Measuring credit risk in a large banking system: econometric modeling and empirics *

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Financial Stability Conference, Washington D.C.

May 30-31, 2013

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*Not necessarily the views of ECB or Sveriges Riksbank.

Motivation

Financial stability assessments are important: Dodd-Frank legislation in U.S., EBA and EC/ECB stress tests in E.U..

Stress tests typically focus on a large cross section (say 15-100) of larger banking groups.

Stress tests are expensive, thus infrequent, and subject to other issues.

Model-based risk/stability assessments are cheap and **fast**, can be done weekly, though potentially less accurate.

Contributions

A novel non-Gaussian framework for financial stability assessment, based on time-varying probabilities of simultaneous financial firm defaults. Focus on extreme tail events.

Overcome substantial difficulties in the econometric modeling of **large-dimensional**, **unbalanced** panels.

Small empirical study: which factors determine systemic correlation and systemic risk?

Literature

1. **Systemic risk**: see e.g. Brownlees and Engle (2011), Acharya, Pedersen, Philippon, Richardson (2012), Billio, Getmansky, Lo, Pelizzon (2012),...
2. **Observation-driven time-varying parameter models**: Creal, Koopman, Lucas (2008), Harvey (2008), Lucas, Schwaab, Zhang (2011),...
3. **Large dimensional covariance matrix models**: Engle, Shephard, Sheppard (2008), Patton and Oh (2011, 2013), Engle and Kelly (2012),...

The firm asset value

Let $y_t = (y_{1,t}, \cdots, y_{N,t})'$ be generated by a normal mean-variance mixture process

$$y_t = (\varsigma_t - \mu_\varsigma)\tilde{L}_t\gamma + \sqrt{\varsigma_t}\tilde{L}_t\epsilon_t, \tag{1}$$

where
1) $\epsilon_t \sim N(0, \mathrm{I}_N)$,
2) ς_t follows a InverseGamma$(\nu/2, \nu/2)$ is an additional risk factor, e.g. for *interconnectedness*,
3) $\tilde{L}_t$ is related to the covariance Σ_t.

The Lévy process driven Merton model

A simple Lévy driven firm default model:

$$y_t \sim p_{ghst}(\Sigma_t, \gamma, \nu), \tag{2}$$

with $\Sigma_t = L_t L_t' = D_t R_t D_t$.

The firm i and j's joint probability of default $p_{ij,t}$:

$$p_{ij,t} = F_{\rho_{ij,t}}(y^*_{i,t}, y^*_{j,t}), \tag{3}$$

define $\rho_{ij,t}$ as one pairwise correlation element in R_t.

The GH (skewed t) distribution

$$
\begin{aligned}
p_{ghst}(y_t;\theta_t) &= \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+n}{2}}}{\Gamma(\frac{\nu}{2})\pi^{\frac{n}{2}}|\tilde{\Sigma}_t|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\nu+n}{2}}\left(\sqrt{d(y_t)\cdot(\gamma'\gamma)}\right) e^{\gamma' \tilde{L}_t^{-1}(y_t-\mu_t)}}{\left(d(y_t)\cdot(\gamma'\gamma)\right)^{-\frac{\nu+n}{4}} d(y_t)^{\frac{\nu+n}{2}}}, \\
\theta_t &= \{\mu_t, \tilde{\Sigma}_t, \gamma, \nu\}, \\
d(y_t) &= \nu + (y_t-\mu_t)'\tilde{\Sigma}_t^{-1}(y_t-\mu_t), \\
\mu_t &= -\frac{\nu}{\nu-2}\tilde{L}_t\gamma.
\end{aligned}
$$

If $\gamma = 0$, the GH skewed t simplifies to a Student's t density.

Time variation in $\tilde{\Sigma}_t$ is driven by the Generalized Autoregressive Score model.

Generalized Autoregressive Score model

Recall that $y_t \sim p_{ghst}(\Sigma_t, \gamma, \nu)$, now assume that $\Sigma_t = G(f_t)$ and that the time varying parameters follow

$$\begin{aligned} f_{t+1} &= \omega + A s_t + B f_t, \\ \text{where } s_t &= S_t \nabla_t \text{ is the scaled score,} \\ \nabla_t &= \partial \log p_{ghst}(y_t | \mathcal{F}_{t-1}; f_t, \theta_t) / \partial f_t. \\ S_t &= E_{t-1}[\nabla_t \nabla_t']^{-1}. \end{aligned}$$

If $f_t = \sigma_t^2$, in a Normal distribution (GARCH):

$$f_{t+1} = \omega + A y_{t+1}^2 + B f_t.$$

Consider the skewed t case:

$$s_t = S_t \cdot \Psi_t' H_t' \mathrm{vec}\left(w_t \cdot y_t y_t' - \tilde{\Sigma}_t - \left(1 - \frac{\nu}{\nu - 2} w_t\right) \tilde{L}_t \gamma y_t' \right),$$

scaling matrix S_t is inverse Fisher information matrix; see Creal, Koopman, Lucas (2013 JAE).

A parsimonious correlation structure

If the dimension N is large, we assume N firms divided into m groups. Group i contains n_i firms with equicorrelation structure.

$$\begin{aligned} R_t &= \begin{bmatrix} (1-\rho_{1,t}^2)\mathrm{I}_{n_1} & \dots & \dots & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_{n_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (1-\rho_{m,t}^2)\mathrm{I}_{n_m} \end{bmatrix} \\ &+ \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell_1' & \rho_{2,t}\ell_2' \dots & \rho_{m,t}\ell_m' \end{pmatrix}, \end{aligned}$$

where $\ell_i \in \mathbb{R}^{n_i \times 1}$ is a column vector of ones and I_{n_i} an $n_i \times n_i$ identity matrix.

Speeding up the score computation

Even medium size dimensions are a severe problem for most multivariate dependence models (think DCC and $N = 30$).

The matrix calculation in large dimension can be done analytically.

A one equicorrelation correlation structure for the dynamic score model, benchmark and a special case:

$$R_t = (1 - \rho_t)\mathrm{I} + \rho_t \ell \ell'.$$

The conditional Law of Large Numbers (1)

The mixture model (1) is a two-factor model with common Gaussian factor κ_t and a mixing factor ς_t:

$$\begin{aligned} y_t &= (\varsigma_t - \mu_\varsigma)\gamma + \sqrt{\varsigma_t} z_t, \\ z_t &= \eta_t \kappa_t + \Lambda_t \epsilon_t. \end{aligned} \tag{4}$$

The vector $\eta_t \in \mathbb{R}^{N \times 1}$ and the diagonal matrix $\Lambda_t \in \mathbb{R}^{N \times N}$ are functions of R_t, γ, ν.

The conditional Law of Large Numbers (2)

The percentage of defaults at time t

$$c_{N,t} = \frac{1}{N}\sum_{i=1}^{N} 1\{y_{i,t} < y^*_{i,t}|\kappa_t, \varsigma_t\},$$

$1\{y_{i,t} < y^*_{i,t}|\kappa_t, \varsigma_t\}$ are conditional independent, as $N \to +\infty$:

$$c_{N,t} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{E}(1\{y_{i,t} < y^*_{i,t}|\kappa_t, \varsigma_t\}) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{P}(y_{i,t} < y^*_{i,t}|\kappa_t, \varsigma_t).$$

We can write the default common factor $\kappa_t = \kappa^*_t(c_{p,t}, \varsigma_t)$, because

$$\mathrm{P}(y_{i,t} < y^*_{i,t}|\kappa_t, \varsigma_t) = \Phi\left(\frac{(y^*_{i,t} + \mu_\varsigma \gamma_i - \varsigma_t \gamma_i)/\sqrt{\varsigma_t} - \eta_{i,t}\kappa_t}{\lambda_{i,t}}\middle|\kappa_t, \varsigma_t\right).$$

Two risk measures

The "Banking Stability Measure" (BSM):

$$p_t = \mathrm{P}(C_{N,t} > c_{p,t}) = \int \mathrm{P}(\kappa_t < \kappa_t^*(c_{p,t}, \varsigma_t))p(\varsigma_t)\mathrm{d}\varsigma_t.$$

The "Systemic Risk Measure" (SRM):

$$\mathrm{P}(C_{N-1,t} > c_{p,t}^{-i} | y_{i,t} < y_{i,t}^*) = \frac{\mathrm{P}(C_{N-1,t} > c_{p,t}^{-i}, y_{i,t} < y_{i,t}^*)}{\mathrm{P}(y_{i,t} < y_{i,t}^*)}$$

$$= \quad \frac{\int \Phi_2(\frac{z_{i,t}^* \sqrt{\mu_\varsigma}}{\sqrt{1-\sigma_\varsigma^2 \gamma' \gamma}}, \kappa_t^*(c_{1,t}^i, \varsigma_t), \eta_{i,t})p(\varsigma_t)\mathrm{d}\varsigma_t}{\int \mathrm{P}(\kappa_t < \kappa_t^*(c_{1,t}^i, \varsigma_t))p(\varsigma_t)\mathrm{d}\varsigma_t},$$

where $c_{p,t}^{-i}$ is the default proportion in the group excluding firm i. The SRM is calculated as the average over N firms.

Illustration with a small dataset

10 banks in Euro Area:
Bank of Ireland, BBVA, Santander, UniCredito, the National Bank of Greece.
BNP Paribas, Commerzbank, Deutsche Bank, Societe Generale, ING.

Data: January 1994 - June 2010,
198 monthly equity returns and EDF observations.

Models:

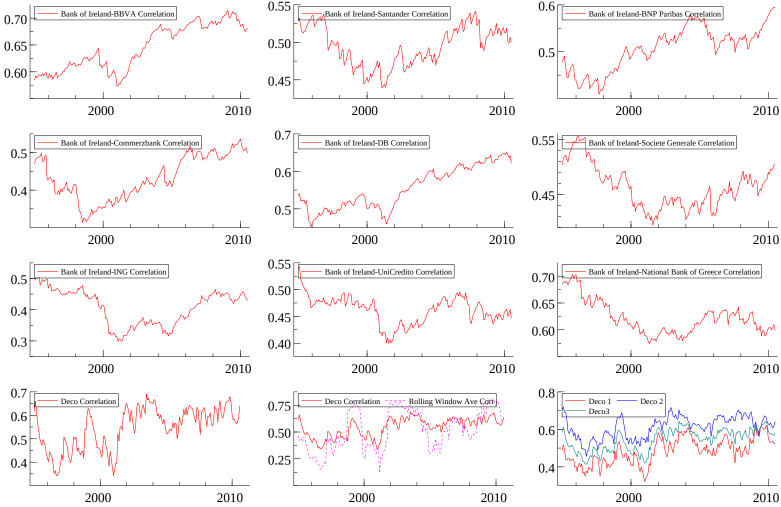
1. Dynamic score model: two-step estimation, correlation targeting.
2. Dynamic equicorrelation model.
3. Dynamic two-block equicorrelation model.

Risk measures: 10,000,000 simulation based, and/or LLN approximations.

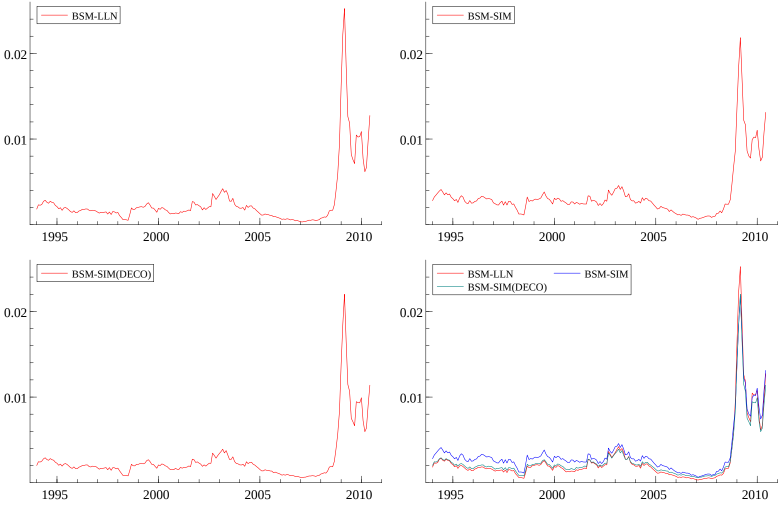
Data descriptions

	Std.Dev.	Skewness	Kurtosis	Minimum	Maximum
Bank of Ireland	1.309	-0.594	16.053	-113.917	106.153
BBVA	0.710	-0.512	3.220	-38.894	37.003
Santander	0.720	-0.725	3.758	-40.720	37.609
BNP Paribas	0.675	-0.502	3.261	-34.001	32.959
Commerzbank	0.940	-1.101	5.474	-67.779	45.536
Deutsche Bank	0.760	-0.421	3.906	-46.588	45.444
Societe Generale	0.777	-0.968	4.110	-53.679	29.201
ING	0.896	-1.647	8.939	-73.367	45.187
UniCredito	0.752	-0.048	3.282	-44.318	36.017
National Bank of Greece	0.938	0.336	2.324	-48.178	53.652

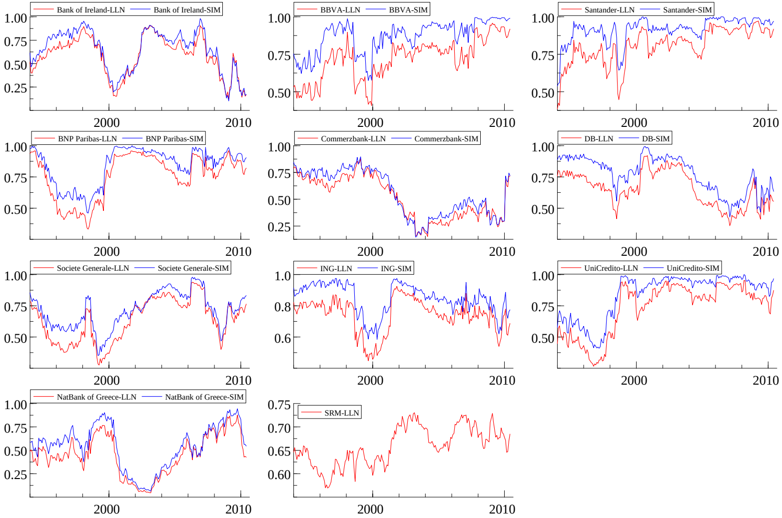
Correlation estimations



The Banking Stability Measure



The Systemic Risk Measure



A study of 73 European financial firms

73 European large financial firms: European banks, insurance companies and investment companies.

Data: January 1992 - June 2010,
monthly equity return and EDF.

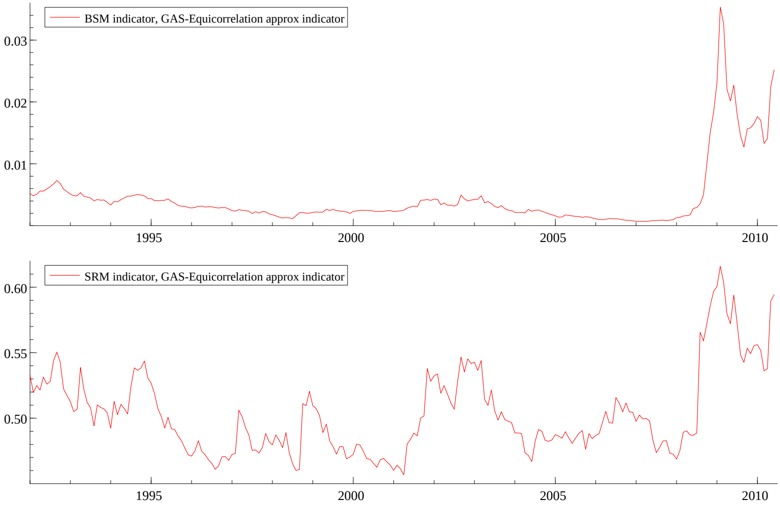
Unbalanced Panel: longest time series contains 488 observations and the shortest one has 10 observations.

Models:

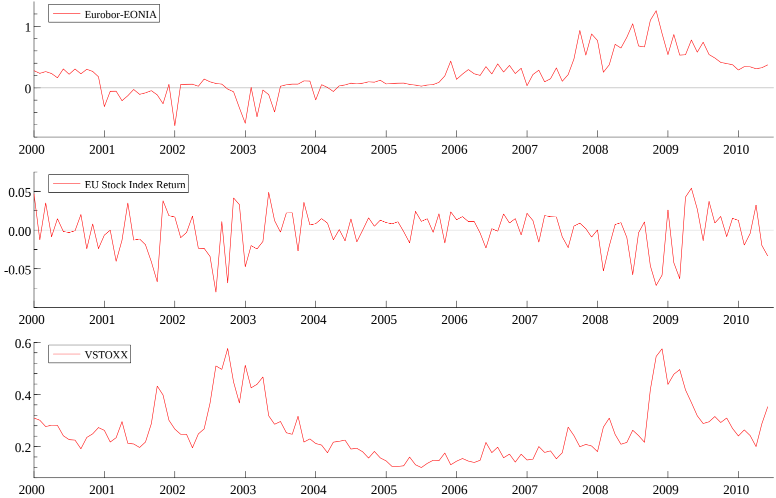
1. Dynamic equicorrelation model.
2. Dynamic equicorrelation model, augmented with economic variables.

Risk measures: Only LLN approximations.

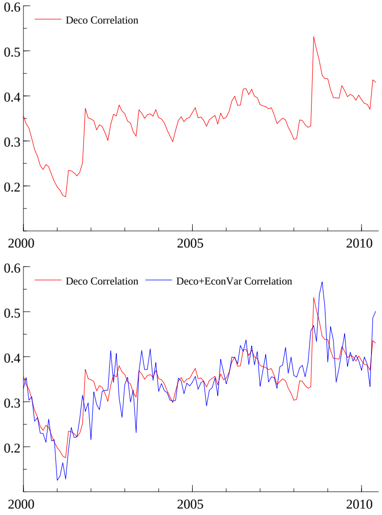
The risk Measure



Economic factors



Economic factors augmented score model



Conclusion and future research

An econometric framework for financial sector risk assessment. Dynamic model with parsimonious correlation structure.

Two risk measures are proposed for large dimensional financial data.

Work in progress: systemic importance/risk contribution of each bank, incorporating additional variables and block correlation structure.

Thank you.