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Safe Assets as Commodity Money

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Abstract

This paper presents a model in which safe assets are systemic because they are the medium of exchange for risky assets. Like commodity money, these assets are costly to produce and have some intrinsic value, resulting in (a) non-neutrality and (b) overproduction. Quantitatively, the welfare consequences of these inefficiencies depend on the costs of producing safe assets, which can be inferred from the equilibrium value of the liquidity premium. When the model is calibrated to plausible liquidity premia the resulting inefficiencies are not large.

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1 Introduction

In the study of the recent financial crisis, many analogies have been drawn between safe assets and money. A common view of the 2007-09 crisis is that it was prompted by a contraction in the effective supply of money-like assets, as various securities that were previously perceived as very safe and liquid were instead perceived as risky, consequently becoming illiquid (Gorton and Metrick (2012)).

The goal of this paper is to formalize the analogy between safe assets and money. In particular, we argue that, since the production of safe assets requires real resources and since safe assets carry coupon payments that are valued regardless of their use as a medium of exchange, the appropriate conceptual framework for understanding their properties is as commodity rather than fiat money. Our model suggests two main implications: (a) changes in the quantity of safe assets can have real effects on the quantity of trading (even absent nominal rigidities), and (b) there is overproduction of safe assets.

These inefficiencies associated with commodity money are often cited as grounds for the superiority of a fiat currency (Barro (1979); Ritter (1995), and Kiyotaki and Wright (1989)). In line with this criticism, our model implies that the use of safe assets as a medium of exchange in financial trading is inefficient. However, a quantitative interpretation of the model suggests that these inefficiencies are not large. In particular, we show that these inefficiencies are increasing in the costs of creating safe assets, which can be inferred from the liquidity premium. Given plausible estimates of the liquidity premium, our simulations suggest that large fluctuations in the quantity of safe assets are associated with only minor changes in welfare, and that there are only minuscule inefficiencies generated by the overproduction of safe assets.

A corollary of this analysis is that the contraction in the stock of safe assets during the 2007-09 crisis is unlikely to be the direct cause of its subsequent severity, at least not through the channels emphasized here. The quantitative interpretation of our model suggests that the contraction in the stock of safe assets was largely offset by an increase in the liquidity premium, rather than by a contraction in the quantity of trading. In fact, while the spike in liquidity premiums during the crisis was viewed by many as cause for alarm, our model suggests that this equilibrium adjustment had an important mitigating effect.

While there has been broad agreement in the literature that the analogy between safe assets and money is potentially useful, there has been some disagreement regarding the extent to which safe assets should be thought of as "nominal balances" or as "real balances" (M and $\frac{M}{p}$ respectively in standard notation). For example, Stein (2012) and Krishnamurthy and Vissing-Jorgensen (2012) consider money-in-the-utility-function models in which safe assets enter the utility function directly, analogously to $\frac{M}{p}$. In contrast, Rocheteau and Wright (2013), Midrigan and Philippon (2011), Hart and Zingales (2015) and Hart and Zingales (2011) consider models in which safe assets are the medium of exchange, analogously to nominal balances. Our paper clarifies the relationship between these two approaches.

This paper contributes to an emerging literature on the systemic importance of safe assets, including Caballero (2006), Caballero et al. (2008), Gourinchas and Jeanne (2012), Gorton and Ordonez (2013), and Dang et al. (2012) (among others). Similar to Rocheteau and Wright (2013), Shen and Yan (2014), Hart and Zingales (2011) and Hart and Zingales (2015), this paper contributes to the discussion by studying the money-like properties of safe assets. Most closely related are Hart and Zingales (2011) and Hart and Zingales (2015). These papers highlight that when safe assets have a transaction role, there is an oversupply of safe assets relative to the social optimum. In this paper, we apply this insight to an environment in which safe assets are used for facilitating trading in risky assets.

This paper is related to an extensive literature that studies pecuniary externalities in constrained environments, such as Geanakoplos and Polemarchakis (1986), Greenwald and Stiglitz (1986), Caballero and Krishnamurthy (2001), Lorenzoni (2008), Bianchi (2011), Farhi et al. (2009), Bengui (2013), Korinek (2011) and Eden (ming) (among others). It is a well-established principle that in the presence of binding constraints, a decentralized equilibrium may be inefficient due to inefficient price impacts on constrained agents. This paper relates this principle to the inefficiency of private money creation in an environment in which the cash-in-advance constraint is binding, and draws implications regarding excessive private creation of safe assets.

Our modeling approach is motivated by important insights from the New Monetarist view of liquidity (see Lagos et al. (2015) for a review). As illustrated by Lagos (2011), there is a theoretical equivalence between collateral assets and assets used as a medium of exchange. In our model, we assume that safe assets are used directly as a medium of exchange, building on this equivalence for the broader interpretation of the model. While our approach resembles a New Monetarist approach in some ways, there are also some differences. In particular, our model departs from the assumption of bilateral trading and assumes the presence of trading posts in which risky assets are exchanged for the safe asset. This precludes the possibility that traders meet by chance and choose to exchange one risky asset for another.[1] We emphasize that the aim of our paper is to apply and quantify insights from monetary theory to the liquidity properties of safe assets, rather than to contribute to the deep understanding of the foundations of the medium of exchange. For this purpose, we adopt a framework that abstracts from multiple equilibria and imposes the use of safe assets as a medium of exchange in risky assets.

[1]This trading structure appears realistic in the context of high-paced financial markets, in which the number of participants is large relative to the number of assets (though perhaps less so in the context of over-the-counter markets, as in Duffie et al. (2005)). In addition, it simplifies the analysis considerably, and generates results that are very much in line with the more carefully microfounded models in the monetary literature.

2 Stylized facts

Before discussing the model, it is useful to illustrate some empirical regularities that form the basis for the analogy between safe assets and money, particularly around the crisis. This section cites some suggestive evidence that safe assets provide liquidity services to the financial system, and that the contraction in safe assets was associated with an increase in liquidity premiums and a decline in trading volumes of illiquid assets.

Pozsar (2013) documents the critical role of safe assets like Treasury bills (t-bills), commercial paper (CP), repurchase agreements (repos), and others in institutional cash pools that are not fully counted in traditional money supply aggregates like M2. Because of limits on Federal Deposit Insurance Corporation (FDIC) insurance, holders of large quantities of cash cannot rely on transaction accounts to store their liquid and transaction funds without credit risk. Under their broader measure of money, classical M2 represents only about half of the total. However, among institutional and securities lender cash pools, non-M2 assets are about 80 percent of cash and cash equivalents. While short term government debt is a significant source of these non-depository safe assets, private actors, notably in shadow banking, meet additional demand for safe assets through securitization and collateral intermediation (Claessens et al. (2012)). During the financial crisis a significant fraction of these privately manufactured liquid safe assets ceased to be safe liquid assets (Gorton and Metrick (2012)) leading to significant increase in the price of liquidity.

Figure 1 shows percentage of yield given up by on-the-run U.S. Treasury holders over holding similar duration off-the-run securities. Since off-the-run U.S. Treasuries are already more liquid than many assets, this premium observed during the crisis is substantial given the actual liquidity difference. When considering less liquid assets, the effect is even more pronounced. Figure 2 shows the price series of government insured Sallie-Mae student loan pool backed bonds around the financial crisis. These bonds have a credit and interest rate risk very similar to U.S. Treasuries but are much less liquid. During the crisis these securities traded at about an 8 percent discount to par and figure 2 documents that the remaining risks do not explain this discount, suggesting illiquidity is the primary reason for depressed prices. Given these pronounced premia over otherwise identical less-liquid safe assets, liquid safe assets were in short supply in the financial crisis.

In addition to these large increases in liquidity premia in assets with no credit risk, some markets for assets with credit risk froze up entirely in the crisis. Hemmerdinger (2012) documents that auction-rate and short-put bond origination declined precipitously as a result of the crisis. Radde (2015) and Covitz et al. (2013) discuss the trading breakdown in repo, inter-bank lending, and asset-backed commercial paper markets. Given the timing coincident to so much asset market disorder, it is natural to wonder if safe assets have an important role in facilitating the efficient trading of illiquid assets and if the welfare losses from this illiquidity are large.

3 Model

This section presents a model in which safe assets are used as a medium of exchange in risky assets. The model features traders with different preferences towards an evolving state of nature. Safe assets are valued similarly by all agents. Though we will interpret the objects in our model as safe and risky assets, it is worth emphasizing that the model also provides a general treatment of economies in which an asset with real returns is used as the medium of exchange.

Time is discrete and indexed $t = 0, 1, ...$ There are $n > 2$ states of nature indexed $\omega = 1, ..., n$. States of nature occur with equal probabilities, and are drawn independently across time.

There are $n+1$ assets, consisting of n risky assets indexed $j = 1, .., n$, and one safe asset. The safe asset delivers 1 unit of the final good each period, regardless of the state. The risky asset indexed j delivers one unit of the final good in state $\omega = j$, and nothing otherwise.[2] The aggregate supplies of the risky assets are denoted $\{A_j\}_{j=1}^n$, and the aggregate supply of the safe asset is denoted A_s. Markets are incomplete in that all assets must be held in weakly positive quantities.

There is a unit measure of traders of each type. Types are indexed $i = 1, .., n$, corresponding to the states of nature. A trader of type i values consumption in state i more than in other states. Types are stochastic and negatively correlated across time: a trader of type i at time t draws his $t+1$ type from $\{1, .., n\} \setminus \{i\}$, where each type occurs with equal probability. The changes in types across time create a motive for trade in assets. The trader's preferences are described by:

$$U(\{c_t, \omega_t, i_t\}_{t=0}^{\infty}) = E_{\omega_t, i_t}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\left(1 + \theta\chi_{\omega_t = i_t}\right)\right] \tag{1}$$

where c_t is time t consumption, $u(\cdot)$ is an increasing and concave function satisfying the Inada conditions, $\beta \in (0, 1)$ is the discount factor, $\theta > 0$ is a preference parameter and $\chi_{\omega_t = i_t}$ is an indicator function that takes a value 1 if $\omega_t = i_t$, and 0 otherwise.[3] Traders are informed of their types one period in advance: information on i_{t+1} arrives at time t. Based on this information, the traders may want to rebalance their asset portfolios. A trader's "favorite" risky asset corresponds to the asset a_j such that $i_{t+1} = j$; all other risky assets will be referred to as "nonfavorite".

We assume that risky assets are less liquid than safe assets, in the sense that the markets for directly trading one risky asset for another are too

[2] To prevent agents from creating safe assets by bundling the n risky assets, it can be assumed that there is an additional state, $\omega = n + 1$, which occurs with negligible probability. Since there is no risky asset that delivers in state $\omega = n + 1$, a portfolio of risky assets cannot be used to create a safe asset.

[3] This specification of preferences is borrowed from the liquidity literature; see, for example, Diamond and Dybvig (1983).

thin and do not open. Instead, trading takes place in n markets in which risky assets are traded for the safe asset. Starr (2008) shows these money-good bilateral markets can lead to the exclusion of good-good markets if bilateral markets are costly to open. Hart and Zingales (2015) offer some microfoundations for the emergence of safe assets as the medium of exchange.

Traders may enter each market only once during the trading period (either as buyers or sellers). The safe asset therefore has a money-like quality, in that it is assumed necessary for trading purposes. The trader allocates a fraction γ_j of his safe assets for the purpose of buying risky assets of type j. The price of risky assets of type j (in terms of safe assets) is denoted p_j. It is assumed for simplicity that dividends cannot be used for trading.[4]

Denote the trader's holdings of assets of type j by a_j, and his holdings of safe assets by a_s. We assume that, in addition to dividends, each agent receives a constant endowment of size e. In recursive form, the trader's problem can be written as follows:

$$V(\{a_j\}_{j=1}^n, a_s, \omega, i, i', \{p_j\}, e) \tag{2}$$

$$= \max_{\{a'_j\}_{j=1}^n, a'_s} u(c)(1 + \theta\chi_{\omega=i}) + \beta E_{\omega', i''}(V(\{a'_j\}_{j=1}^n, a'_s, \omega', i', i'', \{p'_j\}, e'))$$

s.t.

$$c = \sum_{j=1}^n a_j \chi_{\omega=i} + a_s + e \tag{3}$$

$$\sum_{j=1}^n \gamma_j \leq 1 \tag{4}$$

$$\gamma_j \geq 0 \tag{5}$$

$$p_j(a'_j - a_j) \leq \gamma_j a_s \tag{6}$$

[4]Alternatively, we could also assume dividends are paid out after the markets close on holdings before markets open. In this sense, the period in our model where markets are open the ex-dividend date.

$$a'_s, a'_j \geq 0 \tag{7}$$

$$a'_s - a_s = \sum_{j=1}^{n} p_j (a_j - a'_j) \tag{8}$$

The trader's value function, $V(\cdot)$, is a function of $2n+5$ state variables: the initial holdings of each of the n risky assets ($\{a_j\}_{j=1}^n$), their respective market prices ($\{p_j\}$), the initial holding of the safe asset (a_s), the state of nature (ω), the agent's endowment (e), the agent's type, i, and the agent's type next period, i'. The trader consumes dividends and his endowment, and re-optimizes his portfolio by choosing $\{a'_j\}_{j=1}^n$ and a'_s. During the trading period, the trader can choose to allocate a fraction γ_j of his safe assets for the purpose of buying the risky asset of type j. Equation 6 states that the market value of assets of type j purchased by the trader must be less than or equal to the amount of safe assets that he allocates for buying in market j.

Equation 7 states that traders cannot hold negative amounts of assets. Finally, equation 8 describes the evolution of the agent's safe asset holding: the left side is the net increase in safe assets, and the right hand side is net the value of sales of risky assets.

Note that, in this framework, safe assets are valued independently from their use as a medium of exchange: in addition to the liquidity services that they provide (equation 6), they deliver returns in the form of dividends (equation 3). The dividends associated with safe assets correspond, in a standard monetary framework, to interest payments on money. In a model of commodity money, these returns correspond to the consumption value of the commodity (e.g., the utility from looking at shiny gold).

Welfare and efficiency. As a benchmark, it is useful to consider the efficient allocation, defined as the solution to the following problem:

$$\max_{a_{i,s},a_{i,j}} \sum_{i=1}^{n}\sum_{i'=1}^{n} V(\{a_{i,j}\}_{j=1}^{n}, a_{i,s}, \omega, i, i', e) \tag{9}$$

s.t. $\sum_{i=1}^{n} a_{i,j} = A_j$ and $\sum_{i=1}^{n} a_{i,s} = A^s$.

In other words, the efficient allocation is achieved by a planner who assigns equal Pareto weights to all agents and can allocate assets to traders depending on their type. The efficient allocation allocates more consumption to agents with $i = \omega$. Specifically, it equalizes the marginal utility of consumption across agents:

$$u'(\underline{c}^*) = (1+\theta)u'(\bar{c}^*) \tag{10}$$

where $\underline{c}^*$ is the allocation of consumption to agents with $i \neq \omega$ and $\bar{c}^*$ is the allocation of consumption to agents with $i = \omega$.

For simplicity, we restrict attention to the case in which the above allocation cannot be achieved without all agents holding positive amounts of each risky asset. Formally we assume that:

$$e + A_s < \underline{c}^* \tag{11}$$

The left hand side is the supply of safe assets, together with the endowment. In a symmetric equilibrium, this will correspond to the non-state-contingent dividends of all agents. If agents do not hold a positive amount of their nonfavorite risky assets, this will be their consumption in their nonfavorite state. The inequality states that in order to implement the efficient allocation all agents must hold positive quantities of all risky assets.

For the purpose of welfare analysis, social welfare is given by equation 9.

Equilibrium. To define an equilibrium, index the set of traders by $x \in X$, and let the set of trader x's state variables be given by:

$$S_x = (\{a_{x,j}\}_{j=1}^n, a_{x,s}, \omega, i_x, i'_x, \{p_j\}_{j=1}^n, e)$$

A recursive equilibrium of this economy is given by policy functions $a'_{x,j}(S_x)$, $a_{x,s}(S_x)'$, $\gamma_j(S_x)$ and prices $p'_j(\{S_x\}_{x\in X})$ that jointly solve the trader's optimization problem and the market clearing conditions:

$$\int_X (a'_{x,j}(S_x) - a_{x,j})dx = \int_X (a'_{x,s}(S_x) - a_{x,s})dx = 0 \tag{12}$$

The market clearing conditions state that the demand for each asset must equal the supply of each asset; since there is no change in the aggregate supply of assets, the aggregate changes in asset positions must be equal to 0.

Of course, by symmetry, there is an equilibrium in which $p_j = p_i$ for all i and j. We will focus on that symmetric equilibrium and sometimes omit the subscript j ($p = p_j$). Further, we will focus on a steady state equilibrium, in which prices are constant across time ($p = p'$).

In a symmetric steady state, consumption depends on whether or not the trader's favorite state of nature has been realized. Denote equilibrium consumption when $i = \omega$ by $\bar{c}$ and equilibrium consumption when $i \neq \omega$ by $\underline{c}$.

The following proposition characterizes the symmetric steady state equilibrium as a function of safe asset supply. For simplicity, we hold the efficient allocation constant by assuming that $E = e + A^s$ is constant (thus, a change in A_s does not change the aggregate amount of goods in each state[5]):

Proposition 1 *Assume that $e + A^s = E$. Holding E constant:*

1. *For A^s sufficiently large, the steady state implements the efficient allo-*

[5]That is, in a comparative statics sense, at this point A_s is exogenously determined. The next section will explore the consequences of exogenous and costly changes in A_s.

cation, and prices are given by $p_j = \frac{1}{n}$.

2. *Otherwise, the efficient allocation is not implemented, and welfare is increasing in* A^s. *Buyers are constrained:* $(1+\theta)u'(\bar{c}) > u'(\underline{c})$. *Safe assets are associated with a liquidity premium, reflected in the fact that* $p_j < \frac{1}{n}$. *Further,* p_j *and* $(u'(\underline{c}) - (1+\theta)u'(\bar{c}))$ *are increasing in* A^s.

The proof is in Appendix A.

Proposition 1 establishes that when safe assets are scarce, increasing the supply of safe assets increases trading volume and therefore equilibrium welfare. The next section establishes that despite these benefits, there may be excessive resources spent on the private creation of safe assets.

As an aside, it is worth emphasizing that when safe assets are sufficiently abundant, the equilibrium implements the efficient allocation. Thus, this model illustrates a potential advantage of commodity money (or safe assets) over fiat money. While the literature suggests that the implementation of the Friedman rule (Friedman (1969)) may not be feasible in an economy in which fiat money is used as a medium of exchange (see **?**, Wilson (1979), Cole and Kocherlakota (1998) and Ireland (2003)), our model illustrates that it is possible to satiate agents with liquidity given a form of commodity money that is sufficiently abundant. Safe assets that pay dividends can be used in the place of interest-bearing money to satiate the economy with liquidity, effectively implementing the Friedman rule.

4 Creation of safe assets

This section extends the model to study the welfare implications of costly private creation of safe assets. Consider a simple framework in which each trader has a costly technology that transforms his endowment stream $\{e\}_{t=0}^{\infty}$ into additional safe assets. For simplicity, assume that at $t = -1$ (before trading begins), each agent can sacrifice a fraction γ of his endowment stream to create $\epsilon\gamma$ safe assets (where $\epsilon, \gamma \leq 1$). The trader's problem at $t = -1$ can be written as:

$$\max_{\gamma} E_{\omega}(V(\{a_{j,0}\}_{j=1}^{n}, a_{s,0} + \epsilon\gamma e, \omega, i, i', \{p_j\}, (1-\gamma)e)) \tag{13}$$

We assume an interior solution in which traders transform part of their illiquid endowment into safe assets ($\gamma \in (0,1)$). The trader's first order condition with respect to safe asset creation is:

$$\frac{\partial V}{\partial e} = \epsilon \frac{\partial V}{\partial a_s} \tag{14}$$

To characterize the solution to the above equation, it is useful to distinguish between two cases. When $\epsilon = 1$, it is costless to transform e into a_s; thus, in equilibrium, the marginal benefit of a_s is the same as the marginal benefit of e. In other words, there is no liquidity premium. In this case, by proposition 1, the efficient allocation is implemented and the economy is satiated with liquidity.

If, instead, $\epsilon < 1$, then the above equation can have a solution only when there is a liquidity premium, e.g. only when safe assets are scarce.[6] Absent a positive liquidity premium, the trader would be indifferent between safe assets and e, and since $\epsilon < 1$, the trader would optimally set $\gamma = 0$. An interior solution therefore implies a positive liquidity premium.

The above condition further illustrates how the cost of creating liquid safe assets, ϵ, can be inferred from the liquidity premium. Intuitively, the marginal benefit of liquid safe assets ($\frac{\partial V}{\partial a_s}$) is equal to the marginal return to safe assets ($\frac{\partial V}{\partial e}$), plus the liquidity benefits which are valued at the liquidity premium. In the quantitative interpretation of the model in the next section, we will utilize the above relation to calibrate ϵ.

To evaluate the efficiency of the decentralized solution, consider next the

[6] This is analogous to the finding that private money creation is optimal only when the Friedman rule is not implemented, and there is some private seigniorage revenue associated with money creation.

problem of a social planner that can dictate to traders how many safe assets to create. The planner's $t = -1$ problem can be written as:

$$\max_{\gamma} E_{\omega}(\sum_{i,i'=1}^{n} V(\{a_{j,0}\}_{j=1}^{n}, a_{s,0} + \epsilon\gamma e, \omega, i, i', \{p_j\}, (1-\gamma)e)) \qquad (15)$$

s.t. $p_j = p(A_s)$, where $p(\cdot)$ is a function mapping the aggregate supply of safe assets to equilibrium prices (p). The difference between the planner's problem and the trader's (decentralized) problem is that the planner internalizes that an increase in the supply of safe assets may lead to a decline in the liquidity premium (see Proposition 1).

The planner's first order condition is:

$$\frac{\partial V}{\partial e} = \epsilon \frac{\partial V}{\partial A_s} = \epsilon(\frac{\partial V}{\partial a_s} + \frac{\partial V}{\partial p}\frac{\partial p}{\partial A_s}) \qquad (16)$$

Note that the social return to safe assets ($\frac{\partial V}{\partial A_s}$) is weakly lower than the private return to safe assets ($\frac{\partial V}{\partial a_s}$). When there are sufficient safe assets, there is no liquidity premium associated with safe assets and the relative price of risky and safe assets simply reflects the difference in expected asset returns. In this case, $\frac{\partial p}{\partial A_s} = 0$ and $\frac{\partial V}{\partial A_s} = \frac{\partial V}{\partial a_s}$.

Otherwise, buyers of risky assets are constrained and $\frac{\partial V}{\partial p} < 0$. It therefore follows that $\frac{\partial V}{\partial A_s} < \frac{\partial V}{\partial a_s}$, and there is excessive private creation of safe assets relative to the social optimum. This result is a consequence of a pecuniary externality: when traders create safe assets, they do not internalize that an increase in the supply of safe assets reduces the liquidity premium (raises p). As buyers are constrained in each market, the increase in p worsens equilibrium welfare.

It is useful to emphasize that there is no distortion when $\epsilon = 1$. In this case, the supply of safe assets is such that there is no liquidity premium, and the economy is satiated with safe assets. In this case, $\frac{\partial p}{\partial A_s} = 0$ and the planner's solution coincides with the decentralized equilibrium allocation.

This benchmark suggests that when $\epsilon \to 1$, the inefficiency due to excessive private creation of safe assets approaches 0. In the following section, we will argue that this is the empirically relevant case.

5 Quantitative interpretation

This section proposes a quantitative interpretation of the model, with two goals in mind. The first is to quantify the degree of non-neutrality of safe assets, or the real effects resulting from a large contraction in the stock of safe assets. The second is to quantify the inefficiency due to excessive private creation of safe assets.

The baseline model in Section 2 has five parameters: n, θ, A_r, A_s and e. To calibrate the model, we normalize $A_r = 1$ and fix the number of states (n) between 2 and 200,000. As it turns out, our results are not very sensitive to the choice of n. Given A_r and n, we calibrate the remaining parameters θ, A_s and e to match the following targets:

1. We match stock market trading volume or the "velocity" of safe assets. In the model, assuming that A_s is below the saturation threshold level, A_s trades hands in every trading round, in purchasing $p(\bar{c} - \underline{c})$. Note that A_s is the quantity of safe assets, not their value. The net present value of safe assets is $\sum_{t=0}^{\infty} \beta^t A_s = \frac{A_s}{1-\beta}$. We then define the trading volume as a fraction of GDP as:

$$tv_s \equiv \frac{A_s}{(1-\beta)(A_r + e + A_s)} \tag{17}$$

   We target a pre-crisis $tv_s = 3.0$ in line with Singh (2011).

2. The relative liquidity premium is defined as $\frac{\frac{1}{n}}{p} - 1$. To see this, note that when the liquidity premium is 0, $p = \frac{1}{n}$. If p is lower than $\frac{1}{n}$, it is valued at a positive premium. We target a liquidity premium of 2 percent, in the range of values that Elmer (1999) estimates as

the value created by loan securitization as a percent of the original balance.[7] Alternatively, we could take as a liquidity premium the costs of creating an Agency Mortgage-Backed-Securities (MBS). We estimate these costs by adding the Fannie Mae and Freddie Mac guarantee fees (Federal Housing Finance Agency (2014)) to the yield improvement estimates of Sanders (2005) which amount to 0.5 percent of principal. We choose the larger of the two estimates to give the liquidity channel a greater chance to have an effect.

3. The share of safe assets out of total assets ($\frac{e+A_s}{A_r+A_s+e}$) is targeted at $\frac{1}{3}$ to match Gorton et al. (2010).

Using the equilibrium relationship in equation 14, we can then calibrate ϵ as $\epsilon = \frac{\partial V}{\partial e} / \frac{\partial V}{\partial a_s}$.

The equilibrium is characterized by three endogenous variables: p, $\bar{c}$ and $\underline{c}$. To solve for the equilibrium, we therefore rely on three equilibrium conditions. The first is the goods market clearing condition: $\bar{c}+(n-1)\underline{c} = n(A_r+A_s+e)$. The second is an equilibrium condition stating that constrained traders spend all of their safe assets on purchasing their favorite risky asset. The third condition is obtained from the indifference of traders with respect to selling their nonfavorite risky assets (equation 23 in the Appendix A). Appendix B provides the details of the equations used for the simulation.

As a first exercise, we consider the implications of a 30 percent decline in liquid safe assets, which is roughly in line with the share of asset backed securities and mortgage backed securities in safe assets in Gorton et al. (2010). To put the welfare consequences of a liquidity crisis in perspective we also calculate a version of the model under autarchy (no-trade, constant symmetric portfolios).

Figure 4 shows the basic result: both in absolute terms and relative to the gains from trade, the losses from the liquidity crisis are small (regardless

[7] He estimates a range of 1.60-3.00 percent.

of the choice of n). For an $n = 2$ the welfare losses of the liquidity crisis are 14 percent of the gains over autarchy but as n increases they rapidly become a de minimis fraction of the gains from trade.

We next consider the welfare loses from the overproduction of safe sassets. Figure 4 shows that the losses are quantitatively small relative to autarchy and the liquidity crisis. Normalizing the welfare losses by the liquidity crisis losses indicates they are between 0.3 to 1.4 percent of crisis losses.

This quantitative interpretation highlights two striking features. First, safe assets are approximately a neutral medium of exchange. Large changes in the stock of safe assets lead to only minor changes in equilibrium allocations. This is similar to fiat money in an environment in which money is neutral. Second, while there is overproduction of safe assets, the inefficiency is not large since the cost of producing safe assets is small. This too is similar to fiat money, which is costless to produce. Furthermore, the low cost of production results in an equilibrium allocation in which the economy is nearly satiated with liquidity, similar to a monetary economy in which the Friedman rule is implemented.

6 Conclusion

This paper makes several steps towards understanding the properties of safe assets as a medium of exchange in financial trading. We show that while, conceptually, the appropriate way to think about safe assets is as a form of commodity money, in practical terms, they are quantitatively indistinguishable from fiat money in a model in which money is neutral. In particular, we illustrate numerically that large changes in the quantity of safe assets result in only small changes in equilibrium allocations and that, given empirically relevant measures of the liquidity premium, safe assets appear to be nearly costless to produce.

In fact, our analysis suggests that — perhaps unlike traditional forms of commodity money - a monetary system that relies on safe assets as a medium of exchange is relatively efficient. The fact that safe assets are valued independently from their use as a medium of exchange means that, when sufficiently abundant, the economy can be satiated with liquidity.

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A Proof of Proposition 1

To sustain an efficient equilibrium, it must be the case that buyers are indifferent with respect to purchasing an additional unit of their favorite risky asset. For this to be the case, we must have that:

$$\frac{n-1}{n}u'(\underline{c}^*) + \frac{1+\theta}{n}u'(\bar{c}^*) + \beta\frac{\partial V'}{\partial a^s} = \frac{1}{p}\frac{1+\theta}{n}u'(\bar{c}^*) + \beta\frac{\partial V'}{\partial a^s} \tag{18}$$

The left hand side is the return from holding (and keeping) a safe asset. Next period, the safe asset delivers a dividend with certainty; the marginal valuation of that dividend is $u'(\underline{c}^*)$ whenever the realized state is $\omega \neq i$, which occurs with probability $\frac{n-1}{n}$, and $(1+\theta)u'(\bar{c}^*)$ otherwise (with probability $\frac{1}{n}$). The continuation value from holding the safe asset is $\beta\frac{\partial V'}{\partial a_s}$.

The right hand side is the return from using the safe asset to buy $\frac{1}{p}$ "favorite" risky assets, and then selling these assets in the next period in exchange for a safe assets. A favorite risky asset delivers a dividend only when the favorite state is realized, which occurs with probability $\frac{1}{n}$. In that case, the marginal valuation of the dividend is $(1+\theta)u'(\bar{c}^*)$. The trader can then sell his $\frac{1}{p}$ risky assets for a safe asset, which delivers a continuation value of $\beta\frac{\partial V'}{\partial a_s}$. Note that the term $\beta\frac{\partial V'}{\partial a_s}$ cancels out from both sides of equation 18. Further, note that, using equation 10, equation 18 can be rewritten as:

$$u'(\underline{c}^*) = \frac{1}{p}\frac{1}{n}u'(\underline{c}^*) \Rightarrow p = \frac{1}{n} \tag{19}$$

Note that, in this case, assets are priced according to their expected value; there is no liquidity premium associated with safe assets. It is easy to verify that, in this case, sellers of nonfavorite risky assets are indifferent with respect to selling their nonfavorite asset in exchange for a safe asset. To see this, note that the seller's indifference condition is given by:

$$\frac{n-1}{n}u'(\underline{c}^*) + \frac{1+\theta}{n}u'(\bar{c}^*) + \beta\frac{\partial V'}{\partial a^s} = \frac{1}{p}\frac{1}{n}u'(\underline{c}^*) + \beta\frac{\partial V'}{\partial a^s} \tag{20}$$

The left hand side is the marginal benefit from selling $\frac{1}{p}$ nonfavorite risky assets in exchange for a safe asset, and the right hand side is the benefit from holding $\frac{1}{p}$ nonfavorite risky assets (and selling them in exchange for a safe asset in the next period). Given equation 10, this indifference condition is identical to equation 18.

To sustain this equilibrium, the supply of safe assets must be such that the demand for risky assets can sustain the price $p = \frac{1}{n}$. In a symmetric steady state, buyers in the market for risky asset j consist of traders whose favorite asset is j (and their favorite asset in the previous period was not j), and sellers in the market consist of those whose favorite asset in the previous period was j (and their favorite asset in the current period is not j). In each market, buyers allocate at most A_s units of the safe asset towards the purchase of their favorite risky assets. Note that sellers hold $\bar{c} - \underline{c}$ units of asset j. Thus, in equilibrium, it must be the case that:

$$A^s \geq p(\bar{c} - \underline{c}) = \frac{1}{n}(\bar{c}^* - \underline{c}^*) \tag{21}$$

If this inequality is violated, buyers do not hold enough purchasing power to guarantee efficient holdings of their favorite risky assets. Thus, buyers are constrained and $(1+\theta)u'(\bar{c}) > u'(\underline{c})$. The price p is such that sellers are indifferent with respect to selling an additional unit of their nonfavorite asset in exchange for a safe asset. The price p must satisfy the market clearing condition:

$$A^s = p(\bar{c} - \underline{c}) \Rightarrow p = \frac{A^s}{\bar{c} - \underline{c}} \tag{22}$$

This equation defines a decreasing relationship between p and $\bar{c} - \underline{c}$.

$$\begin{gathered}\frac{\partial V}{\partial a^s} = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V'}{\partial a^s} = \\ \frac{1}{p}(\frac{1}{n}u'(\underline{c}) + \beta E(\frac{\partial V'}{\partial a_j}|i) = \frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}}\end{gathered} \tag{23}$$

Where $E(\frac{\partial V'}{\partial a_j}|i)$ is the expected marginal valuation of holding asset j given type $i \neq j$. We claim that this equation defines an increasing relationship between p and $\bar{c} - \underline{c}$. When $\bar{c} - \underline{c}$ is higher, $\underline{c}$ is lower, implying a higher marginal utility of consumption in the nonfavorite state. Given a higher marginal utility of consumption, sellers require a higher price p in exchange for an asset that delivers a return in their nonfavorite state. Thus, the seller's indifference condition defines an increasing relationship between p and $\bar{c} - \underline{c}$.

Equation 22 defines a decreasing relationship between p and $\bar{c} - \underline{c}$, while 23 defines an increasing relationship between p and $\bar{c} - \underline{c}$. Thus, the equilibrium is generated by their unique intersection. An increase in A^s shifts the curve defined by equation 23 upwards, resulting in an equilibrium with higher p and higher $\bar{c} - \underline{c}$. This concludes the proof.

B Simulation procedure

Note that buyers are constrained and $(1 + \theta)u'(\bar{c}) > u'(\underline{c})$. The price p is such that sellers are indifferent with respect to selling an additional unit of their nonfavorite asset in exchange for a safe asset. The price p must satisfy the market clearing condition:

$$A^s = p(\bar{c} - \underline{c}) \Rightarrow p = \frac{A^s}{\bar{c} - \underline{c}} \tag{24}$$

This equation defines a decreasing relationship between p and $\bar{c} - \underline{c}$.

Indifference between the safe asset and selling the nonfavorite risky asset at price p requires that:

$$\frac{\partial V}{\partial a^s} = \frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}} \tag{25}$$

Note that $\frac{\partial V}{\partial a^s}$ is given by the following expression, which is the expected marginal utility of the safe asset in the next period plus its discounted continuation value:

$$\frac{\partial V}{\partial a^s} = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V'}{\partial a^s} \tag{26}$$

Note that $\frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}}$ is given by:

$$\frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}} = \frac{1}{p}(\frac{1}{n}u'(\underline{c}) + \beta E(\frac{\partial V'}{\partial a_j}|i)$$

Where $E(\frac{\partial V'}{\partial a_j}|i)$ is the expected marginal valuation at time t of holding asset j at $t+2$, given type $i \neq j$ at $t+1$.

This requires some manipulation to be in workable format:

Note that:

$$\frac{\partial V}{\partial e} = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V}{\partial e} \tag{27}$$

$$\frac{\partial V}{\partial e} = \frac{1}{1-\beta}(\frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c})) \tag{28}$$

Note that:

$$E(\frac{\partial V'}{\partial a_j}|i) = \frac{1}{n-1}\frac{\partial V}{\partial a_i} + \frac{n-2}{n-1}\frac{\partial V}{\partial a_{j\neq i}} \tag{29}$$

The trader's next period's type is chosen out of $n-1$ types that do not include his current type. With probability $\frac{1}{n-1}$, the current nonfavorite asset becomes next period's favorite asset; with probability $\frac{n-2}{n-1}$ it remains nonfavorite.

The marginal valuation of a favorite asset is given by:

$$\frac{\partial V}{\partial a_i} = \frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V}{\partial a_{j\neq i}} \tag{30}$$

Combining the two:

$$E(\frac{\partial V}{\partial a_j}|i) = \frac{1}{n-1}(\frac{1+\theta}{n}u'(\bar{c}) + \beta E(\frac{\partial V}{\partial a_{j\neq i}})) + \frac{n-2}{n-1}\frac{\partial V}{\partial a_{j\neq i}} \tag{31}$$

$$\Rightarrow E(\frac{\partial V}{\partial a_j}|i) = \frac{1+\theta}{n(n-1)}u'(\bar{c}) + \frac{\beta+n-2}{n-1}E(\frac{\partial V}{\partial a_{j\neq i}}) \tag{32}$$

Using the second line of equation 26,

$$\frac{1}{n}u'(\underline{c}) + \beta E(\frac{\partial V'}{\partial a_j}|i) = \frac{\partial V}{\partial a_{j\neq i}}$$

Combining:

$$\frac{1}{n}u'(\underline{c}) + \beta(\frac{1+\theta}{n(n-1)}u'(\bar{c}) + \frac{\beta+n-2}{n-1}E(\frac{\partial V}{\partial a_{j\neq i}})) = \frac{\partial V}{\partial a_{j\neq i}}$$

$$\frac{1}{n}u'(\underline{c}) + \beta\frac{1+\theta}{n(n-1)}u'(\bar{c}) + \beta\frac{\beta+n-2}{n-1}E(\frac{\partial V}{\partial a_{j\neq i}}) = \frac{\partial V}{\partial a_{j\neq i}}$$

$$\frac{1}{n}u'(\underline{c}) + \beta\frac{1+\theta}{n(n-1)}u'(\bar{c}) = \frac{\partial V}{\partial a_{j\neq i}}(1-\beta\frac{\beta+n-2}{n-1})$$

$$\frac{\frac{1}{n}u'(\underline{c}) + \beta\frac{1+\theta}{n(n-1)}u'(\bar{c})}{1-\beta\frac{\beta+n-2}{n-1}} = \frac{\partial V}{\partial a_{j\neq i}} \tag{33}$$

To calculate $\frac{\partial V}{\partial a_s}$ using the first line of equation 23, we need to calculate $\frac{\partial V'}{\partial a_s}$. Note that $\frac{\partial V'}{\partial a_s} \neq \frac{\partial V}{\partial a_s}$, because the value of holding a unit of a_s at the beginning of a period is different from the value of holding a unit of a_s at the end of the period, after trading concludes. A safe asset at the beginning of the period can be used to purchase $\frac{1}{p}$ units of the favorite risky asset. In the next period, it becomes a nonfavorite risky asset:

$$\frac{\partial V'}{\partial a_s} = \frac{1}{p}(\frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V}{\partial a_{i\neq j}}) \tag{34}$$

The first line of equation 23 then yields:

$$\frac{\partial V}{\partial a_s} = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c}) + \beta(\frac{1}{p}(\frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V}{\partial a_{i\neq j}})) \tag{35}$$

Then, equation 23 is an equation in $\bar{c}$ and $\underline{c}$ (and parameters):

$$\frac{\partial V}{\partial a^s} = \frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}} \Rightarrow \tag{36}$$

$$\frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}} = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c}) + \beta(\frac{1}{p}(\frac{1+\theta}{n}u'(\bar{c}) + \beta\frac{\partial V}{\partial a_{i\neq j}})) \tag{37}$$

$$\frac{1}{p}\frac{\partial V}{\partial a_{j\neq i}}(1-\beta^2) = \frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c})(1+\frac{\beta}{p}) \tag{38}$$

$$\frac{\partial V}{\partial a_{j\neq i}} = \frac{p\frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c})(p+\beta)}{1-\beta^2} \tag{39}$$

Combining with equation 33:

$$\frac{p\frac{n-1}{n}u'(\underline{c}) + \frac{1+\theta}{n}u'(\bar{c})(p+\beta)}{1-\beta^2} = \frac{\frac{1}{n}u'(\underline{c}) + \beta\frac{1+\theta}{n(n-1)}u'(\bar{c})}{1-\beta\frac{\beta+n-2}{n-1}} \tag{40}$$

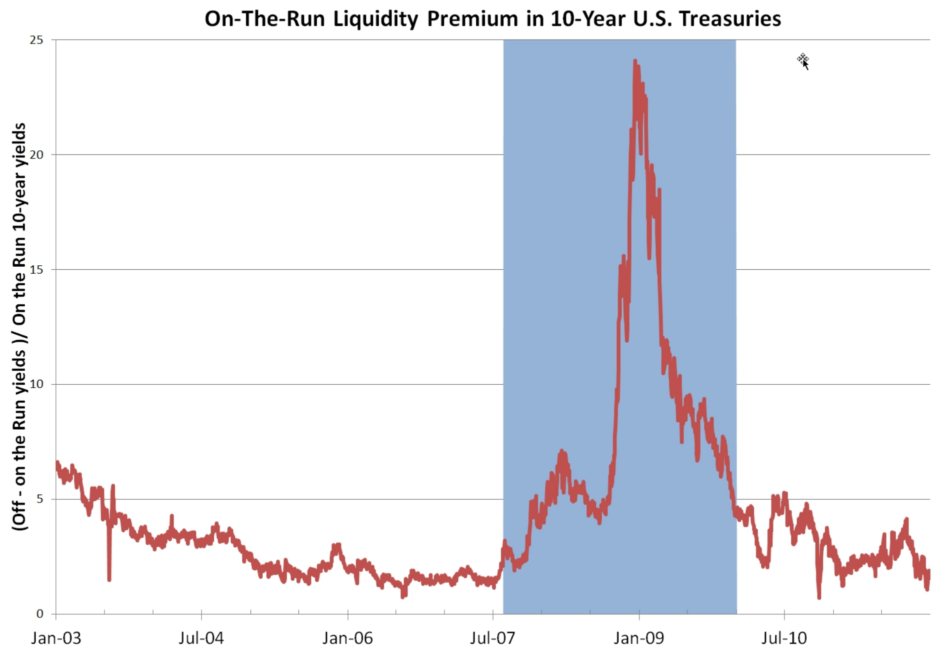
Together with the market clearing condition and $A_s = p(\bar{c} - \underline{c})$ we have three equations in three unknowns (p, $\bar{c}$ and $\underline{c}$). Because the calibration of turnover and velocity are to flows measured in years we choose $\beta = 0.97$ corresponding to an annual discount rate. For each n we choose a θ and fraction of all safe assets ($Y_{safe} \equiv A_s + e$) that are liquid ($\frac{A_s}{Y_{safe}} \equiv \bar{s}$ to satisfy the pre-crisis calibration requirements. The calibration is simplified because $\bar{s}$ does not depend on n or θ.

$$tv_s = \frac{A_s}{(1-\beta)\cdot Y} = \frac{\bar{s}\cdot Y_{safe}}{(1-\beta)\cdot Y} \rightarrow \bar{s} = \frac{tv_s(1-\beta)Y}{Y_{safe}} \tag{41}$$

$$= \frac{3.0\cdot(0.03)\cdot 1.5}{0.5} = 0.27$$

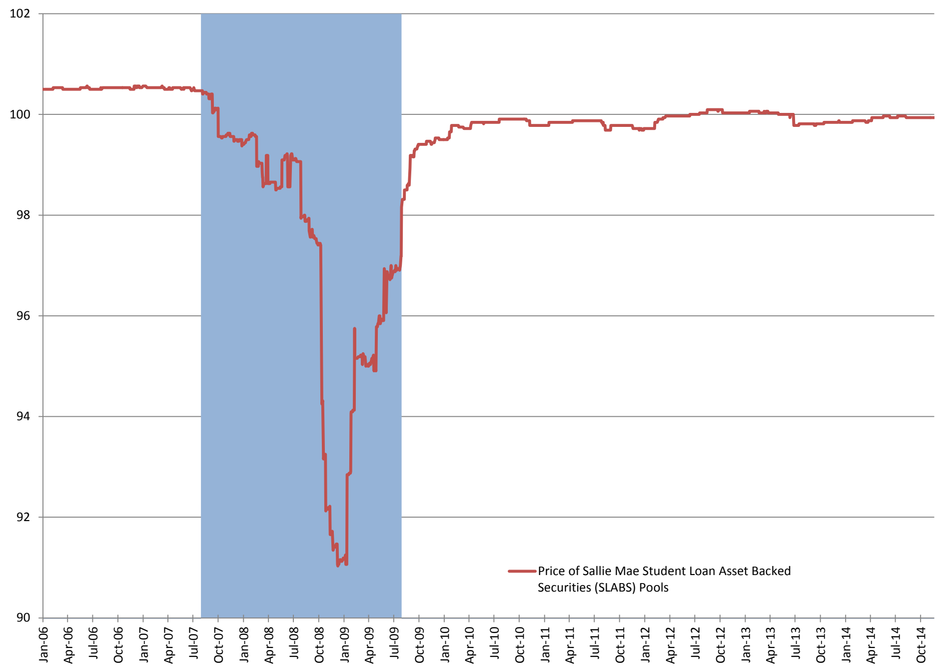
Conditional on $\bar{s}$ and the other parameters, we choose θ to hit the target liquidity premium of 0.02. Figure 5 shows that this relationship between n and θ is nearly linear ($\theta \simeq 0.105\cdot n$).

Figure 1: In the 2007-09 financial crisis even generally liquid securities like off-the-run Treasuries charged a substantial liquidity premium



Source: Grkaynak et al. (2007) and the Board of Governors (2015b)
The liquidity premium is the market implied yield difference in on-the-run and off-the-run Treasuries. It is constructed by bootstrapping two yield curves, one of off-the-run Treasuries (in the manner of Gurkaynak, Sack, Wright (2006)) and one of on-the-run Treasuries (from U.S. Treasury data) and taking their difference at the ten year point. Since these two securities differ only in their liquidity this difference represents the additional yield charged by market participants for holding the less liquid off-the-run securities. We then normalize this spread by the prevailing on-the-run yield to assess how much of the credit-riskless yield on-the-run bondholders give up for superior liquidity. The blue shading indicates the dates of the recent U.S. financial crisis from August 2007 when BNP Paribas halted redemption on three S.I.V.s to December 2009 when the U.S. Treasury Department awarded TARP funds to General Motors and Chrysler.

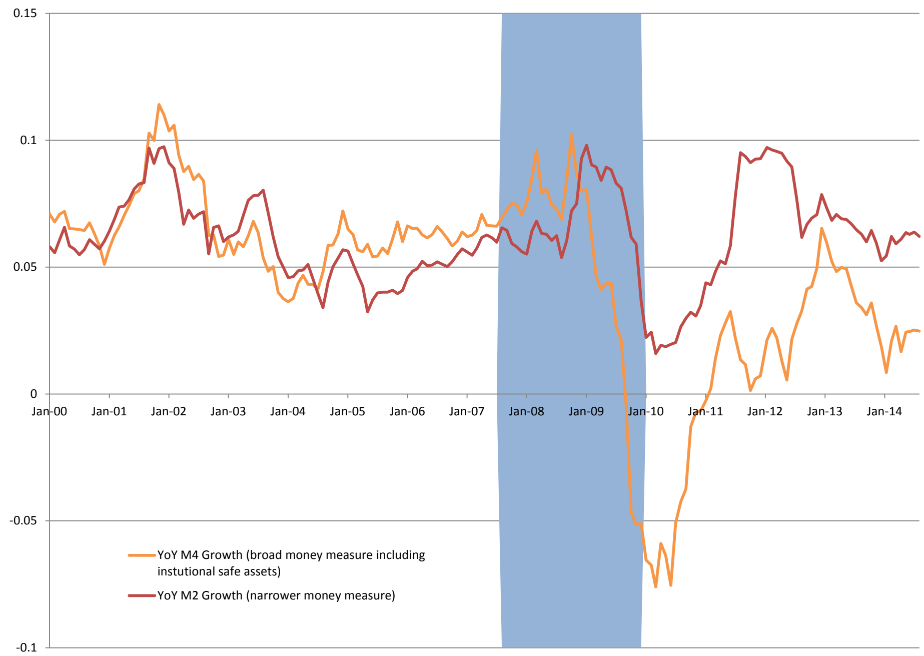
Figure 2: During the 2007-09 financial crisis the liquidity premium on riskless securities rose substantially



Source: Roy (2015)

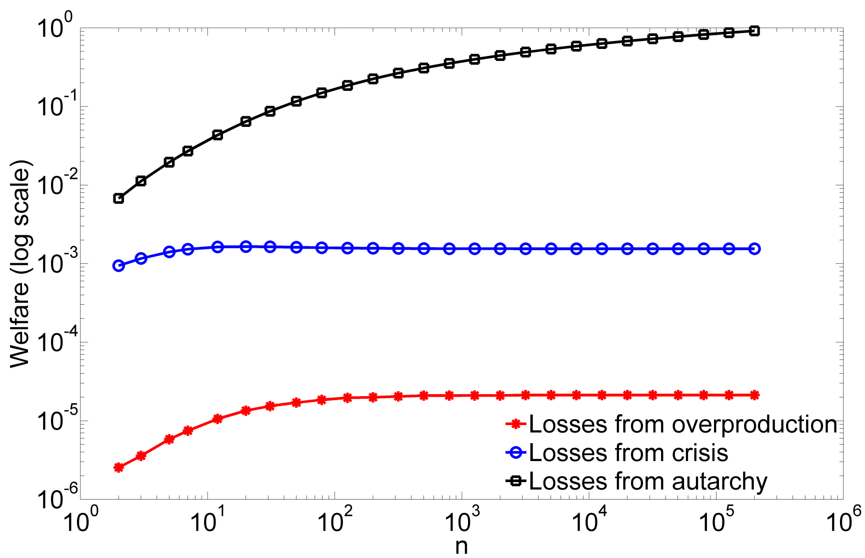
Roy (2015) shows that government-backed (with no credit risk) pools of student loans issued by Sallie Mae traded at a substantial discount to US Treasuries due to their illiquidity. The blue shading indicates the dates of the recent U.S. financial crisis from August 2007 when BNP Paribas halted redemption on three S.I.V.s to December 2009 when the U.S. Treasury Department awarded TARP funds to General Motors and Chrysler.

Figure 3: A contraction in money substitutes during the crisis



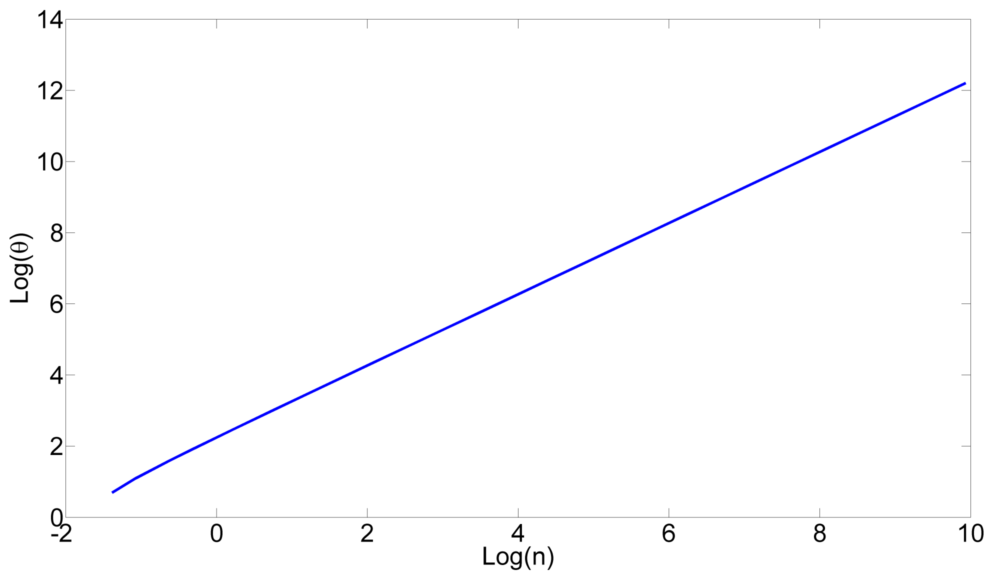
Source: Author's analysis, Center for Financial Stability (2015), and the Board of Governors (2015a)
Monthly growth rates (year-over-year) of a narrow monetary aggregate M2 (Board of Governors) and broader aggregate Divisia M4 (Center for Financial Stability) which includes commercial paper, U.S. Treasuries, and other assets used in money-like ways. The growth rate of the broader Divisia M4 fell faster and more severely than M2, contracting for over a year. The blue shading indicates the dates of the recent U.S. financial crisis from August 2007 when BNP Paribas halted redemption on three S.I.V.s to December 2009 when the U.S. Treasury Department awarded TARP funds to General Motors and Chrysler.

Figure 4: Simulated welfare losses



Source: Authors' analysis
This figure plots the welfare losses from returning to autarchy, the liquidity crisis, and the overproduction of safe assets, as a function of the number of states n. Welfare is the equivalent percent change in permanent consumption.

Figure 5: The calibrated value of $\log(\theta)$ is approximately linear in $\log(n)$



Source: Authors' analysis
This value of θ for each value of n that best matches the empirical estimates of the liquidity premium and trading volume.