Optimal Systemic Risk Mitigation in Financial Networks

Agostino Capponi

School of Industrial Engineering
Purdue University

Joint Conference of the Federal Reserve Bank of Cleveland and OFR
“Financial Stability Analysis: Using the Tools, Finding the Data”
May 30, 2013
(joint work with Peng-Chu Chen)
Outline

1. Introduction
2. Framework
3. Systemic Risk and Bailout Mitigation
4. Systemic Risk Analysis
5. Conclusion
Introduction

Financial network
- Institutions are connected via bilateral exposures originating from trades
- The intricate structure of linkages captured via network representation of financial system

Systemic risk
- The risk of collapse of entire financial network
- Triggered by shock transmission mechanism generated by recursive interdependence in the network

see also Allen & Gale (2001); Eisenberg & Noe (2001); Gai & Kapadia (2010)
Our Goal

Construct a framework which allows

- capturing systemic consequences of the default of a node on the state of the entire network
- mitigating systemic risk through optimal bailout strategies in the form of loans
- providing a tool to support regulator decisions of when/how to intervene to mitigate systemic risk
Financial Network I

- Modeled as a digraph where the set \( V \) of nodes represents financial firms, and edges the liability relationships between nodes
- Multi-period model with finite time horizon \( T \). Each period is a discrete interval \([t, t + 1)\), \( t \in \{0, 1, \ldots, T - 1\}\)

**State of network** in period \([t, t + 1)\) characterized by
- \( L^t \): matrix of interbank liabilities due at \( t \), with \( L^t_{ij} \) denoting liabilities that \( i \) owes to \( j \) at \( t \)
- \( \iota^t \): vector of operating cash inflows at \( t \)
- \( a^t \): vector of illiquid assets owned by nodes at \( t \)
Financial Network II

- $l_i^t := \sum_{j \neq i, j \in V} L_{ij}^t$: total liabilities node $i$ owes to other nodes at $t$
- $\Pi_{ij}^t := \frac{L_{ij}^t}{l_i^t} 1_{l_i^t > 0}$: fraction of nodes $i$'s total liabilities owed to $j$ at $t$
- $p_i^t$: payment node $i$ makes to other nodes at $t$
- $v_i^t$: cash available to node $i$ at $t$
- $c_i^t := \sum_{j \neq i, i \in V} \Pi_{ji} p_j^t + \nu_i^t + v_i^t$
Bailout Rules

- Lender of Last Resort (LLR) can provide bailout loans to **illiquid but solvent** nodes so to mitigate financial distress caused by default.
- Each node in network uses cash left after paying due liabilities to his creditors, to repay currently owed bailout amount.
- Define the bailout quantities
  - $o^t_i$: bailout loan granted by LLR to rescue node $i$ at $t$
  - $b^t_i$: amount that node $i$ uses to repay his outstanding bailout loan at $t$.
  - $q^t_i$: portion of bailout loan node $i$ still needs to repay at $t$, i.e. $q^{t+1} = (1 + r_c)(q^t + o^t - b^t)$. 
Default Modeling I

- A node $i$ at $t$ is said to be
  - **illiquid** if it cannot repay in full its liabilities, i.e. $p^t_i < l^t_i$.
  - **solvent** if its net asset value is positive.
  - **default** if it is either
    1. illiquid and insolvent at $t$ or
    2. illiquid and solvent at $t$ but not rescued by LLR.

- The **net assets** of node $i$ at time $t$ is

\[ e^t_i = c^t_i + a^t_i + \alpha^t_i (1 - \gamma^t_i) - \sum_{\tau=t}^{T-1} (1 + r)^{-(\tau-t)} \mathbb{E}_t [ l^\tau_i ] , \]

where $\alpha^t_i$ denotes expected value of total debt owed to $i$ at $t$, and $\gamma^t_i$ the expected loss rate caused by nodes defaulted before $t$. 
Default Modeling II

- Automatic stay for debtors in possession, in accordance with Chapter 11 procedures of U.S..

- Repay outstanding debt with available cash and illiquid assets until it is cleared.

- Grace period of not repaying debt and obtaining acceptance of reorganization plan.

- Remain in the default state, but still retains control of business, keep generating operating cash inflows, collecting interbank payments, and receiving illiquid assets transferred from defaulted nodes.
Unpaid Debt

The default time of node $i$ is

$$\eta_i := \inf \{ t : p^t_i < l^t_i \}.$$ 

Define the unpaid debt of node $i$ to $j$ as

$$W^{t+1}_{ij} = (1 + r) \left[ 1_{\eta_i = t} \left( \sum_{\tau = t}^{T-1} (1 + r)^{-(\tau - t)} E_t[L^T_{ij}] - \Pi^t_{ij} p^t_i \right) 
+ 1_{t - \theta + 1 \leq \eta_i < t} W^t_{ij} + 1_{\eta_i < t - \theta + 1} \left( W^t_{ij} - \frac{W^t_{ij}}{w^t_i} (c^t_i + a^t_i) \right)^+ \right]$$

$$w^t_i = \sum_{j \neq i, j \in V} W^t_{ij}$$
The expected value of debt owed to \(i\) is computed as

\[
\alpha^t_i = \sum_{j \neq i, j \in V} \left( 1_{\eta_j < t} W^t_{ji} + 1_{\eta_j \geq t} \sum_{\tau = t+1}^{T-1} (1 + r)^{-\tau - t} \mathbb{E}_t[L^T_{ji}] \right).
\]

The expected loss rate of \(i\) at \(t\) is given by

\[
\gamma^t_i = \frac{1}{\alpha^t_i} \left( \sum_{j \neq i, j \in V} 1_{\eta_j < t} (1 + r)^{-T - t} \mathbb{E}_t \left[ W^T_{ji} \right] \right) \quad \text{unpaid debt by}\ T
\]
Building on Eisenberg & Noe (2001), define the multi-period clearing payment system, which allows specifying default events:

**Definition**

A sequence of \((p^{t*}, o^{t*})_{t=0}^{T}\) is a clearing sequence if it satisfies

- **Systemically efficient rescuing.** The LLR provides bailout loans to illiquid yet solvent nodes so to

\[
\text{maximize } \left\{ o^t \right\}_{t=0}^{T-1} \sum_{t=0}^{T-1} (1 + r)^{-t} \sum_{i \in V} p^t_i.
\]
Definition (cont.)

- **Proportional repayment of liabilities.** A node $i$ pays $\Pi_{ij} p_i^t$ to node $j \in V, j \neq i$ at $t$.

- **Absolute priority.** $p_i^t = \begin{cases} l_i^t & \text{if } \eta_i > t \\ c_i^t & \text{if } \eta_i = t \\ 0 & \text{if } \eta_i < t \end{cases}$

- **Admissible bailout.** LLR provides bailout loans only to illiquid yet solvent nodes, i.e.
  $o_i^t > 0 \Rightarrow c_i^t < l_i^t, e_i^t \geq 0, \eta_i > t$.

- **Just enough bailout.** Nodes are rescued with the minimum needed amount, i.e. $o_i^t > 0 \Rightarrow o_i^t = l_i^t - c_i^t$. 
A Markov Decision Process (MDP) Formulation I

Define
- $X^t = (L^t, i^t) \in \mathcal{X}$: stochastic process
- $o^t \in \mathcal{O}^t$: decision process; $\mathcal{O}^t$: feasible policies.
- $s^t = (v^t, a^t, q^t, \eta^t, W^t) \in S$: state at $t$.
- $f(s^t, o^t, X^t) = s^{t+1}$: state transition function
- $\mathbb{P}^s[s^{t+1} | s^t, o^t] = \mathbb{P}^X [\omega : s^{t+1} = f(s^t, o^t, X^t(\omega))]$: transition probability.
A Markov Decision Process (MDP) Formulation II

Objectives function:

\[ Z^0(s^0) = \max_{\pi \in \Pi} \mathbb{E} \left[ \sum_{\tau=0}^{T-1} (1 + r)^{-\tau} z^\tau(s^\tau, o^\tau_\pi, X^\tau) \right], \]

\[ z^\tau(s^\tau, o^\tau_\pi, X^\tau) := \sum_{i \in V} \rho^\tau_i \]
Constraints: The bailout loan vector $o^t_\pi$ must satisfy the following constraints:

$$O^t(s^t) = \{o^t_\pi \in O^t :$$

$$p^t_i = (1 - 1_{\eta_i < t}) \min \{l^t_i, c^t_i + o^t_i\},$$

$$o^t_i > 0 \Rightarrow c^t_i < l^t_i \text{ and } e^t_i \geq 0 \text{ and } \eta_i > t,$$

$$o^t_i > 0 \Rightarrow o^t_i = l^t_i - c^t_i$$

$$\sum_{i \in V} (q^t_i + o^t_i) \leq B(1 + r_c)^t\}$$
For high dimensional financial networks, MDP becomes computational intractable.

Develop a suboptimal approach:
- Choose heuristic bailout allocation rules
- Approximate the objective value $Z^0(s^0)$ of each heuristics via Monte-Carlo simulations
- Suitably combine the heuristics so to select the best suboptimal policy in each decision epoch

Analyze behavior of heuristic algorithms in this talk (see paper for suboptimal strategies)
Myopic Heuristic

- Computes solution maximizing the single period payment function

\[
\begin{align*}
\mathbf{o}_{\pi_1}^t &= \arg \max_{\mathbf{o}^t \in \mathcal{O}^t} z^t(s^t, \mathbf{o}^t, X^t) \\
\ z_1^t &= \sum_{i \in V} p_i^t(s^t, \mathbf{o}_{\pi_1}^t, X^t)
\end{align*}
\]
(1) Computes the ratio of unpaid debt caused by defaults in decision epoch $t$, i.e.

$$\mathcal{R}_t = \frac{\sum_{i \in \mathcal{V}} \mathbf{1}_{\eta_i=t} w_i^T (1 + r)^{-T}}{\sum_{\tau=0}^{T-1} \sum_{i \in \mathcal{V}} \mathbf{1}_{\eta_i=\tau} w_i^T (1 + r)^{-T}}$$

(2) Preallocate budget $B_t = B \times \mathcal{R}_t$ to decision epoch $t$.

(3) Compute

$$o^{t}_{\pi_2} = \arg \max_{o^t \in \mathcal{O}^t} z^t(s^t, o^t, X^t) \quad z^t_2 = \sum_{i \in \mathcal{V}} p^t_i(s^t, o^{t}_{\pi_2}, X^t)$$
For $Y \subseteq V$, define

$$UL_Y = \frac{\sum_{t=0}^{T-1} (1 + r)^{-t} \sum_{i \in Y} (l_i^t - p_i^t(s^t, o^t, X^t)) \sum_{t=0}^{T-1} (1 + r)^{-t} \sum_{i \in V} l_i^t}{\sum_{t=0}^{T-1} (1 + r)^{-t} \sum_{i \in V} l_i^t},$$

and residual systemic risk generated by $Y$ as,

$$RS_Y = \mathbb{E}[UL_Y],$$

i.e. the percentage of liabilities unpaid by the nodes in $Y$, after accounting for optimal bailout policy.
Systemic Graph

- **C1** is the Source Cluster at time 1.
- **C3** is the Contaminated Cluster at time 2, with **C3** being contaminated by **C1**.
- **C3** is the Source Cluster at time 2.

Legend:
- ○ not default
- ● defaulting
- ■ defaulted
- □ contaminated
Let $S$ and $C$ denote respectively the set of nodes in source and contaminated clusters. The residual systemic risk attributed to time $t$ is

$$RS_V^t = \mathbb{E} \left[ R^t UL_V \right],$$

The amount generated by source clusters is

$$RS_S^t = \mathbb{E} \left[ R^t UL_S \right],$$

while the amount due to contaminated clusters is

$$RS_C^t = \mathbb{E} \left[ R^t UL_C \right].$$
Consider two networks, **homogeneous** and **heterogeneous**, with $n = 20$, $T = 20$, and $\Delta t = 1$

- **Homogeneous** network: liabilities and operating cash inflows are i.i.d. Gaussian
- **Heterogeneous** network: liabilities and operating cash inflows are independently distributed Gaussian
Homogeneous Network
Homogeneous Network: Systemic Risk I
Homogeneous Network: Systemic Risk II

Graphs showing the impact of different bailout strategies on systemic risk. The graphs compare the systemic risk under three scenarios:

1. **No bailout**
2. **Myopic control**
3. **Pre-allocation control**

The x-axis represents the in-and-out degree of nodes, while the y-axis represents systemic risk. The graphs illustrate how each strategy affects the overall systemic risk and the connectivity of the network over time.
Homogeneous Network: Systemic Risk III

Connectivity = 10/19

Connectivity = 19/19

Connectivity = 10/19

Connectivity = 10/19
Heterogeneous Network

- Operating cash inflow < liabilities
- Small/large: low/high balance sheet size
- Weak/strong: low/high initial available cash
Heterogeneous Network: Systemic Risk 1

Connectivity = 19/19
Heterogeneous Network: Systemic Risk II

Connectivity = 10/19

- Systemic Risk without Mitigation
- Connectivity = 19/19

- Total
- Contaminated
- Source

Connectivity = 19/19

- Systemic Risk under Myopic Control
- Systemic Risk under Pre-allocation Control

Time vs. Systemic Risk
Conclusion

- Developed a multi-period framework to quantify systemic risk propagation and mitigation effects.
- Clearing payments and bailout strategies recovered as the solution of Markov decision process.
- Homogeneous network: systemic risk has an inverted U shape, and can be significantly reduced using myopic strategies.
- Heterogeneous network: systemically important nodes may change over time depending on the state of network.