complex domain $\mathbb{C}$. By (18), $H(l)$ equals zero whenever l is a purely imaginary. The identity theorem of analytic functions in the complex domain (Rudin[50], Theorem 10.18) then implies that $H(l) \equiv 0$ for all $l \in \mathbb{C}$. Accordingly, $h(l) = 0$ for all $l \in \mathbb{C} \setminus \{-\eta_f, -\eta_m\}$.

If we choose $l = -\gamma_j$, $j = 1, 2, 3$, then $G(l) = 0$, and the equation $h(l) = 0$ becomes

$$
\begin{aligned}
e^{-\gamma_j x} &= \mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_0}|X_0 = x]e^{-\gamma_j b} + e^{-\gamma_j b}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_1}|X_0 = x]\frac{\eta_f}{\eta_f-\gamma_j} \\
&\quad + e^{-\gamma_j b}\mathbb{E}[\exp(-a\tau_b)\mathbf{1}_{J_2}|X_0 = x]\frac{\eta_m}{\eta_m-\gamma_j},
\end{aligned}
\tag{19}
$$

for $j = 1, 2, 3$. This give us a system of three linear equations in the three unknown quantities $\mathbb{E}[e^{-a\tau_b}\mathbf{1}_{J_i}|X_0 = x]$, $i = 1, 2, 3$. Using the solution to the linear equations in (14)–(16), we get $\mathbb{E}[e^{-a\tau_b+\theta X_{\tau_b}}\mathbf{1}_{F_i}|X_0 = x]$, $i = 1, 2, 3$. □

Iterated expectations of the form $\mathbb{E}[e^{-a_1\tau_c+\theta_1 X_{\tau_c}}\mathbb{E}[e^{-a_2(\tau_b-\tau_c)+\theta_2 X_{\tau_b}}\mathbf{1}_{F_i}|X_{\tau_c}]]$ can be evaluated the same way, and this is what we need for (9).

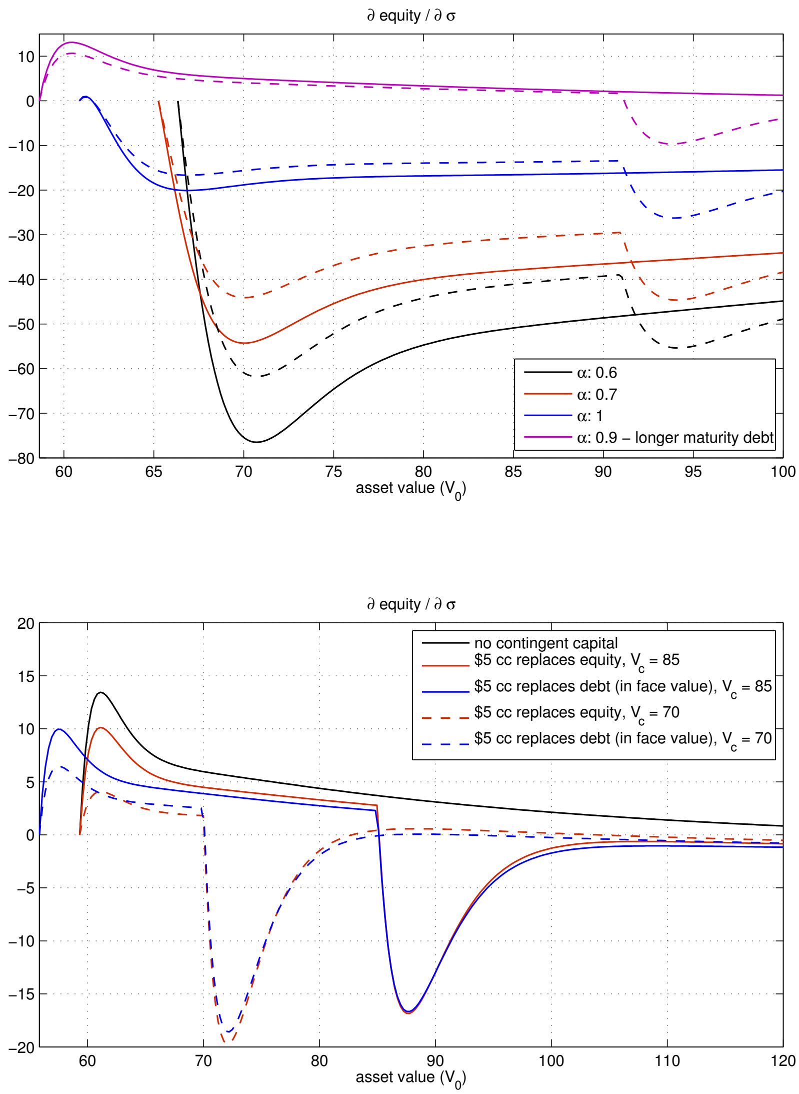

Figure 5: Sensitivity of equity value to diffusive volatility σ. With longer maturity debt, equity holders have a positive risk-shifting incentive. CoCos tend to reverse this incentive.

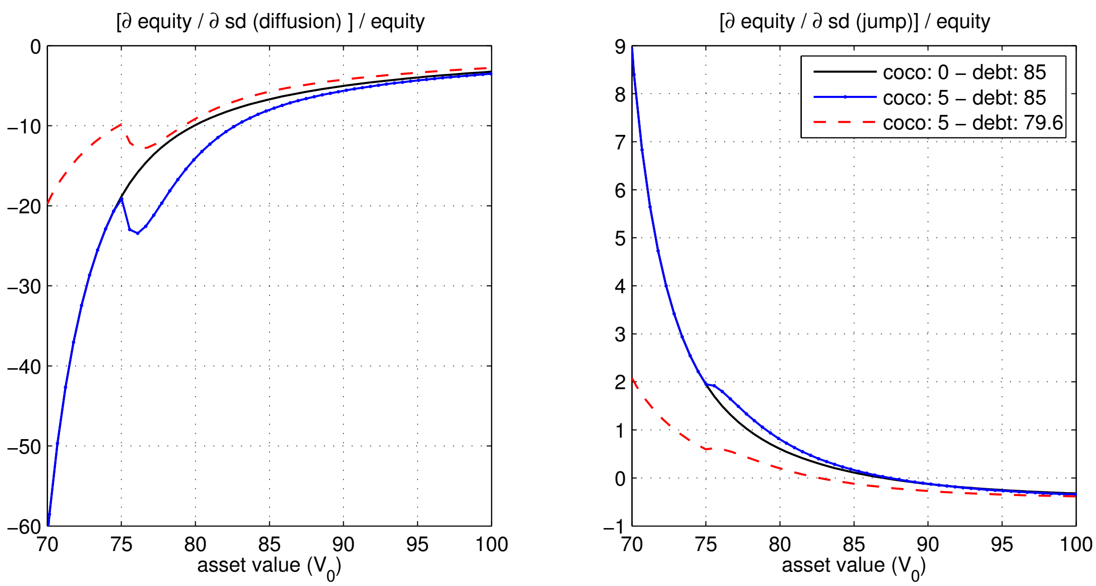

Figure 6: Sensitivity of equity value to diffusive volatility and jump risk in assets.

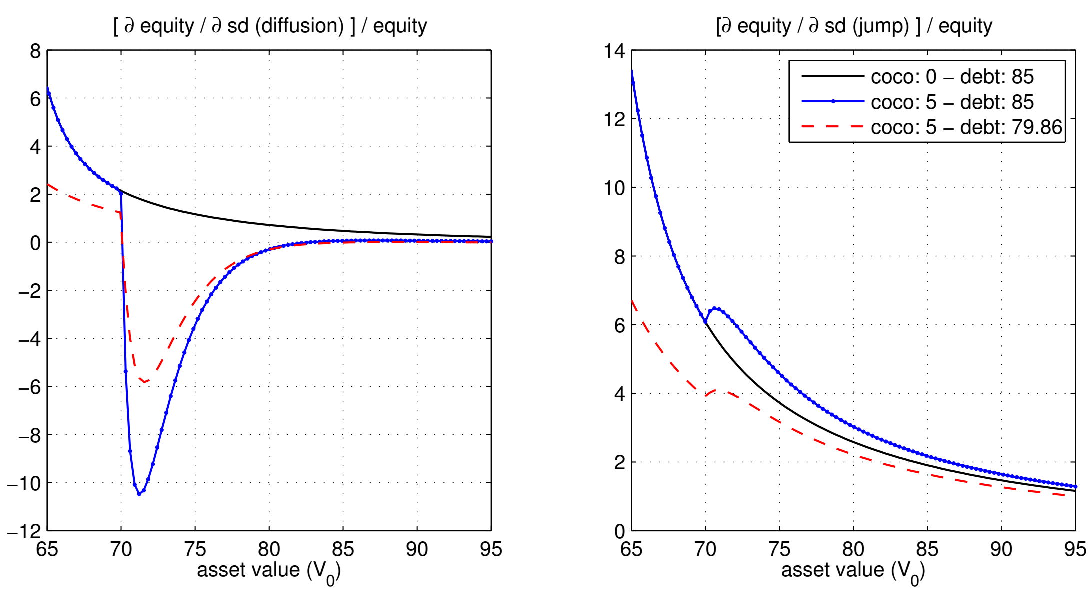

Figure 7: Same comparisons as Figure 6 but with with longer average maturity. In all plots, at the same asset level the dashed line corresponds to a larger distance to default due to less outstanding regular debt.

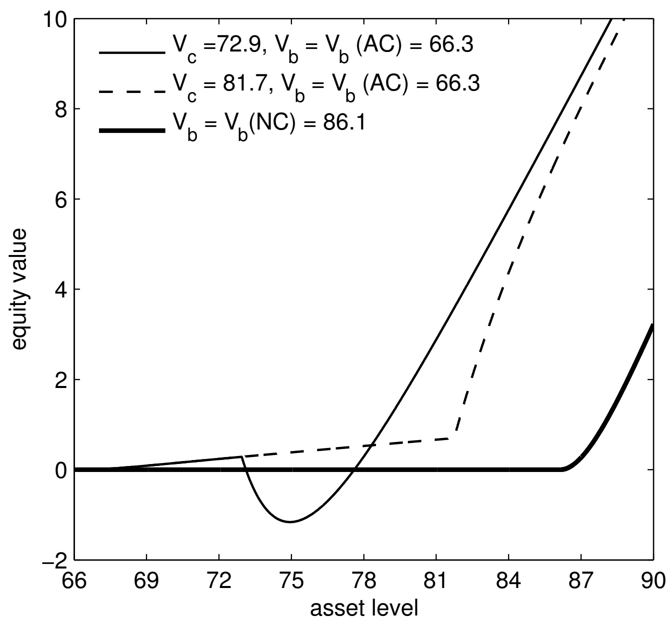

Figure 8: Candidate equity value as a function of asset value in three scenarios. The heavy solid line reflects default at $V_b(NC) = 86.1$, prior to conversion. The other two lines reflect default at $V_b(AC) = 66.3$ with two different conversion triggers. With $V_c = 72.9$, equity becomes negative so $V_b(AC)$ is infeasible and default occurs at $V_b(NC)$. With $V_c = 81.7$, default at $V_b(AC)$ is feasible, and it is optimal because it yields higher equity than $V_b(NC)$.

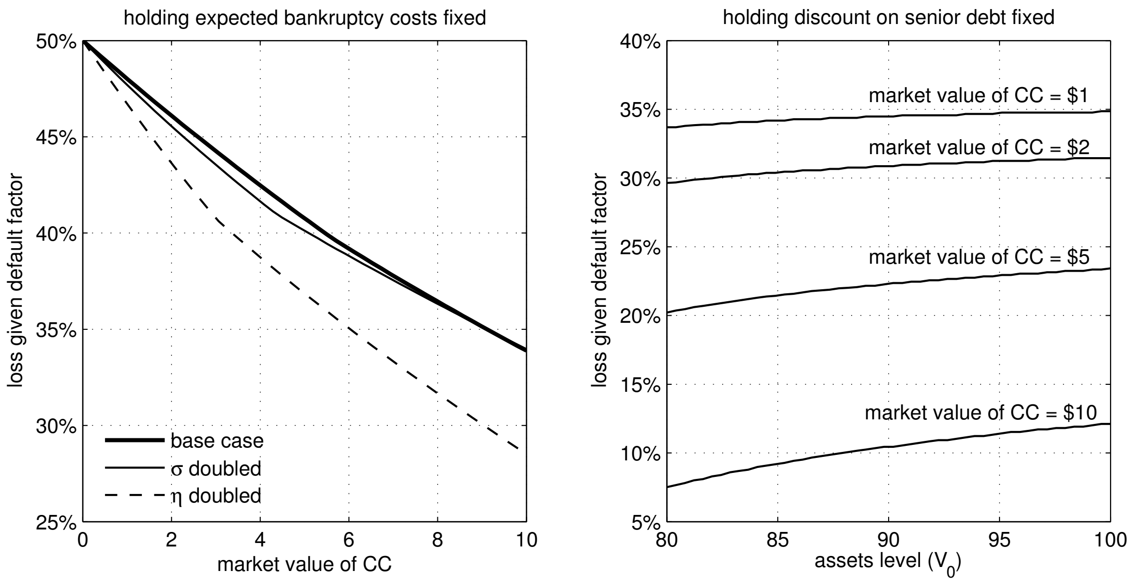

Figure 9: The left panel shows how much the loss given default would have to decrease to achieve the same expected bankruptcy costs as replacing straight debt with CoCos. The heavy solid line is our base case, and the other two lines double either σ or η. In the right panel, we show the corresponding tradeoff holding the discount on senior debt fixed.

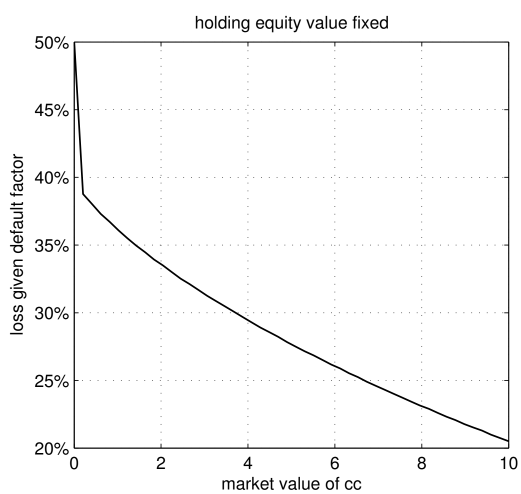

Figure 10: The figure shows how much the loss given default would have to decrease to achieve the same increase in equity value as replacing straight debt with CoCos.

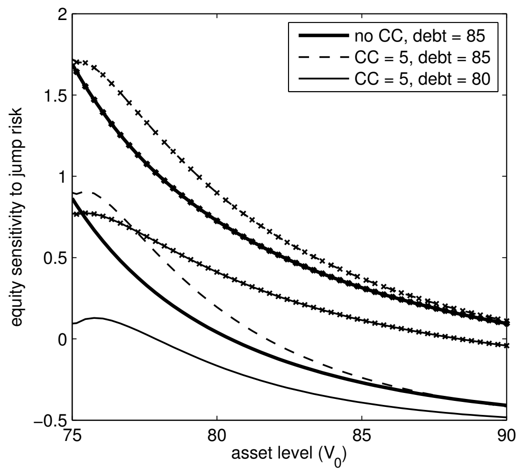

Figure 11: The figure revisits the example on the right side of Figure 5 with a lower recovery rate for defaults that occur at market-wide jumps.

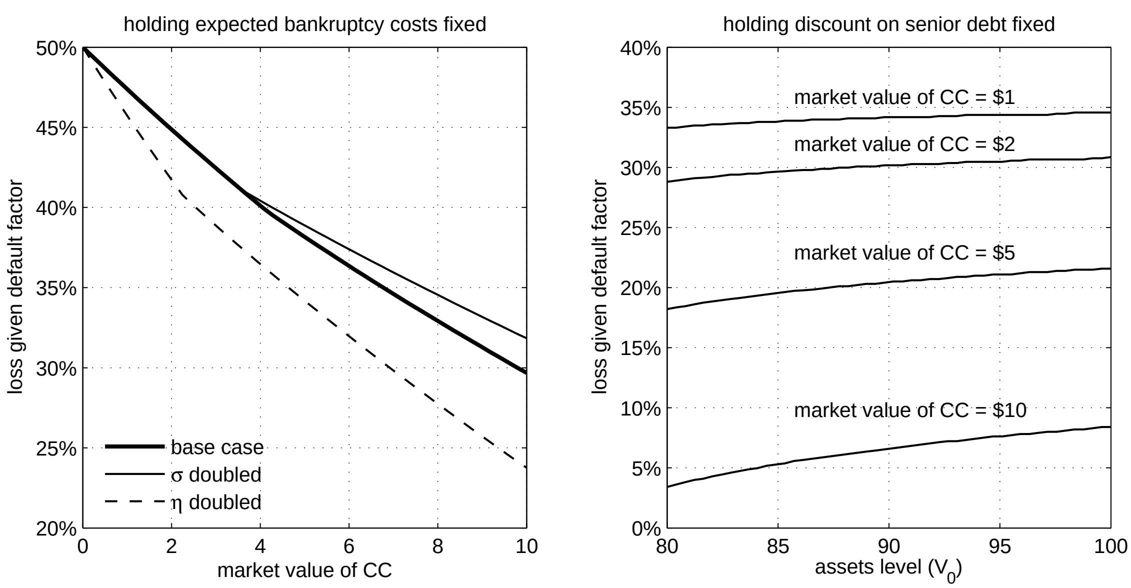

Figure 12: The figure revisits the example of Figure 9 with the restriction that resolution authority does not affect the recovery rate for defaults that occur at a market-wide jump.

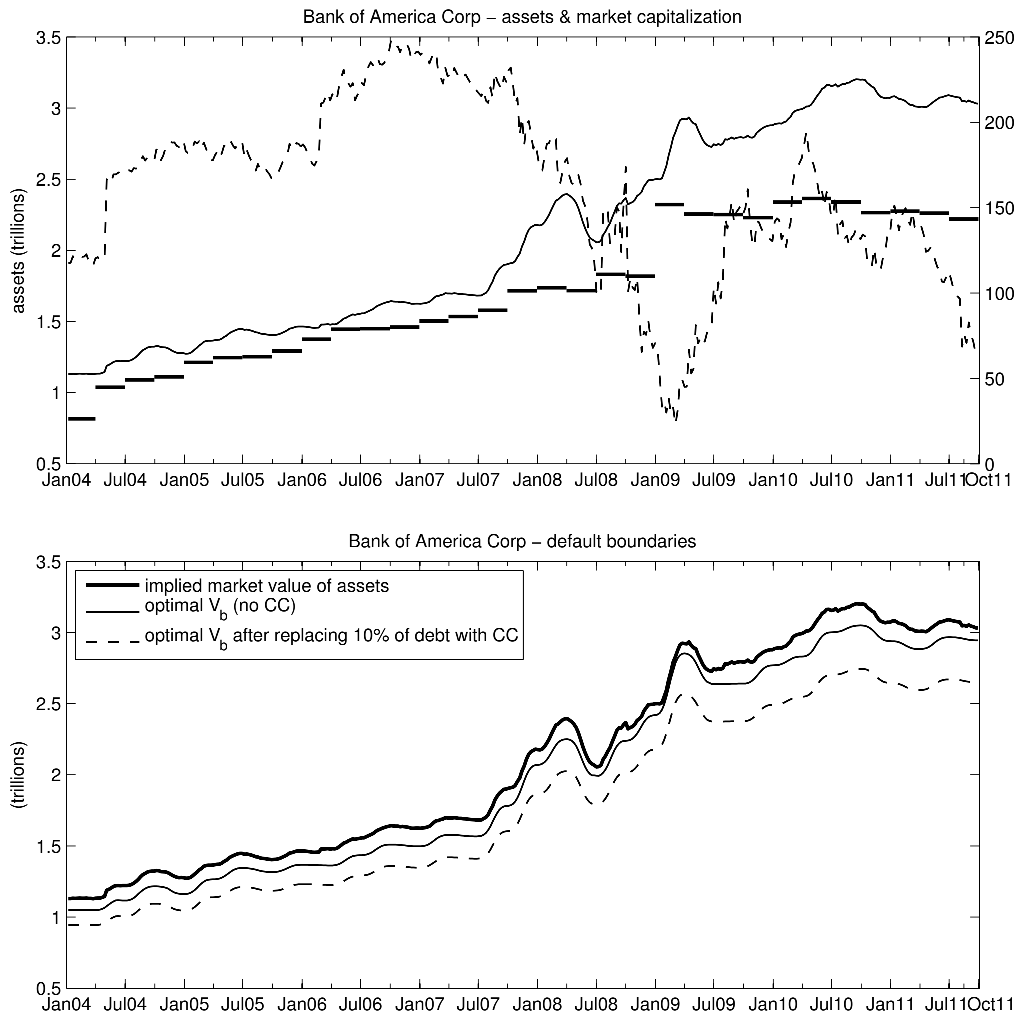

Figure 13: Calibration results for Bank of America.

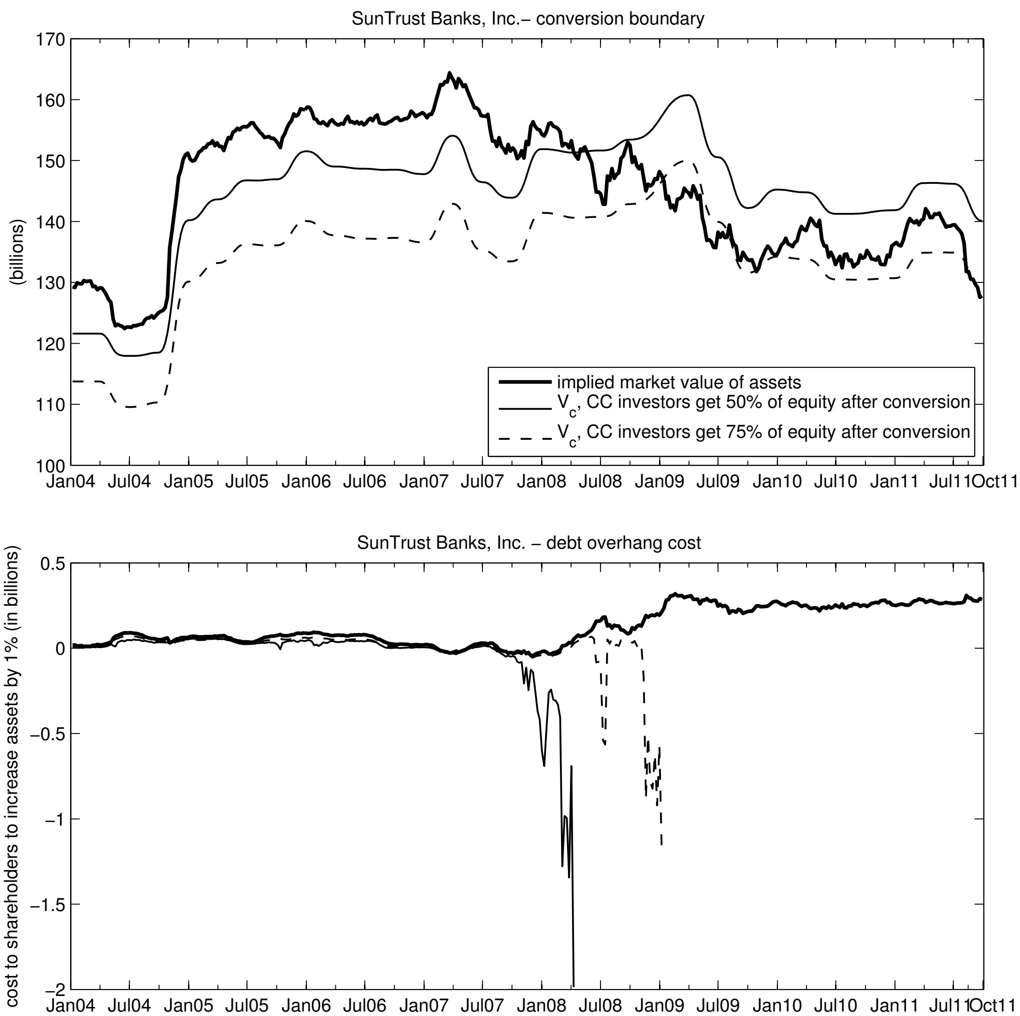

Figure 15: The top figure shows calibrated conversion boundaries for SunTrust at 50 percent and 75 percent dilution. The lower figure shows debt overhang costs without CoCos (heavy solid line) and with CoCos at 50 percent (thin solid line) and 75 percent dilution (dashed line).