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The Transition to Alternative Reference Rates in the OFR Financial Stress Index

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Abstract

The OFR Financial Stress Index (OFR FSI) is a daily market-based snapshot of stress in global financial markets that is constructed from 33 financial market variables. As of the time of writing, seven of these variables rely on obsolete reference rates. Since its inception, the OFR FSI was intended to allow for the periodic replacement of obsolete variables as the need arises. In this paper, I introduce replacements for these seven obsolete variables, and I make explicit the procedure with which the OFR FSI incorporates these new variables. Furthermore, I demonstrate generally that this replacement procedure produces an index with the following desirable properties: (1) the index is a weighted sum of the presently included variables; (2) removed variables no longer directly affect the index, and newly included variables do not modify historical values of the index; (3) the index uses all available historical data on the newly included variables to train the model; and (4) the volatility of the index is roughly comparable before and after the replacement.

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1 Introduction

The OFR Financial Stress Index (OFR FSI or FSI) is a daily market-based snapshot of stress in global financial markets. Originally, the OFR FSI was constructed from 33 financial market variables that are correlated with some form of financial stress. Seven of these variables are based on LIBOR or other ceasing and/or already-ceased benchmark interest rates. As such, these seven variables are now obsolete. However, since its inception, the OFR FSI was intended to allow for the periodic replacement of obsolete variables as the need arises. In this paper, I introduce replacements for these seven obsolete variables, along with the reasoning behind their selection. Furthermore, in an effort to supplement the original description of the OFR FSI's construction found in Monin (2019)[1] and to provide transparency, I make the procedure for this replacement strategy (as well as the underlying algorithm used to construct the FSI) as explicit as possible, thus increasing the transparency and reproducibility of the index. I also make some slight revisions to this procedure that have no impact on previous calculations of the OFR FSI but will improve the way in which the FSI transitions from LIBOR-based variables to the new variables. These revisions will also improve the transition procedure should any additional replacements need to be made in the future.

As outlined in Monin (2019), the OFR FSI was designed to accommodate both missing data and the periodic replacement of obsolete variables over time. The methodology allows for periodic substitutions that are not necessarily envisioned

[1]See also the Working Paper version, published on the OFR's website (Monin, 2017).

ahead of time, while also maintaining a degree of consistency in the index over time. The FSI uses a factor model as the underlying statistical model and is estimated via least squares. It accommodates missing values by dropping these observations from the least-squares problem, as do other approaches in the literature on estimating factor models on unbalanced panels (see Bańbura and Modugno 2014; or Hatzius et al. 2010). Periodic replacement of obsolete variables can be handled in a number of ways, and the method used may dictate the answers to the following questions: Will historical values of the index be revised as new data arrives? When the decision to include a new economic variable is made, will historical values be updated to reflect the new data? How will historical data on newly included variables from before the inclusion date be used? All of these questions are important. I answer them by making the underlying procedure for constructing the OFR FSI, as well as how it accommodates the transition from obsolete variables to new variables, as transparent as possible.

In addition to giving a full mathematical description of how the index is constructed, I demonstrate that the procedure produces an index that has several desirable properties. Specifically, I show that (1) the OFR FSI is a weighted sum of the presently included variables; (2) removed variables no longer directly affect the index, and newly included variables do not modify historical values of the index; (3) the index uses all available historical data on the newly included variables to train the model; and (4) the volatility of the index is roughly comparable before and after the replacement. These properties describe an index that is simple to interpret; provides estimates that are free from look-ahead bias (in that the inclusion of the new

Table 1: Variables in the Funding Category and Their Benchmark Rate Dependencies

Variable	Reference Rate Dependencies
2-Year EUR/USD Cross-Currency Swap Spread	Euribor, USD LIBOR
2-Year USD/JPY Cross-Currency Swap Spread	USD LIBOR, JPY LIBOR
2-Year US Swap Spread	USD LIBOR
3-Month EURIBOR - EONIA OIS	Euribor, EONIA
3-Month Japanese LIBOR - OIS	JPY LIBOR
3-Month US LIBOR - OIS	USD LIBOR
3-Month TED Spread	USD LIBOR

This table lists the variables included in the funding category of the original version of the OFR Financial Stress Index, and it lists the benchmark rates on which each variable depends. Given the eventual cessation of all LIBOR settings, as well as the cessation of EONIA, all variables in this category must be replaced. *Source: author's creation.*

variables does not revise the historical values of the FSI); efficiently uses all available data; and maintains some degree of consistency, even after variables are periodically replaced.

The other main goal of this paper is to choose replacements for the seven variables that are based on LIBOR or other obsolete reference rates, as well as to assess the impact of the inclusion of the new variables on the OFR FSI. In the following subsection, I discuss the context of these obsolete variables in more detail.

LIBOR cessation and the transition to alternative reference rates. In response to cases of attempted manipulation of reference rates and declining liquidity in key interbank unsecured funding markets, the Financial Stability Board (FSB) undertook a review of major interest rate benchmarks. In 2014, the FSB published recommendations for the development of alternative reference rates (Financial Stability Board, 2014). Following this review, the UK's Financial Conduct Authority

Table 2: Replacement Variables

Old Indicator	New Indicator
2y EUR/USD Swap Spread (Euribor v LIBOR)	2y EUR/USD Swap Spread (ESTR v SOFR)
2y USD/JPY Swap Spread (LIBOR)	2y USD/JPY Swap Spread (SOFR v TONAR)
USD LIBOR Swap Spread, 2y	SOFR Swap Spread, 2y
3m Euribor - EONIA OIS	3m Euribor - ESTR OIS
3m JPY LIBOR - OIS	3m TIBOR - TONAR OIS
3m USD LIBOR - OIS	3m Commercial Paper - SOFR OIS
TED Spread	3m Commercial Paper - 3m Treasury

As of January 1, 2022, the old variables are no longer included in the OFR Financial Stress Index and the new variables are included. The 3-month commercial paper rates use the US 3-month AA Financial Commercial Paper Interest Rates published by the Board of Governors of the Federal Reserve System. *Source: author's creation.*

declared in 2017 that it would no longer compel banks to continue making LIBOR submissions after December 31, 2021. This began what is now known as the interbank offered rates (IBOR) transition. In the US, the Alternative Reference Rate Committee (ARRC) was convened by the Federal Reserve Board and the Federal Reserve Bank of New York to identify a suitable replacement for USD LIBOR. Similar efforts were undertaken in other currency jurisdictions around the world.

In the OFR FSI, the seven variables that compose the funding category are all based on ceasing or ceased benchmark rates. These variables and the benchmark rates that they depend on are listed in Table 1. The ceasing benchmark rates upon which the FSI relies are USD LIBOR, JPY LIBOR, and EONIA. Each of these seven variables will be replaced. Their replacements are listed in Table 2. The selection process behind these replacements is detailed in Section 3. These replacements are included in the FSI's calculations for January 1, 2022, and the old variables are removed from the calculations as of the same day.

The methodology for variable replacements in the OFR FSI. As described earlier, there are several ways in which an index constructed using a factor model estimation procedure could incorporate the periodic replacement of the index's underlying variables. The methodology that I specify in this paper is one of several options. Here, I'll discuss in a little more depth how the specific replacements for the LIBOR-based variables, introduced above, helped to motivate the choice of a variable replacement methodology.

To begin with, there are several related questions that arise when weighing various variable replacement strategies. The first is, how will the FSI treat historical data associated with the new variables when the data comes from a period of time before those variables were included in the index? The new variables introduced above replace the old variables as of January 1, 2022. The histories of all the new variables constitute important information that is needed to train the model, and the methodology should incorporate these histories.

There are several potential ways to construct the FSI so that these full variable histories are used. One potential option would be to retrain a new factor model based only on the new set of variables. In the past, the FSI worked by retraining a new factor model each day, using the new day's data. The estimated factor score associated with the new day was designated as the FSI for that day. Thus, the variable replacement could potentially be implemented by retraining the factor model after the transition, using only the new set of variables. However, there are some disadvantages to this approach. The first and most important disadvantage is that the history of data available for the chosen replacement variables is relatively short.

Table 3: Earliest Observation of New Variables

New Indicator	First Observation Date
2y EUR/USD Swap Spread (ESTR v SOFR)	2021-11-24
2y USD/JPY Swap Spread (SOFR v TONAR)	2021-11-24
SOFR Swap Spread, 2y	2018-06-07
3m Euribor - ESTR OIS	2019-09-26
3m TIBOR - TONAR OIS	2009-12-14
3m Commercial Paper - SOFR OIS	2018-06-07
3m Commercial Paper - 3m Treasury	1997-01-02

As of January 1, 2022, the old variables are no longer included in the OFR FSI and the new variables are included. The 3-month commercial paper rates use the US 3-month AA Financial Commercial Paper Interest Rates published by the Board of Governors of the Federal Reserve System. The table shows the earliest observation of each variable that the OFR has available for use in the FSI. *Source: Refinitiv Datastream, Office of Financial Research*

Table 3 lists the earliest observation of each of the new variables available in the data used by the OFR FSI. Some of these series, such as the EUR/USD Cross-Currency Swap and the USD/JPY Swap Spreads, have histories going back only to November 24, 2021. If we were to retrain the model using only the new variables, we would essentially be throwing away a lot of the information contained in the IBOR-based variables that was an important part of estimating the factor scores before the transition. The loss of this information could drastically change the FSI, even after the transition.

By contrast, the methodology that I construct will still allow for the estimation procedure to use the old variables to affect the estimation of the factor scores during the time periods in which those old variables were included in the FSI, but it will limit the old variables' influence on the FSI after the date of their removal. In the methodology that I describe, I prove that once a variable is removed from the index, it will have no direct effect of the FSI, in the sense that the FSI is a weighted sum of only the presently included variables. However, the old variables are still used to train the weights of this sum—which is desirable given the reasons I describe above.

The timing of how each variable is allowed to affect the FSI is illustrated in Figure 1. The IBOR-based variables are shown in the bottom seven rows. While these variables are included in the FSI, they directly affect its calculation in the sense of being included in the weighted sum of which the FSI consists. After January 1, 2022, new observations from the old variables are not used in any capacity (though observations of the old variables from before January 1 are still used to inform the model weights, even after the transition). In addition, notice that historical observations from the new variables—those that occurred before January 1, 2022—are shown to have an indirect effect of the FSI only after January 1. As described previously, this allows the FSI to use these histories to train the model weights. In summary, for each new variable included, we wish to use its full history to train the model upon inclusion. Also, when a variable is removed, we wish to completely cease using it to directly affect the model.

Another question that arises is, will the introduction of new variables change the historical values of the FSI? The methodology that I introduce makes it clear how the expanding window of data maps to each day's calculation of the FSI. This allows me to describe in detail the information sets that are used for each calculation. The FSI and similar indexes work by re-estimating a factor model every day. This estimation procedure produces estimated factor scores, but importantly, these factor scores and the value of the index are not the same objects. This is because the estimates for the factor scores, $\widehat{f}_{t|\tau}$ associated with day t but estimated using data up until day $\tau > t$, may change as τ increases. However,, the value of the index at day t is FSI_t and is set equal to the value of the score at time t estimated with data only up to time t. Thus, $\text{FSI}_t = \widehat{f}_{t|t}$. This distinction allows the values of the FSI to remain fixed over time, even as the set of variables used to inform the FSI changes over time.

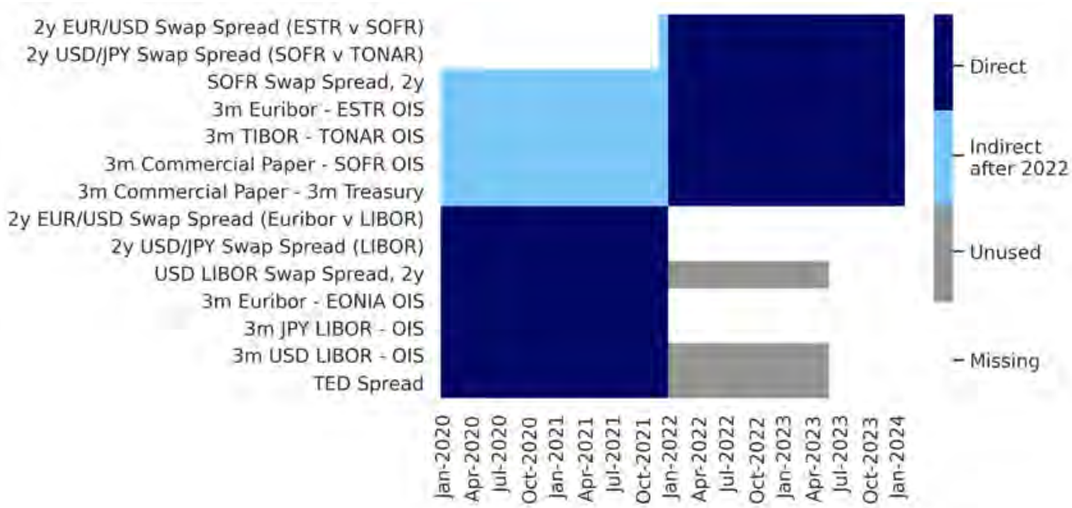
All these questions are ultimately related to the following general concern: will the index behave in a comparable fashion after the new variables are introduced? We can refer to this as a concern over the *time consistency* of the index. As described above, it is important to keep the information that is contained in the old variables, even after they have been removed from the index. Each substitution of variables must not throw this information away since that would likely cause drastic changes in the behavior of the FSI after each transition. However, it is not enough to ensure that this information stays. Time consistency of the index depends on the covariance structure of these new variables, which places importance on the actual choices made for each replacement, and also on some of the details with regard to the identification scheme used in the factor model.[2] Concerning whether the new replacement variables that I have chosen behave similarly to the old variables, Section 3 describes the economics behind the new variables and describes why they are appropriate choices. Section 4 analyzes the effect of these new variables on the FSI by backtesting the outcomes against an alternative, hypothetical scenario in which the substitutions are

[2]Factor models are typically only identified "up to a rotation" of the factor loadings and scores. Thus, a set of additional constraints is required and the choice of these constraints is somewhat arbitrary.

not made. These analyses show that the methodology and new variables chosen result in an index that behaves consistently across time, allowing for good comparability of the index across time.

In summary, the methodology that I propose here will make use of the full history of data from each time series to inform the FSI, the transition will not cause a revision of historical values of the FSI, and the index will behave roughly similarly both before and after the replacement. In the next section, I provide the full mathematical description. I will then show that the methodology has the properties I describe above by demonstrating that (1) the OFR FSI is a weighted sum of only the currently included variables; (2) removed variables no longer directly inform the index, and newly included variables do not modify historical values of the index; (3) the index uses all available historical data on the newly included variables to train the model; and (4) the weights used to construct the FSI are designed to remain roughly stable over time so that the level and volatility of the OFR FSI is roughly comparable over time.

Figure 1: The OFR FSI Indirectly Incorporates Historical Data from the New Variables Only After the Transition Date of January 1, 2022



As of January 1, 2022, the OFR FSI replaces the old variables with the new variables. The OFR FSI allows the historical data o the new variables to indirectly inform the FSI—but only after the transition date of January 1, 2022. This indirect effect works by allowing the historical data to "train" the factor loadings as soon as the variables are included. Without this, the estimation of the loadings would be very noisy until enough time had passed after the new variables' inclusion date. Furthermore, the traditional method would throw away a lot of valuable information that could be used to discipline the FSI. Also, even though a few variables htat are based on 3-month USD LIBOR are available until at least June 30, 2023, that data will not inform the FSI in any way. Historical data from old variables from before the transition will still be used to train the model. *Source: Refinitiv Datastream, Office of Financial Research*

2 Model

Here, I give a full mathematical description of the OFR FSI and demonstrate that the index, as defined, has the desirable properties described previously.

Defining the information sets in the model. Let X_{it} be a panel of normalized economic variables $i \in \mathcal{I}$ over time $t = 1, \ldots, T$. Let each variable be normalized to have mean zero and unit variance. Suppose that observations of some variables are missing or unavailable over certain time periods. Define M_{it} as a binary variable that is equal to 1 when a variable is not missing and is equal to 0 otherwise,

$$M_{it} \coloneqq \begin{cases} 1 & \text{variable } i \text{ is not missing at time } t \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Furthermore, suppose that one wishes to exclude certain variables i from the analysis on and after a particular time t. Let I_t be the set of variables $i \in I_t$ included in the analysis at time t, which I call the set of *included variables* or the *inclusion set.*

The distinction here between missing data and nonincluded variables will allow me to define the OFR FSI in such a way that when a new economic variable i on some date τ enters into the FSI (i.e., $i \in I_\tau$ but $i \notin I_{\tau-1}$), the full history of i will inform or "train" the FSI from date τ onward. Without this distinction, this hypothetical variable i would have to be artificially designated as missing at time $\tau - 1$ and not missing at time τ. This is problematic because, if we are to use the value $X_{i,\tau-1}$ to train the model weights of the FSI, the time $\tau - 1$ value's designation as missing or not missing would have to change over time. Hence, I treat *missing* and *included* here as separate concepts.

The desire to allow historical data to train the model weights of the FSI upon the inclusion of a new economic variable motivates the definition of a set that I call the set of *informing variables,*

$$\mathcal{I}_{t|\tau} = \bigcup_{s=t}^{\tau} I_s. \tag{2}$$

As an example, note that $\mathcal{I}_{\tau|\tau} = I_\tau$ and $\mathcal{I}_{\tau-1|\tau} = I_{\tau-1} \bigcup I_\tau$. Furthermore, let $\mathcal{I} \coloneqq \mathcal{I}_{1|T}$. The set of informing variables lists the economic variables $i \in \mathcal{I}_{t|\tau}$ that will be used to estimate the time t factor scores, which are in turn used to estimate the value of the FSI at time τ. Note that there will be a distinction made between estimated factor scores and the FSI. The FSI will be defined such that it does not use information on future values and is not revised as time progresses or as new variables are substituted in.

To further understand the definition of this set of informing variables, note that it is defined in (2) such that $\mathcal{I}_{t|\tau}$ is weakly increasing in τ. That is, for all τ, we have $\mathcal{I}_{t|\tau-1} \subseteq \mathcal{I}_{t|\tau}$. On each day τ that the FSI is published, the analysis of the covariation among historical observations, X_{it} with $t < \tau$, uses a weakly increasing set of information as new economic variables are included in the analysis. By contrast, note that $\mathcal{I}_{t|\tau}$ is weakly decreasing in t. That is, for all t, we have $\mathcal{I}_{t-1|\tau} \supseteq \mathcal{I}_{t|\tau}$. This captures the idea that historical estimates conditional on data up to $\tau \geq t$ can use newly included economic variables and potentially use currently nonincluded variables that were included in the past. This enforces our requirement that as soon as a variable is removed from the included set, it is no longer allowed to inform the current FSI but can still inform the training of the model in historical time periods in which the variabes were included.

In summary, for each new variable included, we wish to use its full history to train the model upon the variable's inclusion. Also, when a variable is removed, we wish to completely cease using it to inform the model, as illustrated in Figure 1.

Defining the FSI and its relationship to the underlying factor model. I now turn to defining the factor model that underlies the FSI and its associated expanding data window. Given observations of the economic variables X_{it}, suppose the variables follow a factor model with a single common factor,

$$X_{it} = w_i \cdot f_t + \varepsilon_{it}, \tag{3}$$

where ε_{it} are independent, identically distributed, mean-zero random variables and w_i and f_t are parameters to be estimated. Here, the common factor f_t is treated as a set of parameters to be estimated. In the case of the OFR FSI, it is assumed that there is only a single common factor, so that $w_i, f_t \in \mathbb{R}$ are scalar values.

Now, to make explicit the *expanding window* used in the index's computation, fix a time τ such that $1 < \tau \leq T$. Let T be some time period far in the future and let τ represent a day on which we would like to compute the FSI. The FSI computed on that day will be based on the estimate of the common factor at time τ, using data from $t = 1, \ldots, \tau$.

The estimate of this common factor, along with all other parameters of the model, is constructed as the least-squares estimate of the model, (3). Given a day τ on which we would like to construct the FSI, define the least-squares estimator of the factor model as

$$\{\widehat{w}_{i|\tau}\}_{i\in\mathcal{I}_{1|\tau}}, \{\widehat{f}_{t|\tau}\}_{t=1,\ldots,\tau} := \begin{cases} \underset{\{w_i\}_{i\in\mathcal{I}_{1|\tau}}, \{f_t\}_{t=1,\ldots,\tau}}{\text{argmin}} & \sum_{t=1}^{\tau} \sum_{i\in\mathcal{I}_{t|\tau}} M_{it}(X_{it} - w_i \cdot f_t)^2 \\ \text{s.t.} & 1 = \sum_{i\in I_\tau} M_{it} w_i^2 \\ & 0 < f_{t^*}, \end{cases} \tag{4}$$

where $t^* =$ September 15, 2008. Note that the least-squares problem does not have a unique solution unless the two constraints are added. The first constraint normalizes the size of the weights, so that the factor variance is unconstrained. The first constraint, however, is still not enough to ensure a unique solution, as the signs of the weights and factor scores may be reversed. Thus, the second constraint ensures that positive values of the estimated factor scores can be interpreted as heightened financial stress. In general, the choice of the day to use in this constraint is arbitrary, but one should boose a day on which it is generally agreed upon that financial stress was elevated. September 15, 2008, is a natural choice.

Having defined the underlying factor model as well as the expanding window, I now define the OFR FSI.

Definition: OFR Financial Stress Index. The OFR Financial Stress Index (FSI) on today's date is defined as the most recent value of the estimated factor score, given an expanding window of data up to the current date. That is, the FSI at time t is defined as

$$\text{FSI}_t = \widehat{f}_{t|t}. \tag{5}$$

Solution algorithm. The solution to (4) can be computed via an iterative procedure similar to an Expectation-Maximization algorithm. Given a time τ on which we would like to compute the FSI, the solution algorithm uses the following procedure:

1. Begin with guesses for the factor scores $\widehat{f}_{t|\tau}^{(0)}$ at each time $t = 1, \ldots, \tau$, ensuring that $\widehat{f}_{t^*|\tau}^{(0)} > 0$. The superscript on the guesses represents that this is step $k = 0$. After making these guesses, begin step $k = 1$.

2. Update the estimate of the factor loadings at step k by projecting the variables onto the previous step $k-1$ estimates of the factor scores, and then scale the loadings to satisfy the loading size constraint. Do this using the equation,

$$\widehat{w}_{i|\tau}^{(k)} = \frac{\sum_{t=1}^{\tau} M_{it}\,\mathbf{1}_{i\in\mathcal{I}_{t|\tau}}\,X_{it}\widehat{f}_{t|\tau}^{(k-1)}}{\lambda_\tau \mathbf{1}_{i\in I_\tau} + \sum_{t=1}^{\tau} M_{it}\,\mathbf{1}_{i\in\mathcal{I}_{t|\tau}}\,(\widehat{f}_{t|\tau}^{(k-1)})^2}, \tag{6}$$

   where $\mathbf{1}_{i\in\mathcal{I}_{t|\tau}}$ is an indicator function that is equal to 1 when the subscripted condition is satisfied, and is equal to 0 otherwise, and λ_τ is a Lagrange multiplier that is chosen so that $1 = \sum_{i\in I_\tau} M_{it}\,(\widehat{w}_i^{(k)})^2$. This step follows from the first-order condition for the weights, which is derived in Appendix A.2. Note that the Lagrange multiplier also follows from the first-order conditions and the associated constraint in which the loadings of the currently active, non-missing variables are scaled so that their norm equals 1.

3. Now, update the estimate for the factor scores at step k using the step k estimates of the factor loadings. This comes from projecting the variables onto

the loadings,

$$\widehat{f}_{t|\tau}^{(k)} = \frac{\sum_{i \in \mathcal{I}_{t|\tau}} M_{it}\, X_{it} \widehat{w}_{i|\tau}^{(k)}}{\sum_{i \in \mathcal{I}_{t|\tau}} M_{it}\, (\widehat{w}_{i|\tau}^{(k)})^2}. \tag{7}$$

This follows from the first-order condition for the factor scores, as derived in Appendix A.2.

4. Check whether $f_{t^*}^{(k)} > 0$. If it is not, reverse the signs of the loadings and the factor scores.

5. Check for convergence. Given a small, predetermined tolerance tol > 0, check whether

$$\max_{t \in 1,\ldots,\tau} |f_{t|\tau}^{(k)} - f_{t|\tau}^{(k-1)}| \overset{?}{<} \text{tol}. \tag{8}$$

If this condition is not met, return to step 2. Otherwise, the algorithm concludes.

◦

Using the first-order conditions associated with (4), it can easily be shown that the procedure above converges to the unique solution, given an arbitrarily small tolerance.

Now, having characterized the solution, I demonstrate some selected properties of the FSI. Each of these properties shows that the FSI, as defined, possesses certain desirable mathematical properties that we expect of an index that aims to provide daily estimates of the level of financial stress in the financial system. The first of these properties that I describe helps us to understand the nature of the solution to the defining least-square problem, (4), and it also helps us to understand the properties that follow.

Property 1 (The FSI at time t is a weighted sum of the values of the economic variables at time t.)**.** From the first-order condition, (11), we see that estimated factor scores are constructed by summing over the contemporaneous non-missing variable values. Since the constraint forces $\sum_{i\in I_t} M_{it}\left(\widehat{w}_{i|t}\right)^2 = 1$ and $\mathcal{I}_{t|t} = I_t$, we have

$$\mathrm{FSI}_t = \sum_{i\in I_t} \widehat{w}_{i|t} M_{it} X_{it}. \tag{9}$$

∘

Property 1 follows from the first-order conditions that characterize the solution to the least-squares problem. These are derived in Appendix A.2. This property allows for an easy decomposition of the FSI into the direct contributions of each variable. Furthermore, we can easily quantify the contribution of different categories of variables. The OFR FSI has 33 variables included at any given time. On January 1, 2022, the seven old variables were removed from the inclusion set I_t and the new variables took their place. These 33 variables are separated into 5 groups: credit, equity valuation, funding, safe assets, and volatility. Because the FSI is a weighted sum of these, we can decompose the weighted sum into the weighted sums within each group to evaluate how much each group contributes to the overall index. This is illustrated in Figure 2. Note that this figure represents these weighted sums as a stacked plot. When the value of a particular category is above zero, variables in that category push the overall value of the FSI in the positive direction. When two categories are both positive, their effects on the FSI are cumulative, and this is represented by stacking the two plotted areas on top of each other. Note that some variables may have a negative effect on the FSI while others simultaneously have a positive effect. For that reason, the OFR FSI, represented by the bold black line, may lie in the middle of the shaded regions.

Property 2 (Non-included economic variables don't directly affect the FSI.)**.** For all $i \notin I_t$, the partial derivative of the FSI to a perturbation in the variable X_{it}, fixing the factor loadings, is zero:

$$\left.\frac{\partial \mathrm{FSI}_t}{\partial X_{it}}\right|_{w_{i|t}} = 0. \tag{10}$$

∘

This property follows from a simple application of the envelope theorem and from the first-order condition associated with the factor scores

$$\mathrm{FSI}_t = \widehat{f}_{t|t} = \frac{\sum_{i \in I_t} M_{it}\, X_{it} \widehat{w}_{i|t}}{\sum_{i \in I_t} M_{it}\, (\widehat{w}_{i|t})^2}, \tag{11}$$

since $\mathcal{I}_{t|t} = I_t$ and $i \notin I_t$, by assumption. This property ensures that variables that have been removed from the FSI no longer directly affect the FSI. They may, however, still indirectly affect the FSI via their effects on the estimation of the factor loadings (the weights). However, Property 2 calculates the effect of the perturbation, holding the weights constant.

Property 3 (Past observations of currently included variables that were previously not included *do* affect the current FSI.)**.** Suppose that a variable i is included currently, $i \in I_\tau$, but was not included at a previous time t, with $t < \tau$. Suppose further that $M_{it} = M_{i\tau} = 1$, that $X_{i\tau}$, $f_{t|\tau}$, and that $\widehat{w}_{i|\tau}$ are all non-zero. Then,

$$\frac{\mathrm{dFSI}_\tau}{\mathrm{d}X_{it}} \neq 0. \tag{12}$$

∘

This property follows again from the same first-order condition and an application of the envelope theorem (see Appendix A.2). The effect works through the effect of the historical data on the estimation of the factor loadings,

$$\frac{\mathrm{dFSI}_\tau}{\mathrm{X}_{it}} = \frac{\partial \mathrm{FSI}_t}{\partial w_{i|\tau}} \frac{\partial \widehat{w}_{i|\tau}}{\partial X_{it}} = X_{i\tau} \frac{\partial \widehat{w}_{i|\tau}}{\partial X_{it}},$$

where $\frac{\partial \widehat{w_{i|\tau}}}{\partial X_{it}}$ is nonzero because $i \in \mathcal{I}_{t|\tau}$, even though $i \notin I_\tau$. The benefit of this property is that when a new variable is included at a time τ, historical information about that variable is then used to compute the factor loadings but has no effect of the FSI before its inclusion. This means that historical, non-missing data is not thrown away, even when a variable is not included in the FSI. In the case of the chosen replacement variables, while several of the new variables have only very short histories before their inclusion in the FSI on January 1, 2022, others have long histories, as can be seen in Table 3. This property of the FSI allows this history to start informing the FSI as soon as the variable is included in the FSI. This property is illustrated in Figure 1. Each row of the graph illustrates the data associated with each variable. White-shaded regions illustrate missing data, and dark blue regions illustrate periods of time in which the values of those variables are included in the

OFR FSI and directly affect its calculation, in the sense of Property 1. As can be seen, the replacement variables begin directly affecting the FSI after January 1, 2022, and the old variables cease affecting the FSI at the same time. The gray regions illustrate the period of time in which the non-missing data on the variables is ignored by the FSI because the variables are not included in the FSI at that time. This is the case with the variables based on USD LIBOR. The light blue regions illustrate non-missing historical data from the new variables that are allowed to indirectly affect the FSI after January 1, 2022, in the sense of Property 3 (through their effect on the training of the factor loadings).

Property 4 (The volatility of the FSI is roughly comparable before and after the transition, as long as the number of variables in I_t stays constant and the new variables behave similarly to the old variables.)**.** This property is suggestive, based on the following argument. Suppose that the factor scores are computed by giving equal weight to all economic variables and that there are no missing data points, so that

$$\mathrm{FSI}_t = \sum_{i \in I_t} \widehat{w}_{i|t} X_{it}.$$

Given that each variable is given equal weight, in order to satisfy the normalization constraint $1 = \sum_{i \in I_t} \widehat{w}_{i|t}^2$, we must have $\widehat{w}_{i|t} = 1/\sqrt{N}$. This implies that

$$\mathrm{Var}(\mathrm{FSI}_t | \widehat{w}_{i|t}) = N\bar{\sigma}, \tag{13}$$

where

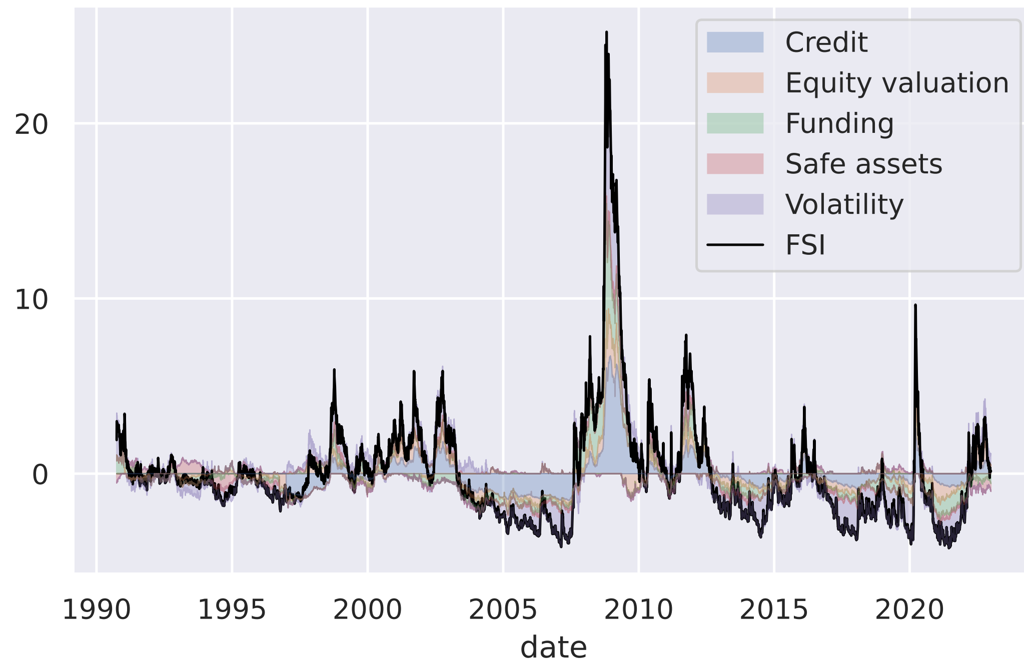
$$\bar{\sigma} = \frac{1}{N^2} \sum_{i \in I_t} \sum_{j \in I_t} \mathrm{Cov}(X_{it}, X_{jt}).$$

This suggests that the variance of the FSI will be roughly comparable over time, as long as the covariance structure of the new economic variables is similar to that of the old ones and as long as the total number of variables included in I_t stays the same over time (N does not change over time). $\circ$

This property is built into the definition of the constraint on the weights specified in (4). One could imagine specifying the constraint to sum over all variables $i \in \mathcal{I}_{1|\tau}$ or $i \in \mathcal{I}$, rather than the chosen set $i \in I_t$. Summing the weights over these other sets would cause the weights used in the FSI's calculation to systemically shrink over time as new variables are substituted in. The currenlty chosen constraint avoids this problem, thus better preserving the ability to compare the levels and volatility of the OFR FSI over time.

Properties 1 – 3 support the claim in Property 4 that the OFR FSI will maintain some degree of smoothness and comparability in its transition to the new variables. At present, the OFR FSI has $N = 33$ economic variables included. If whenever a variable is removed another is substituted in, then the FSI will always be a weighted sum of exactly $N = 33$ variables. As long as the correlation structure of the new variables is roughly similar to that of the old variables (and it is), the volatility of the FSI should remain roughly the same. I will elaborate on this further in the remaining sections. In the next section, I discuss the process of selecting the new variables In the section following that, I discuss the effects of these substitutions on the calculation of the FSI.

Figure 2: The Five Categories of Variables and Their Contributions to the OFR FSI



Property 1 of the OFR FSI demonstrates that the FSI is the weighted sum of 33 variables. This allows us to easily visualize how much each variable and each category of variables contributes to the overall value of the FSI. This figure represents these weighted sums as a stacked plot. When the value of a particular category is above zero, variables in that category push the overall value of the FSI in the positive direction. When two categories are both positive, their effects on the FSI are cumulative, and this is represented by stacking the two plotted areas on top of each other. Note that some variables may have a negative effect on the FSI while others simultaneously have a positive effect. For that reason, the OFR FSI, represented by the bold black line, may lie in the middle of the shaded regions. *Source: Refinitiv Datastream, Office of Financial Research*

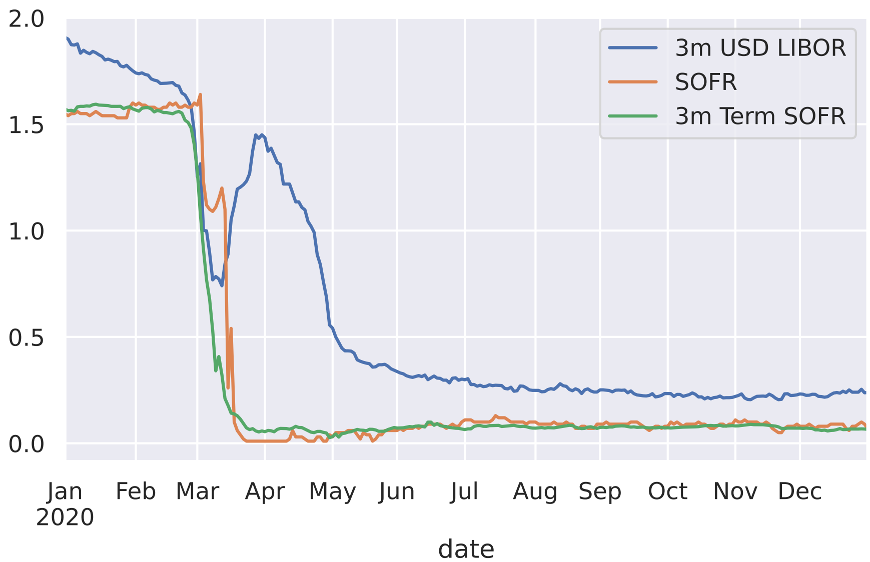
3 Selection of Replacement Variables

The initial version of the OFR FSI used 33 economic variables across 5 variable categories. There are 7 of these variables that depend on LIBOR or other discontinued reference rates. The 7 variables comprise the entire set of variables in the funding category. These variables and their dependencies are listed in Table 1. In this section, I will describe the replacement variables and the process by which they were selected. The chosen replacement variables are listed in Table 2. The goal of the selection process was to adhere to the recommendations of the governing body within each currency jurisdiction and to choose variables with economic properties similar to those of the old variables, in order to preserve the continuity of the OFR FSI. To demonstrate the effects of these changes, I will recalculate the FSI using different variable replacement strategies to evaluate the effect of these replacements on the index. For these exercises, I obtain data on each of these economic variables from Refinitiv Datastream or from Bloomberg.

Recommended alternative reference rates in various currency jurisdictions. As described previously, interbank offered rates (IBORs), including the London Interbank Offered Rate (LIBOR), have for many years served as widely accepted benchmark interest rates. Following the 2013 review of interest rate benchmarks by the Financial Stability Board, the Financial Conduct Authority declared in 2017 that it would no longer compel banks to continue making LIBOR submissions after December 31, 2021. This began what is now known as the IBOR transition. In the US, the Alternative Reference Rate Committee (ARRC) identified the Secured Overnight Financing Rate (SOFR) as the recommended alternative to USD LIBOR. The work of the ARRC to identify a new benchmark rate was complemented by parallel efforts in other currency jurisdictions. For the purposes of replacing dependencies on USD LIBOR, JPY LIBOR, and EONIA, our focus is on the recommendations of the ARRC, the Financial Services Agency, the Bank of Japan, the Cross-Industry Committee on Japanese Yen Interest Rate Benchmarks, the European Central Bank, and the Working Group on Euro Risk-Free Rates. Accordingly, in Japan, the Tokyo O/N Average rate (TONA rate or TONAR) is the recommended main benchmark for JPY interest rate swaps. However, market participants are not precluded from using other benchmarks, including the Tokyo Term Risk Free Rate (TORF) and the Japanese Bankers Association Tokyo Interbank Offered Rate (TIBOR), as deemed necessary (see Cross-Industry Committee on Japanese Yen Interest Rate Benchmarks 2021). Furthermore, in Europe, the Euro Short-Term Rate (ESTR) is the recommended replacement for the Euro Overnight Index Average (EONIA). Note that the term rates of Euribor are expected to continue to be used. These recommendations serve as a guide to identifying appropriate replacement variables within the OFR FSI.

The economic characteristics of the variables. The goal of the OFR FSI is to provide daily updates on financial conditions. In order to provide a measure of how financial stress changes over time, and to provide a time-consistent metric, it is important to maintain the economic characteristics and composition of the underlying variables over time. Most visibly, this applies to the decision to include

Figure 3: SOFR and USD LIBOR Briefly Diverged in March 2020

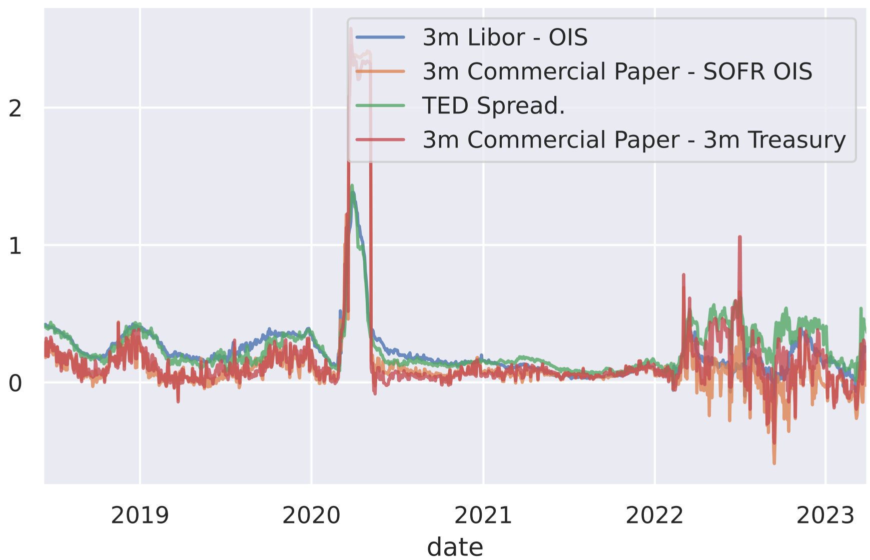


The 3-month USD LIBOR rate compared to SOFR in March 2020 illustrates the difference in behavior that may arise from the differences in the economic properties of different variables. Since LIBOR represents unsecured borrowing, it contains a credit-sensitive component that SOFR does not. During March 2020, it was reasonable to assume that the credit risks of large banks were heightened, even while at the same time, risk-free rates were decreasing—explaining the temporary divergence of these two rates. This difference cannot be attributed to the term nature of LIBOR, as a similar increase was not present in Term SOFR rates either. *Source: Refinitiv Datastream, Bloomberg, Office of Financial Research* commercial paper as a variable. The OFR FSI uses the US 3-month AA financial commercial paper rate (published by the Board of Governors of the Federal Reserve System) in the place of 3-month USD LIBOR.

Commercial paper is unsecured, short-term debt issued for a specified amount to be paid at a specified date. USD LIBOR purported to measure the cost of unsecured, short-term borrowing between large banks. SOFR, on the other hand, is a transactions-based measure of secured, overnight borrowing within a broadly defined market. Figure 3 illustrates the differences that can arise between SOFR and LIBOR due to these differences. Since LIBOR represents unsecured borrowing, it contains a credit-sensitive component that SOFR does not. During March 2020, concerns over broad macroeconomic conditions led to an increase in credit risk. However, at the same time, decreases in the Fed Funds rate pushed SOFR lower. This led to a temporary divergence between these two rates in early 2020. On the other hand, commercial paper rates increased in early 2020 and demonstrated elevated volatility in 2022, mirroring LIBOR. This can be seen in Figure 4. Note that whenever values of the commercial paper rate are missing, I use the last observed value.

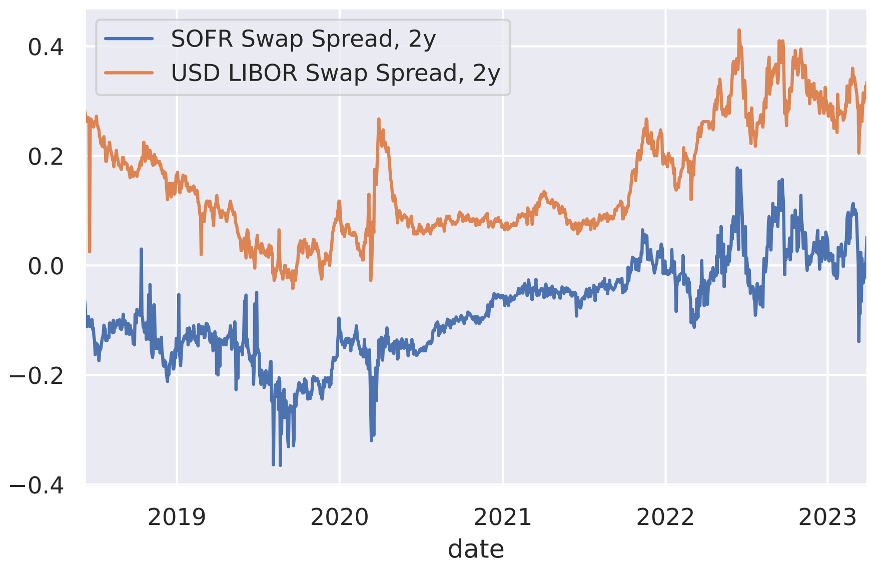
As a reference rate upon which to base financial contracts, SOFR is superior to LIBOR in many ways. Besides SOFR's robustness to potential manipulation, a perhaps underappreciated fact about SOFR is that it has likely been more correlated with banks' overall average funding costs than LIBOR, even during the 2007-2009 crisis (Bowman, Scotti, and Vojtech, 2020). However, banks fund themselves in a variety of ways, and unsecured borrowing remains an important, if decreasing, component of their liability exposures. Accordingly, the funding category of the FSI

Figure 4: Commercial Paper and LIBOR



This plot compares two old variables (3-month USD LIBOR - OIS spread and the TED spread) against their replacements (3-month commercial paper - SOFR OIS and 3-month commercial paper - 3-month Treasury yields). The 3-month USD LIBOR rate and 3-month US AA Financial Commercial Paper rate share the same rough characteristics. The both spiked during March 2020 and exhibited elevated volatility during 2022. Note that whenever values of the commercial paper rate are missing, I use the last observed value. *Source: Refinitiv Datastream, Office of Financial Research*

Figure 5: SOFR and LIBOR Swap Spreads



The 2-year USD LIBOR Swap Spread and the 2-year SOFR Swap Spread behave similarly, with a correlation of 74% over the period from mid-2019 to 2023. The greatest differences in behavior, occurring in early 2020, mirror the temporary divergence of SOFR and LIBOR at the same time. *Source: Refinitiv Datastream, Office of Financial Research*

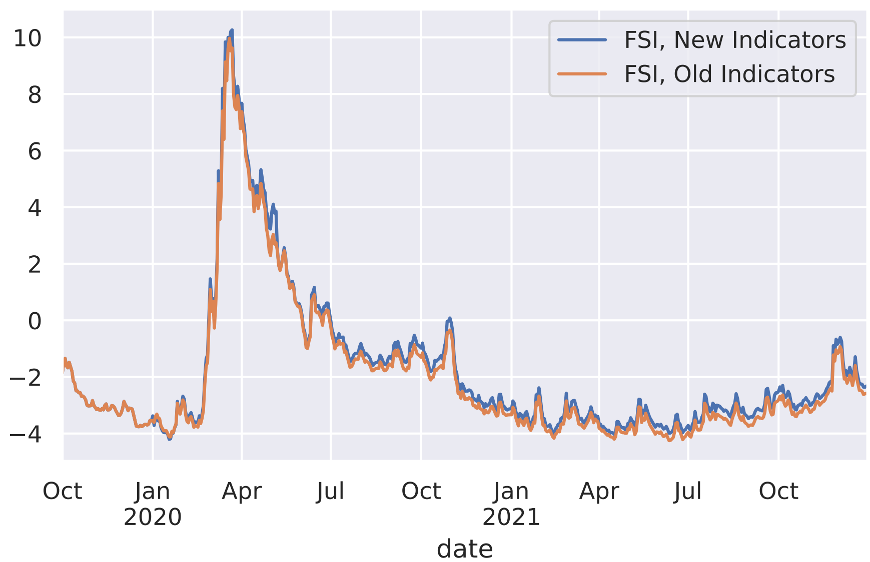
includes variables that measure stress in both secured (e.g., the 2-year SOFR Swap Spread) and unsecured (e.g., commercial paper) funding markets.

Aside from directly replacing 3-month USD LIBOR with commercial paper rates, the OFR FSI replaces variables that are based on products referencing a US benchmark rate with variables based on products referencing SOFR. This includes the cross-currency swap spreads and the overnight index swaps (OIS). Given the importance of the market for repurchase agreements (repo market) as a source of financing, these SOFR-based variables improve the FSI in many ways. Consider, for example, the 2-year US Swap Spread variable. The previous variables used the difference between the rate on a LIBOR swap and the yield on a US Treasury. However, using a SOFR swap, in this case, is a more natural fit for the FSI. The 2-year SOFR swap spread is the difference between the 2-year SOFR OIS rate and the yield on a 2-year Treasury bill. To understand the economic meaning of this spread, consider a firm that owns a 2-year bill and needs to raise cash. The firm could sell the bill, foregoing the yield. Alternatively, the firm could borrow cash in the repo market and use the bill as collateral, rolling over the repo over the two-year period. However, since rates in the repo market fluctuate, the SOFR OIS rate is used to hedge the interest rate risk and thus reflects the expected cost of rolling over the repo over this period. Given the replicating nature of these two strategies, this spread should in practice remain near zero. However, various sources of financial stress will tend to move this spread away from zero. Figure 5 shows how the 2-year SOFR swap has remained near zero over its existence. It also shows how LIBOR and SOFR swaps behave similarly, with a correlation of 74% over the period from mid-2019 to the present.

4 The Effect of the New Variables on the OFR FSI

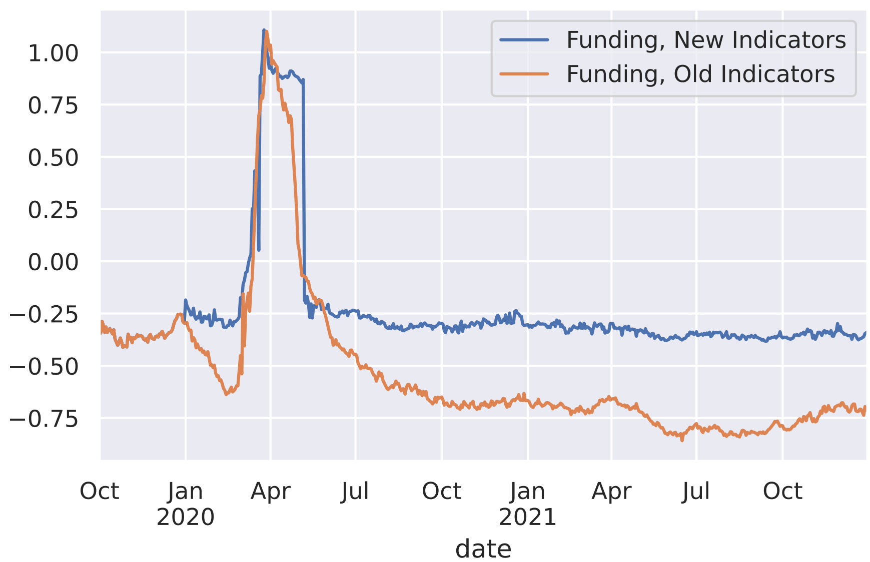
In this section, I will consider how the inclusion of these new variables affects the FSI. Since many of the new variables do not have available data that goes back very far in time (see Table 3), I will consider a hypothetical scenario in which the replacement variables were substituted in for the old variables on January 1, 2020. Note that the earliest available observations for the cross-currency swap spread variables

Figure 6: The New Variables Produce an FSI that Behaves Like The Old One



Here, I calculate the values of the FSI under a hypothetical scenario in which the replacement variables (with the exception of the cross-currency swap variables) were substituted in on January 1, 2020 (SOFR-based cross-currency swaps are not available in the data until the end of 2021). The result is an FSI that looks like the old FSI, with the two series having a correlation of 99.75%. *Source: Refinitiv Datastream, Office of Financial Research*

Figure 7: The Effect of the New Variables on the Funding Component of the FSI Captures the Spike in Early 2020, But Not the Slow Decline in Late 2020



Here, I calculate the values of the funding component of the FSI under the hypothetical scenario in which the replacement variables were substituted in January 1, 2020, again with the exception of the cross-currency swap variables. Focusing on this component of the FSI magnifies the effect of the substitution. Although the two series are somewhat different, they have a correlation of 87.11%. *Source: Refinitiv Datastream, Office of Financial Research*

Table 4: Correlation of Replacement Variables with Old Variables and the Old FSI

Old Indicator	New Indicator	Corr(Old, New)	Corr(Old, FSI)	Corr(New, FSI)
2y EUR/USD Swap Spread (Euribor v LIBOR)	2y EUR/USD Swap Spread (ESTR v SOFR)	0.92	-0.50	-0.56
2y USD/JPY Swap Spread (LIBOR)	2y USD/JPY Swap Spread (SOFR v TONAR)	0.63	-0.20	-0.52
USD LIBOR Swap Spread, 2y	SOFR Swap Spread, 2y	0.58	0.29	-0.45
3m Euribor - EONIA OIS	3m Euribor - ESTR OIS	1.00	0.69	0.69
3m JPY LIBOR - OIS	3m TIBOR - TONAR OIS	0.59	0.62	0.46
3m USD LIBOR - OIS	3m Commercial Paper - SOFR OIS	0.87	0.83	0.76
TED Spread	3m Commercial Paper - 3m Treasury	0.88	0.78	0.75

This table reports the correlation between each pair of old and new variables, as well as the correlation of each variable with the old version of the FSI. These correlations are calculated over the intersection of dates for which the FSI, old variable, and the new variable are all available so that the reported correlations across the columns each row are comparable. *Source: Refinitiv Datastream, Office of Financial Research*

is on November 24, 2021, so this hypothetical scenario will replace all variables except for these two cross-currency swap variables. We can then assess the impact of substituting in the remaining five replacement variables over the period of January 1, 2020 – January 1, 2022. The comparison of the FSI computed with the original variables to the FSI computed with these five new variables is shown in Figure 6. As can be seen, the resulting differences are very small. The two versions of the FSI have a correlation of 99.75%. This is unsurprising, given how similar the behavior of the new variables is to that of the old variables. This can be seen in Table 4, which reports the correlation between each pair of old and new variables, as well as the correlation of each variable with the old version of the FSI. These correlations are calculated over the intersection of dates for which the FSI, old variable, and new variable are all available, so that the reported correlations across the columns each row are comparable.

Now consider the effect of these substitutions only on the funding component of the FSI. The hypothetical substitution has replaced five of the seven variables in this category. This comparison is plotted in Figure 7. Focusing on this component of the FSI magnifies the effect of the substitution. Although the two series are somewhat different, they have a correlation of 87.11% and both exhibit a large spike in March 2020. However, the old funding component of the FSI suggests a slow decline in the degree of financial stress in funding markets since mid-2020, whereas the new variables show a much milder decline. Part of this is explained by the fact that, over this sample period, the USD LIBOR swap spread variable loads positively on the FSI while the SOFR swap spread variable loads negatively, as can be seen in Table 4. As more data becomes available, this may change. Regardless, the inclusion of the SOFR swap spread as a variable focuses on an important dimension that was not as prominent in the old version of the FSI.

5 Conclusion

In this paper, I have detailed the construction of the OFR Financial Stress Index and made explicit how it will transition away from LIBOR-based variables, in addition to any other ceasing benchmark rates. The original version of the OFR FSI was designed to accommodate missing data and unbalanced panels, as outlined in Monin (2019). This paper expands on the methodology by demonstrating how the OFR FSI will allow for historical data on new variables to inform the index without introducing look-ahead bias. This paper makes explicit how the expanding window in the FSI works and how the transition to new variables will allow historical data to affect the calculation of the FSI only after the date of inclusion of the new variables in the index.

I have also discussed the decision process by which the new variables were chosen. Where possible, I have followed the recommendation of the relevant governing bodies. I have also chosen variables that will preserve the time-consistent interpretation of the index as best as possible. I have also shown that the inclusion of these new variables has only a small effect on the FSI and that the interpretation of the FSI will remain the same going forward.

6 References

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A Appendix

A.1 Economic Variables in the OFR FSI

Here, I provide the full list of economic variables used in the OFR FSI. These are listed in Table 5, along with the dates when they enter the FSI and exit the FSI, as well as the financial and geographic categories to which they are assigned. Note that a series is only included on days that occur strictly *before* the listed end date but is included in the index starting *on* the listed start date. Thus, as written, there are only 33 series included in the FSI at any one time. Note also that when a variable is assigned to multiple categories, its effect on the overall FSI is divided equally among the assigned categories.

Here, I also provide the ticker expressions used to pull each data series from Refinitiv Datastream. These expressions are listed in Table 6.

Table 5: Economic Variables in the OFR FSI

Indicator Name	Start Date	End Date	Financial Category	Geographic Category
CEMBI Strip Spread(bps)			Credit	EM
EMBIGLOBAL Strip Spread(bps)			Credit	EM
Euro Corp Bond OAS			Credit	AE
Euro Corp High Yield OAS			Credit	AE
HY Cash OAS			Credit	US
IG Cash OAS			Credit	US
JP Corporate OAS			Credit	AE
MSCI EM Equities			Equity valuation	EM
MSCI Europe Equity			Equity valuation	AE
Nikkei			Equity valuation	AE
SPX			Equity valuation	US
Dollar			Safe assets	US
Germany 10 YR			Safe assets	AE
Gold			Safe assets	US, AE, EM
Swiss Franc			Safe assets	AE
US Treasuries (10y)			Safe assets	US
Yen			Safe assets	AE
2y EUR/USD Swap Spread (Euribor v LIBOR)		2022-01-01	Funding	US, AE
2y EUR/USD Swap Spread (ESTR v SOFR)	2022-01-01		Funding	US, AE
2y USD/JPY Swap Spread (LIBOR)		2022-01-01	Funding	US, AE
2y USD/JPY Swap Spread (SOFR v TONAR)	2022-01-01		Funding	US, AE
3m Euribor - EONIA OIS		2022-01-01	Funding	AE
3m Euribor - ESTR OIS	2022-01-01		Funding	AE
3m JPY LIBOR - OIS		2022-01-01	Funding	AE
3m TIBOR - TONAR OIS	2022-01-01		Funding	AE
3m USD LIBOR - OIS		2022-01-01	Funding	US
3m Commercial Paper - SOFR OIS	2022-01-01		Funding	US
TED Spread		2022-01-01	Funding	US
3m Commercial Paper - 3m Treasury	2022-01-01		Funding	US
USD LIBOR Swap Spread, 2y		2022-01-01	Funding	US
SOFR Swap Spread, 2y	2022-01-01		Funding	US
Brent Front Month			Volatility	US, AE, EM
EM-VXY (3m)			Volatility	EM
EU 3m swaption vol			Volatility	AE
EURUSD 6m vol			Volatility	US, AE
US 3m swaption vol			Volatility	US
USDJPY 6m vol			Volatility	US, AE
V2X			Volatility	AE
VIX			Volatility	US
VNKY			Volatility	AE

This table lists all economic variables used in the OFR Financial Stress Index. The Equity Valuation variables are transformed using the logarithm of the ratio of the variable to its 250-trading day moving average. The Gold/USD real spot exchange rate, the Japanese Yen/USD spot exchange rate, the Swiss Franc/USD spot exchange rate, and the US Dollar Index (DXY) are also computed using the logarithm of the ratio of the variable to its 250-trading day moving average. The 10-year US Treasury yield and the 10-Year German Bond yield, are computed as the difference between the variable and its 250-trading day moving average. A blank value under Start Date indicates that the series uses data from as early as is available. A blank value under End Date indicates that the series will be used indefinitely unless further changes are announced. *Source: author's creation.*

Table 6: Ticker Expressions Used to Pull Data for Each Series

Indicator Name	Datastream Query
CEMBI Strip Spread(bps)	JPCCECE(BSPRD)
EMBIGLOBAL Strip Spread(bps)	JPMGTOT(BSPRD)
Euro Corp Bond OAS	MLEC00L(OAS)
Euro Corp High Yield OAS	MLHEUCL(OAS)
HY Cash OAS	MLHMACL(OAS)
IG Cash OAS	MLCORML(OAS)
JP Corporate OAS	MLJPCPL(OAS)
MSCI EM Equities	MSEMKF$/@:M2EMG(AB12TB)
MSCI Europe Equity	MSEROP$/@:M1EROP(AB12TB)
Nikkei	WIJPAN$/@:JPFTSP(AB12TB)
SPX	S&PCOMP/@:USSP500(AB12TB)
CPI	USCONPRCE
Dollar	NDXYSPT
Germany 10 YR	TRBD10T
Gold	GOLDBLN
Swiss Franc	TDCHFSP
US Treasuries (10y)	TRUS10T
Yen	TDJPYSP
2y EUR/USD Swap Spread (Euribor v LIBOR)	.5*(EUCBS2Y(IB) +EUCBS2Y(IO))
2y EUR/USD Swap Spread (ESTR v SOFR)	EUUSC2Y
2y USD/JPY Swap Spread (LIBOR)	.5*(JPCBS2Y(IB) +JPCBS2Y(IO))
2y USD/JPY Swap Spread (SOFR v TONAR)	JPUSC2Y
3m Euribor - EONIA OIS	EIBOR3M-OIEUR3M
3m Euribor - ESTR OIS	EIBOR3M-EESTR3M
3m JPY LIBOR - OIS	BBJPY3M-.5*(OIJPY3M(IB) +OIJPY3M(IO))
3m TIBOR - TONAR OIS	JPIBK3M-.5*(OIJPY3M(IB) +OIJPY3M(IO))
3m USD LIBOR - OIS	BBUSD3M-.5*(OIUSD3M(IB) +OIUSD3M(IO))
3m Commercial Paper - SOFR OIS	FRCPF3M-USDSR3M
TED Spread	(BBUSD3M-TRUS3MT)*100
3m Commercial Paper - 3m Treasury	FRCPF3M-TRUS3MT
USD LIBOR Swap Spread, 2y	TRUSS2Y(IB)
SOFR Swap Spread, 2y	USDSR2Y-TRUS2YT
Brent Front Month	LLCC.01
EM-VXY (3m)	JPVXYEM
EU 3m swaption vol	(RFE3A2S(V2)*.20)+(RFE3A5S(V2)*.20) +(RFE310S(V2)*.40)+(RFE330S(V2)*.20)
EURUSD 6m vol	FVEUR6M
US 3m swaption vol	SMOVE3M
USDJPY 6m vol	FVJPY6M
V2X	VSTOXXI
VIX	CBOEVIX
VNKY	VXJINDX

This table lists all economic variables used in the OFR Financial Stress Index along with the Refinitiv Datastream ticker expression used to pull each series. Note that some historical data gaps are filled with data from other sources. Also, note that CPI is used to compute the real exchange rate between Gold and USD. On each day, the last available measurement of the CPI is used. *Source: author's creation.*

A.2 First-order Conditions for the Least-squares Solution to the Factor Model

Here, I derive the first-order conditions that characterize the solution to the least-squares solution to the factor model described in (4).

Proof. Define the Lagrangian for the time τ problem

$$\mathcal{L}_\tau = \sum_{t=1}^{\tau} \sum_{i \in \mathcal{I}_{t|\tau}} M_{it}(X_{it} - w_i \cdot f_t)^2 + \lambda_\tau \left(\sum_{i \in I_\tau} w_i^2 - 1 \right). \tag{14}$$

Note that in the following, I suppress some of the notation for the time τ problem for brevity. Thus, I write $w_{i|\tau}$ as w_i and $f_{t|\tau}$ as f_t. The first-order conditions for the weights $w_i \in \mathcal{I}_{1|\tau}$ are

$$0 = \frac{\partial \mathcal{L}}{\partial w_i} = -2 \sum_{t=1}^{\tau} \mathbf{1}_{i \in \mathcal{I}_{t|\tau}} M_{it}(X_{it} - \widehat{w}_i \widehat{f}_t) f_t + 2\lambda_\tau \, \mathbf{1}_{i \in I_\tau} \widehat{w}_i, \tag{15}$$

where $\mathbf{1}_{i \in I_\tau}$ is an indicator function that is equal to 1 when the subscripted condition is satisfied and is equal to 0 otherwise. $\mathbf{1}_{i \in \mathcal{I}_{t|\tau}}$ is defined likewise. The first-order conditions associated with the factor scores are

$$0 = \frac{\partial \mathcal{L}}{\partial f_t} = -2 \sum_{i \in \mathcal{I}_{t|\tau}} M_{it}(X_{it} - \widehat{w}_i \widehat{f}_t) \widehat{w}_i. \tag{16}$$

These can be solved in terms of the weights and factor scores,

$$\widehat{w}_i = \frac{\sum_{t=1}^{\tau} \mathbf{1}_{i \in \mathcal{I}_{t|\tau}} M_{it} X_{it} \widehat{f}_t}{\mathbf{1}_{i \in I_\tau} \lambda_\tau + \sum_{t=1}^{\tau} \mathbf{1}_{i \in \mathcal{I}_{t|\tau}} M_{it} \widehat{f}_t^2} \tag{17}$$

$$\widehat{f}_t = \frac{\sum_{i \in \mathcal{I}_{t|\tau}} M_{it} X_{it} \widehat{w}_i}{\sum_{i \in \mathcal{I}_{t|\tau}} M_{it} \widehat{w}_i^2}. \tag{18}$$

Together with the constraints,

$$1 = \sum_{i \in I_\tau} \widehat{w}_i^2 \tag{19}$$

$$f_{t^*} > 0, \tag{20}$$

these four equations uniquely characterize the solution.

$\square$