Measuring credit risk in a large banking system: econometric modeling and empirics *

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*Not necessarily the views of ECB or Sveriges Riksbank.
Motivation

Financial stability assessments are important: Dodd-Frank legislation in U.S., EBA and EC/ECB stress tests in E.U..

Stress tests typically focus on a large cross section (say 15-100) of larger banking groups.

Stress tests are expensive, thus infrequent, and subject to other issues.

Model-based risk/stability assessments are cheap and fast, can be done weekly, though potentially less accurate.
Contributions

A novel non-Gaussian framework for financial stability assessment, based on time-varying probabilities of simultaneous financial firm defaults. Focus on extreme tail events.

Overcome substantial difficulties in the econometric modeling of large-dimensional, unbalanced panels.

Small empirical study: which factors determine systemic correlation and systemic risk?
1. **Systemic risk**: see e.g. Brownlees and Engle (2011), Acharya, Pedersen, Philippon, Richardson (2012), Billio, Getmansky, Lo, Pelizzon (2012),...


The firm asset value

Let \( y_t = (y_{1,t}, \ldots, y_{N,t})' \) be generated by a normal mean-variance mixture process

\[
y_t = (\varsigma_t - \mu_\varsigma)\tilde{L}_t \gamma + \sqrt{\varsigma_t \tilde{L}_t} \epsilon_t,
\]

(1)

where

1) \( \epsilon_t \sim N(0, I_N) \),
2) \( \varsigma_t \) follows a \text{InverseGamma}(\nu/2, \nu/2) is an additional risk factor, e.g. for \textit{interconnectedness},
3) \( \tilde{L}_t \) is related to the covariance \( \Sigma_t \).
The Lévy process driven Merton model

A simple Lévy driven firm default model:

\[ y_t \sim p_{ghst}(\Sigma_t, \gamma, \nu), \]  

(2)

with \( \Sigma_t = L_t L_t' = D_t R_t D_t. \)

The firm \( i \) and \( j \)'s joint probability of default \( p_{i,j,t}: \)

\[ p_{i,j,t} = F_{\rho_{i,j,t}}(y_{i,t}^*, y_{j,t}^*), \]  

(3)

define \( \rho_{i,j,t} \) as one pairwise correlation element in \( R_t. \)
The GH (skewed t) distribution

\[
p_{ghst}(y_t; \theta_t) = \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+n}{2}}}{\Gamma(\frac{\nu}{2}) \pi^{\frac{n}{2}} |\tilde{\Sigma}_t|^\frac{1}{2}} \cdot \frac{K\nu^{\frac{n}{2}}}{\nu^{\frac{n}{2}} + \nu + (y_t - \mu_t)'\tilde{\Sigma}_t^{-1}(y_t - \mu_t)} \cdot e^{\gamma'\tilde{L}_t^{-1}(y_t - \mu_t)},
\]

\[
\theta_t = \{\mu_t, \tilde{\Sigma}_t, \gamma, \nu\},
\]

\[
d(y_t) = \nu + (y_t - \mu_t)'\tilde{\Sigma}_t^{-1}(y_t - \mu_t),
\]

\[
\mu_t = -\frac{\nu}{\nu - 2} \tilde{L}_t \gamma.
\]

If \( \gamma = 0 \), the GH skewed t simplifies to a Student’s t density.

Time variation in \( \tilde{\Sigma}_t \) is driven by the Generalized Autoregressive Score model.
Generalized Autoregressive Score model

Recall that \( y_t \sim p_{ghst}(\Sigma_t, \gamma, \nu) \), now assume that \( \Sigma_t = G(f_t) \) and that the time varying parameters follow

\[
\begin{align*}
    f_{t+1} &= \omega + A s_t + B f_t, \\
    \text{where } s_t &= S_t \nabla_t \text{ is the scaled score,} \\
    \nabla_t &= \partial \log p_{ghst}(y_t | F_{t-1}; f_t, \theta_t) / \partial f_t. \\
    S_t &= E_{t-1}[\nabla_t \nabla_t']^{-1}.
\end{align*}
\]

If \( f_t = \sigma^2_t \), in a Normal distribution (GARCH):

\[
f_{t+1} = \omega + A y^2_{t+1} + B f_t.
\]

Consider the skewed \( t \) case:

\[
s_t = S_t \cdot \Psi_t' H_t' \text{vec} \left( w_t \cdot y_t y'_t - \tilde{\Sigma}_t - \left( 1 - \frac{\nu}{\nu - 2} w_t \right) \tilde{L}_t \gamma y'_t \right),
\]

scaling matrix \( S_t \) is inverse Fisher information matrix; see Creal, Koopman, Lucas (2013 JAE).
A parsimonious correlation structure

If the dimension $N$ is large, we assume $N$ firms divided into $m$ groups. Group $i$ contains $n_i$ firms with equicorrelation structure.

$$R_t = \begin{bmatrix} (1 - \rho_{1,t}^2)I_{n_1} & \ldots & \ldots & 0 \\ 0 & (1 - \rho_{2,t}^2)I_{n_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & (1 - \rho_{m,t}^2)I_{n_m} \end{bmatrix} + \begin{pmatrix} \rho_{1,t\ell_1} \\ \rho_{2,t\ell_2} \\ \vdots \\ \rho_{m,t\ell_m} \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t\ell_1'} & \rho_{2,t\ell_2'} & \ldots & \rho_{m,t\ell_m'} \end{pmatrix},$$

where $\ell_i \in \mathbb{R}^{n_i \times 1}$ is a column vector of ones and $I_{n_i}$ an $n_i \times n_i$ identity matrix.
Speeding up the score computation

Even medium size dimensions are a severe problem for most multivariate dependence models (think DCC and $N = 30$).

The matrix calculation in large dimension can be done analytically.

A one equicorrelation correlation structure for the dynamic score model, benchmark and a special case:

$$R_t = (1 - \rho_t)I + \rho_t \ell \ell'.$$
The conditional Law of Large Numbers (1)

The mixture model (1) is a two-factor model with common Gaussian factor $\kappa_t$ and a mixing factor $\varsigma_t$:

$$
\begin{align*}
y_t &= (\varsigma_t - \mu_\varsigma)\gamma + \sqrt{\varsigma_t}z_t, \\
z_t &= \eta_t\kappa_t + \Lambda_t\epsilon_t.
\end{align*}
$$

(4)

The vector $\eta_t \in \mathbb{R}^{N \times 1}$ and the diagonal matrix $\Lambda_t \in \mathbb{R}^{N \times N}$ are functions of $R_t, \gamma, \nu$. 
The conditional Law of Large Numbers (2)

The percentage of defaults at time $t$

$$c_{N,t} = \frac{1}{N} \sum_{i=1}^{N} 1\{y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t\},$$

$1\{y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t\}$ are conditional independent, as $N \to +\infty$:

$$c_{N,t} \approx \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}(1\{y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t\}) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{P}(y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t).$$

We can write the default common factor $\kappa_t = \kappa_t^* (c_{p,t}, \varsigma_t)$, because

$$\mathbb{P}(y_{i,t} < y_{i,t}^* | \kappa_t, \varsigma_t) = \Phi \left( \frac{(y_{i,t}^* + \mu_{\varsigma} \gamma_i - \varsigma_t \gamma_i)/\sqrt{\varsigma_t - \eta_{i,t} \kappa_t}}{\lambda_{i,t}} \right | \kappa_t, \varsigma_t).$$
Two risk measures

The “Banking Stability Measure” (BSM):

\[ p_t = P(C_{N,t} > c_{p,t}) = \int P(\kappa_t < \kappa_t^*(c_{p,t}, \varsigma_t))p(\varsigma_t)d\varsigma_t. \]

The “Systemic Risk Measure” (SRM):

\[
\begin{align*}
P(C_{N-1,t} > c_{p,t}^- | y_{i,t} < y_{i,t}^*) &= \frac{P(C_{N-1,t} > c_{p,t}^-, y_{i,t} < y_{i,t}^*)}{P(y_{i,t} < y_{i,t}^*)} \\
&= \frac{\int \Phi_2\left(\frac{z_{i,t}^* \sqrt{\mu_s}}{\sqrt{1-\sigma_s^2 \gamma' \gamma}}, \kappa_t^*(c_{1,t}, \varsigma_t), \eta_{i,t}\right)p(\varsigma_t)d\varsigma_t}{\int P(\kappa_t < \kappa_t^*(c_{1,t}, \varsigma_t))p(\varsigma_t)d\varsigma_t},
\end{align*}
\]

where \( c_{p,t}^- \) is the default proportion in the group excluding firm \( i \). The SRM is calculated as the average over \( N \) firms.
Illustration with a small dataset

10 banks in Euro Area:
Bank of Ireland, BBVA, Santander, UniCredito, the National Bank of Greece.
BNP Paribas, Commerzbank, Deutsche Bank, Societe Generale, ING.

Data: January 1994 - June 2010,
198 monthly equity returns and EDF observations.

Models:

1. Dynamic score model: two-step estimation, correlation targeting.
2. Dynamic equicorrelation model.
3. Dynamic two-block equicorrelation model.

Risk measures: 10,000,000 simulation based, and/or LLN approximations.
## Data descriptions

<table>
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<tr>
<th></th>
<th>Std.Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Minimum</th>
<th>Maximum</th>
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<td>Bank of Ireland</td>
<td>1.309</td>
<td>-0.594</td>
<td>16.053</td>
<td>-113.917</td>
<td>106.153</td>
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<td>BBVA</td>
<td>0.710</td>
<td>-0.512</td>
<td>3.220</td>
<td>-38.894</td>
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<td>Santander</td>
<td>0.720</td>
<td>-0.725</td>
<td>3.758</td>
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<td>37.609</td>
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<tr>
<td>BNP Paribas</td>
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<td>32.959</td>
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<td>Commerzbank</td>
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<tr>
<td>Deutsche Bank</td>
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<td>Societe Generale</td>
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<tr>
<td>ING</td>
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<td>8.939</td>
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<tr>
<td>UniCredito</td>
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<td>3.282</td>
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<tr>
<td>National Bank of Greece</td>
<td>0.938</td>
<td>0.336</td>
<td>2.324</td>
<td>-48.178</td>
<td>53.652</td>
</tr>
</tbody>
</table>
Correlation estimations

- Bank of Ireland-BBVA Correlation
- Bank of Ireland-Santander Correlation
- Bank of Ireland-BNP Paribas Correlation
- Bank of Ireland-Commerzbank Correlation
- Bank of Ireland-DB Correlation
- Bank of Ireland-Societe Generale Correlation
- Bank of Ireland-ING Correlation
- Bank of Ireland-UniCredito Correlation
- Bank of Ireland-National Bank of Greece Correlation
- Deco Correlation
- Deco Correlation Rolling Window Ave Corr

Deco 1
Deco 2
Deco 3
The Systemic Risk Measure
A study of 73 European financial firms

73 European large financial firms: European banks, insurance companies and investment companies.

Data: January 1992 - June 2010, monthly equity return and EDF.

Unbalanced Panel: longest time series contains 488 observations and the shortest one has 10 observations.

Models:

1. Dynamic equicorrelation model.
2. Dynamic equicorrelation model, augmented with economic variables.

Risk measures: Only LLN approximations.
The risk Measure

- BSM indicator, GAS-Equicorrelation approx indicator
- SRM indicator, GAS-Equicorrelation approx indicator
Economic factors
Economic factors augmented score model
Conclusion and future research


Two risk measures are proposed for large dimensional financial data.

Work in progress: systemic importance/risk contribution of each bank, incorporating additional variables and block correlation structure.
Thank you.