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Technology Shocks and Predictable Minsky Cycles

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Why These Findings Are Important

This paper offers an economical and internally consistent model to rationalize macrofinancial boom-bust cycles. The authors present a simple model that can clarify the interaction of optimism with capital reallocation and demonstrate how this interaction can generate predictable boom-bust financial cycles. This clarification enhances our understanding of the channels through which credit markets could threaten financial stability. The model is also a starting point for future research that could have important implications for welfare and policy. Adding more features to the model could help identify optimal policy responses to mitigate the boom's initial size and thus mitigate the bust's size.

Key Findings

We provide a model to understand what types of economic shocks might cause a predictable boom followed by a bust in credit markets. The model reveals the following findings:

1. News leads to a rise in asset prices via the financial markets, allowing for more immediate leverage.
2. Because the shock primarily impacts a secondary sector, capital is reallocated from its primary users to its secondary users.
3. The fact that the primary users are leveraged leads to a bust when users reallocate capital to repay debts.

How the Authors Reached These Findings

The authors use a theoretical macro model with credit markets to investigate what types of shocks can generate boom-bust cycles. They solve out the equilibrium dynamics in a standard macrofinance model and consider the consequences for output and welfare. The analysis allow us to differentiate between different types of shocks.

The baseline model is identical to the model proposed by Kiyotaki and Moore (1997). The authors' model's only addition to this standard building block is an anticipated technology shock to an innovative sector.

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Abstract

Big technological improvements in a new, secondary sector lead to a period of excitement about the future prospects of the overall economy, generating boom-bust dynamics that propagate through credit markets. Increased future capital prices relax collateral constraints today, leading to a boom before the realization of the shock. But reallocation of capital toward the secondary sector when the shock hits leads to a bust going forward. These cycles are perfectly foreseen in our model, making them markedly different from the typical narrative about unexpected financial shocks that is used to explain crises. In fact, these cycles echo Minsky's original narrative for financial cycles, according to which, "financial trauma occur as normal functioning events in a capitalistic economy." (Minsky, 1980, p. 21)

Keywords: cyclical propagation, endogenous cycles, boom-bust dynamics, optimism, credit markets, predictability

JEL classification: E22, E23, E32, E44

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1 Introduction

There is an old idea in macroeconomics that major technological advances generate dramatic boom-bust cycles that typically finish in a pronounced and somewhat predictable financial retrenchment. There is growing recent evidence that market economies are subject to cyclical boom-bust dynamics, with times of expansion sowing the seeds of the slump that follows (see Beaudry, Galizia, and Portier, 2017, 2020). The Great Recession, the Great Depression, and the Japanese slump of the 1990s were all preceded by periods of major technological innovation (Cao and L'Huillier, 2018), and it is easy to find similar evidence for the case of sudden stops in emerging markets (Boz, 2009).

It is well known that these major episodes often feature credit expansions during the boom and financial contractions, or even crises, during the subsequent slump. Economists since Fisher (1933), Keynes (1936), and Minsky (1982, 1986) have seen the behavior of financial markets as playing a central role in economic downturns. There remains considerable debate about the causes and consequences of recessions, and still less is known about the role (if any) of endogenous boom-bust dynamics emphasized by, for example, Kindleberger and Aliber (2011) and Minsky (1980, 1982).

The objective of this paper is to rationalize these ideas in a canonical macro-finance framework. We consider a standard model with collateral constraints following Kiyotaki and Moore (1997). We add two main ingredients to this model. The first ingredient is the presence of news, in the form of advance information, about a positive technology shock that will hit the economy in the future. The second ingredient is the presence of an innovative sector. The technology shock we consider does not primarily impact agents who are already investing with leverage, but instead primarily affects a new, secondary sector that is not *already* leveraged. This sector may take on leverage in order to invest, but it does not have preexisting debts.

As we show, the combination of these novel ingredients makes the task of obtaining predictable Minsky cycles straightforward. In fact, our baseline model features perfect foresight and rational expectations, and therefore, we do not need to introduce any type of sluggishness in beliefs or behavioral failure to anticipate the equilibrium consequences of the productivity shock. Thus, the mechanisms of the Minsky cycle in our model are, at face value, quite distinct from the "Wile E. Coyote" moment in the literature (see Eggertsson and Krugman, 2012); financial retrenchment in our model is completely predictable and foreseen.

Our paper offers a parsimonious and internally consistent model to rationalize macro-financial boom-bust cycles. Two essential channels are responsible for this, each owing to the novel ingredients emphasized above. First, news leads to a rise in asset prices via the financial market, immediately allowing for more leverage. Second, because the shock primarily impacts a secondary sector, capital is reallocated from its primary users to its secondary users. Since the primary users are leveraged, this leads to a bust when reallocation takes place to repay debts. Importantly, the boom is fueled entirely by an expansion of credit, but borrowers do not have additional resources with which to repay their debts. This means that the present value of the credit boom must be matched by a credit *bust* of equal present value (see Proposition 1).

We describe these equilibrium dynamics as an endogenous boom-bust cycle. Our story is about the endogenous *propagation* of a shock into a boom and bust as follows. A shock directly affects the non-leveraged sector in the future. Due to endogenous forces acting through credit markets, the economy immediately experiences a boom in the leveraged sector in advance of the shock. Then, when credit markets endogenously tighten in the future, it is the leveraged sector that experiences a bust going forward. Thus, the boom-bust dynamics occur in a sector unaffected by the shock, and this propagation is due to endogenous forces in financial markets.

The two channels result in the following narrative of a rational and predictable Minsky cycle: The economy experiences news of a positive productivity boom in some new technology or secondary sector, which will lead to a reallocation of resources toward the new technology. In anticipation of future growth, asset prices increase right away, which fuels a credit expansion affecting the entire economy, not just the sector that will experience the technological innovation. This credit-filled boom is primarily driven by leveraged users of capital. However, the positive shock in the new sector pulls resources away from the economy's primary producers, who have taken on more debt during the credit expansion, and the primary producers are forced to cut their capacity to repay debts. This deleveraging process leads to a persistent bust after the transitory positive shock dissipates. Our narrative matches the stylized facts of emerging market sudden stop episodes, as well as of the Great Recession in the U.S., which was preceded by new innovations in the information technology (IT) sector as well as a boom in the housing market (see Section 5.3).[1]

There are several reasons why news of a reallocative technology shock is an attractive candidate to make sense of Minsky cycles. First, news of a future reallocation endogenously leads to boom-

[1] For simplicity, we start with a transitory shock as a way of capturing the dynamics that typically occur when a new technology or innovation arises and investment flows in quickly, perhaps exceeding the steady-state level. However, whether the shock is transitory or permanent does not matter for the argument, as we show in an extension. Our focus is not on modeling the behavior of investment in the innovative technology, but to focus on how such an innovation affects the broader economy.

bust dynamics in asset prices and output. Second, a reallocative technology shock can lead output to fall while asset prices are still high. This disconnect between real and financial variables, together with the subsequent convergence, generates predictable dynamics similar to a "Minsky moment," when asset prices suddenly correct after an unsustainable period of exuberance.

Second, a reallocative technology shock better matches the dynamics of a Minsky cycle than a technology shock that primarily affects leveraged users. A technology shock that primarily affects leveraged users of capital unequivocally leads to a persistent *boom*. In order for a bust to occur, good news must be followed by bad news, in which case the shocks (rather than inherent dynamics) are truly driving the "cycle." This point underlines the relevance of technological innovation (or, in more abstract terms, productivity improvements in a secondary sector) for generating Minsky cycles.

Finally, and perhaps most importantly, a reallocative technology shock is, perhaps surprisingly, a much better candidate than a shock to financial conditions directly. A reallocative technology shock produces dynamics reminiscent of Minsky's narrative, but a shock that directly relaxes financial conditions (e.g., a "financial liberalization") produces very different dynamics in output and asset prices. A temporary relaxation of collateral constraints produces a cycle *without* an increase in asset prices (i.e., asset prices stay the same and then fall going forward), but high asset prices are central to the narratives of Kindleberger, Minsky, Fisher, and Keynes.

While our baseline model does not require the introduction of departures from a full information rational expectations framework in order to generate these cycles, we nevertheless explore the implications of departures from this theoretical baseline. Indeed, whereas we feel that the evidence strongly suggests that periods of great financial and macroeconomic excitement may be rooted in something as fundamental as a technological revolution or a structural reform, there is also a large body of evidence suggesting that the associated rosy beliefs about the future are partly flawed. A growing empirical literature suggests that these beliefs could be the result of behavioral biases leading to excessive extrapolation (Bordalo et al., 2018; Krishnamurthy and Li, 2020) or neglect of rare systemic events (Gennaioli et al., 2012; Mian et al., 2017; Greenwood et al., 2020). In an extension considering a rise in expectations of the future not warranted by fundamentals (i.e., "noise shocks"), we find that our results regarding reallocation are completely unchanged, with a larger bust in output and the asset price. In an extension considering belief extrapolation akin to diagnostic expectations, we find that extrapolation neatly interacts with our baseline channels by amplifying the boom-bust cycle.

Related Literature Our paper is far from the first to consider cyclical propagation in market economies. Beaudry, Galizia, and Portier (2017, 2020) provide evidence of medium-run cyclical behavior in aggregate variables. Beaudry et al. (2018) propose a model that includes Hayekian mechanisms of over-investment and liquidation, with Keynesian mechanisms working through aggregate demand. Rognlie, Shleifer, and Simsek (2018) consider how over-investment in one sector (as an initial condition), together with nominal rigidities at the ZLB, lead to investment hangover during the recovery. We show how initial over-investment is likely to occur given the nature of productivity news we think is relevant in the data. Boissay et al. (2016) offer a landmark quantitative analysis of crises that follow credit booms. Similar to us, they focus on productivity as the main driver of these cycles. Caramp (2017) shows how the interaction between adverse selection and asset creation generates boom-bust cycles.

Our analysis is most closely related to Kiyotaki and Moore (1997), who extend the insight from Bernanke and Gertler (1989) that changes in borrower net wealth and agency costs create *persistence* in business cycles, to show that borrowing constraints also *amplify* business cycles precisely because the values of borrowers' assets are procyclical. Kiyotaki and Moore (1997) consider a temporary shock to productivity that affects all agents—but most importantly, the shock increases the funds available to experts ("farmers," in their terminology). The initial increase in output leads experts to buy more capital, increasing their output next period, and increasing asset prices next period. The increase in future asset prices relaxes current collateral constraints, leading to amplifications in current output and increases in asset prices that are an order of magnitude larger than would occur in a frictionless model. An extended version of the model with investment features internal propagation mechanisms that can lead to credit cycles in response to the aforementioned shock. However, when such models are estimated, the implied parameters generally do not generate quantitatively meaningful endogenous cyclical behavior. In contrast, our model generates cyclical behavior in the baseline Kiyotaki and Moore (1997) setup because of the timing and nature of the productivity shock. The bust following the shock is of the same order of magnitude as the initial shock itself and is not driven by amplification mechanisms, but by reallocations to the new sector and following expansion by the main sector during the credit expansion.

Our paper relates to the literature on over-investment, which includes Caballero and Krishnamurthy (2001), Lorenzoni (2008), He and Kondor (2016), and Korinek and Simsek (2016). Closely related to our focus on collateral constraints, Akıncı and Chahrour (2018) consider an open economy with occasionally binding collateral constraints and find that positive productivity shocks increase leverage, thus increasing the probability of a future sudden stop. On average, good news is realized, but higher leverage exposes agents to a greater risk that an *unfavorable future shock* will eventually lead the constraint to bind. Our model considers a single positive productivity shock and does not rely on the possibility of unfavorable future shocks. Bhattacharya et al. (2015) provide a model of rational learning in which periods of good times lead to more optimism and greater leverage. Eggertsson and Krugman (2012) assume a Minsky moment when borrowing constraints suddenly tighten and study the aggregate consequences. Simsek (2013) considers how belief disagreements increase leverage. Farhi and Werning (2020) study optimal coordination of monetary and macroprudential policy when Minsky cycles are caused by excessive optimism (extrapolative expectations). Gorton and Ordonez (2020) find evidence to support the hypothesis that financial cycles can be thought of as medium-run phenomena.

A growing body of empirical evidence supports the pattern of boom-bust investment cycles as well as the predictability of asset price busts (or financial crises). A seminal contribution is the important paper by Schularick and Taylor (2012), who assemble a new historical dataset to assess this predictability. Gulen et al. (2019) find that elevated credit market sentiment correlates with a boom in corporate investment over the subsequent year, followed by a long-run contraction. López-Salido et al. (2017) find that elevated credit market sentiment predicts lower GDP growth two years later. More recently, Greenwood et al. (2020) find that financial crises are predictable, as is the case in our paper.

2 The Model

The baseline model is identical to the model proposed by Kiyotaki and Moore (1997). Our only addition to this standard building block is an *anticipated* technology shock to an innovative sector. We introduce this shock in Section 2.2.

2.1 Baseline Model

Setup Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The economy contains a single durable factor of production, which we call capital. The aggregate supply of capital is fixed at $\bar{K}$. Capital trades at a price q_t per unit of output.

There are two types of agents, experts and non-experts, who for simplicity have linear utility over consumption. Non-experts discount future consumption using discount factor $\underline{\beta}$ (we underline non-expert variables). Experts are strictly more impatient, discounting with $\beta < \underline{\beta}$. Hence, experts and non-experts have utility $\sum_{t=0}^{\infty} \beta^t x_t$ and $\sum_{t=0}^{\infty} \underline{\beta}^t \underline{x}_t$, where $x_t, \underline{x}_t$ denote consumption.

Technology There are two types of production technologies. Non-experts have production function G with decreasing returns to scale: a non-expert with $\underline{k}_t$ units of capital at t produces

$$\underline{y}_{t+1} = G(\underline{k}_t) \tag{1}$$

units of output in $t+1$. Experts have linear technology: an expert with k_t units of capital at t produces

$$y_{t+1} = (a+c)k_t \tag{2}$$

units of output in $t+1$, where ak_t units are tradable and can be used to purchase capital but ck_t units are non-tradable and must be consumed by experts. As in Kiyotaki and Moore (1997), we suppose that c is sufficiently large so that experts will not consume any of the tradable output. Specifically, $c > (1/\beta - 1)a$, or equivalently, $\frac{a}{a+c} < \beta$ (the tradable fraction of output is not too large). This condition implies that, local to the steady state (see below), experts prefer to invest additional resources in capital rather than to consume. This is a weak condition if $\beta \approx 1$.

As explained below, experts are subject to a collateral constraint limiting their credit, which will bind in equilibrium. We make assumptions on G so that, in equilibrium, experts' marginal productivity is above non-experts' marginal productivity, and thus the optimal allocation gives capital to experts (i.e., $G'(0) > a/\underline{\beta} > G'(\bar{K})$).

Budget and Collateral Constraints All borrowing must be collateralized by capital. Since experts are more productive and more impatient, experts will borrow from non-experts in equilibrium. Because non-experts are unconstrained with linear utility, their discount factor pins down the rate to $R = 1/\underline{\beta}$. At date t an expert with capital k_t can borrow up to the value of the capital in $t+1$. That is,

$$Rb_t \leq q_{t+1}k_t, \tag{3}$$

where R is the gross interest rate, b_t is the amount borrowed, q_{t+1} is the future asset price, and k_t is present capital holdings. An expert borrowing b_t at interest rate R must repay Rb_t tomorrow. The capital tomorrow has value $q_{t+1}k_t$.

Given the assumptions, experts borrow up to the collateral constraint and use all tradable output to buy capital. The condition $\frac{a}{a+c} < \beta$ implies that experts do not want to consume any of the tradable output but want to invest all of it in capital; they do not want to save output because $\beta R < 1$. An expert's budget constraint is

$$q_t k_t = (a k_{t-1} + q_t k_{t-1} - R b_{t-1}) + b_t.$$

Plugging in for b_t using the collateral constraint yields

$$k_t = \frac{(a k_{t-1} + q_t k_{t-1} - R b_{t-1})}{q_t - \frac{q_{t+1}}{R}} = \frac{(a k_{t-1} + q_t k_{t-1} - R b_{t-1})}{u_t},$$

where $u_t \equiv q_t - \frac{q_{t+1}}{R}$ is the user cost or the down payment for a unit of capital.

Non-experts are not credit constrained, which means they will hold capital until the marginal value of capital equals the opportunity cost R:

$$R = \frac{G'(\underline{k}_t) + q_{t+1}}{q_t}, \quad \Longrightarrow \quad \frac{1}{R} G'(\underline{k}_t) = u_t. \tag{4}$$

Aggregate Equations We can aggregate by summing over experts to get

$$K_t = \frac{1}{u_t} \left(a K_{t-1} + q_t K_{t-1} - R B_{t-1} \right), \tag{5}$$

$$B_t = \frac{q_{t+1} K_t}{R}, \tag{6}$$

where K_t and B_t are aggregate capital holdings and borrowing by experts. The user cost is given by

$$u_t = \frac{1}{R} G'(\bar{K} - K_t). \tag{7}$$

Total output is given by

$$Y_{t+1} = (a+c) K_t + G(\bar{K} - K_t). \tag{8}$$

Denoting steady-state variables by $*$, in the steady-state equilibrium we have

$$u_* = a, \quad q_* = \frac{aR}{r}, \quad Ra = G'(\bar{K} - K_*), \quad Y_* = (a+c) K_* + G(\bar{K} - K_*),$$

where $r = R - 1$ is the net interest rate. Thus, the tradable output just covers the interest on experts' debt, and the down payment equals the tradable output.

Discussion In order to clearly highlight our proposed mechanism, we intentionally consider a simple model, even though some of the ingredients, like linear utilities and non-tradable output, are somewhat nonstandard. As in Kiyotaki and Moore (1997), these simplifications can be relaxed and replaced by standard ingredients with suitable parameter restrictions. Assuming linear utilities, a linear production function for experts, and a fraction of non-tradable output ensure that the collateral constraint is binding local to the steady state and that experts reinvest a constant fraction of output, which aid in characterizing equilibrium dynamics. In their appendix, Kiyotaki and Moore (1997) sketch a model with log utility to generate similar results, and Cordoba and Ripoll (2004) quantitatively consider a similar model with standard features. In Appendix D we sketch such a model and provide conditions so that our results continue to hold even when interest rates fluctuate in response to the shock.

As in Kiyotaki and Moore (1997) and Geanakoplos (1997), our collateral constraint depends on future prices q_{t+1} and not contemporaneous prices q_t. This matters critically for our main mechanism to generate a boom in response to news about the future. If instead the contemporaneous price q_t determined the collateral constraint, our mechanism would not operate (see also Proposition 4).

Finally, we assume that agents trade debt. In a deterministic model, there is no distinction between debt and equity claims (there is no risk); the collateral constraint operates as a pledgeability constraint on any type of financial claim. However, the distinction matters in a richer model with anticipated aggregate shocks. The inability to issue equity generally has quantitative consequences for amplification and propagation (see Cao and Nie, 2017). We formally consider this issue in Appendix C.4. In our setting, in response to our shock of interest, if agents borrowed with state-contingent claims (equity), then in equilibrium there would be a smaller endogenous boom but a larger bust.

2.2 News Shocks to an Innovative Sector

Central to our boom-bust narrative is that the positive productivity shock leading to the expansion affects some innovative or secondary sector—not the experts, who are already leveraged. In light of the behavior of the motivating events as discussed in the Introduction, we model this innovation as offering temporarily higher productivity.[2] Initially, this innovative sector is secondary to the most

[2] Indeed, as discussed in the Introduction, the evidence suggests that these episodes of major technological improvement or structural reform lead to a period of sustained higher growth, followed by sharp reversals. For the purposes of our argument, it does not matter whether in the end the shock does or does not have permanent effects on the level of productivity. What matters is that the increased *growth* is temporary.

productive uses of capital in the economy, though in reality the most productive uses of capital may also benefit from this sector (examples include IT, innovations in housing finance, and real estate more broadly). Precisely mapping this stylized shock to the data requires care. Section 5.3 illustrates how one could map our proposed shock, and resultant dynamics, to the U.S. economy during the 1990s–2000s and also discusses related empirical issues.

Our goal is to present a simple model that can clarify the interaction of optimism with capital reallocation and demonstrate how this interaction has the ability to generate predictable financial cycles. Since in reality there are many factors driving optimism, credit expansion, and reallocation, we remain in this paper agnostic about how exactly to map the innovative sector to any given episode. (In the Conclusion, we offer a few thoughts about the challenges and promises present when empirically testing the predictions of the theory.) We prefer to interpret our model as highlighting the key mechanisms at play in these events.

Accordingly, we model the productivity shock of interest as the temporary entrance of an innovative technology. The temporary entrance is only for simplicity; we show that this can generalize to persistent or permanent shocks, as well.[3] The innovative technology is a linear production function

$$y_{t+1} = a^I k_t^I, \tag{9}$$

with $Ra < a^I < a+c$, and superscript I denotes variables associated with the innovative technology. The productivity of the innovative technology is higher than non-experts' marginal productivity in steady state but lower than the experts' marginal productivity. As a result, experts endogenously choose not to use the innovative technology.[4]

We suppose that in future periods, non-experts have access to this more-productive innovative technology for one period. For simplicity, we suppose each non-expert has access to the innovative technology with a capacity constraint K^I. Since the innovative technology has a higher marginal

[3]In addition, all of our results regarding the dynamics of capital and the asset price would also be captured by a shock directly to the non-experts' production function G. For example, suppose non-expert output is given by $z_t G$ and z_t increases. A shock directly to G imposes additional constraints because it increases productivity for all the output generated by non-experts, rather than only for the marginal investments.

[4]In reality, new technologies are often funded with leverage. What matters for our analysis is not that they are funded by non-constrained agents, but that they are separate from the initial experts. The shock could also represent a new set of entrants with wealth that gets leveraged into innovative capital. We continue to assume that a fraction $\frac{c}{a+c}$ of output is non-tradable for experts. If the technology were not available to experts, then productivity a^I could exceed $a + c$ without affecting our results. When the innovative technology is run by non-experts, who are unconstrained, we need not distinguish between tradable and non-tradable production for the innovative technology. However, it is sensible to suppose that a units are tradable so that the innovative technology has the same marginal product of *tradable* output as the expert technology.

product than G in steady state (which is $G' = Ra$ in steady state), non-experts will invest as much as possible in the new technology. Hence, the non-expert technology can be written

$$\underline{y}_{t'+1} = G(\underline{k}_{t'} - K^I) + a^I K^I,$$

for any period t' in which the technology is available. Aggregate output at $t' + 1$ is

$$Y_{t'+1} = (a + c)K_{t'} + a^I K^I + G(\bar{K} - K_{t'} - K^I). \tag{10}$$

Crucially, a credit expansion requires that *future* asset prices increase. Hence, at $t = 0$, agents receive news of the future innovative technology becoming available in future periods. In our main specifications, the technology is only available for one period in t'. As a result, we refer to access to the innovative technology in period t' as "the shock."

2.3 Linearized Equilibrium

Similar to Kiyotaki and Moore (1997), we solve for the log-linearized dynamics around the steady state. For a variable X_t, we denote log-deviations from steady state by $\hat{X}_t \equiv \log(X_t) - \log(X_*)$, where X_* denotes the steady-state value. (We use the terms "log-linearization" and "linearization" interchangeably.) Following the notation in Kiyotaki and Moore (1997), let $1/\eta$ denote the elasticity of the user cost to changes in aggregate non-expert capital. By definition, $\eta = -\frac{G'(\bar{K}-K_*)}{G''(\bar{K}-K_*)K_*}$, where K_* is capital held by experts. It is useful to define $\gamma \equiv \frac{\eta}{1+\eta}$. Through the elasticity of non-expert demand, γ will determine the size and persistence of equilibrium dynamics ($\gamma < 1$ since $\eta > 0$).

We define $\hat{K}^I \equiv \frac{K^I}{K_*}$, which normalizes the demand for innovative capital by the fraction of experts' steady-state capital holdings. It is convenient to define $\hat{z} \equiv \frac{1}{\eta}\hat{K}^I$, which captures the change in non-experts' marginal product (G') that would occur when shifting K^I units of capital away from their primary technology G towards the innovative technology. We characterize dynamics in terms of $\hat{z}$.

Asset Prices and Capital Allocations The behavior of asset prices and the evolution of expert capital holdings are crucial for our story. We provide the relevant linearized equations here to provide intuition regarding how the shock affects prices and allocations each period. The remaining equations are in the Appendix.

Linearizing the non-experts' optimality condition in equation (4) delivers

$$\hat{u}_t = \frac{1}{\eta}\hat{K}_t, \, \forall t \neq t', \quad \text{and} \quad \hat{u}_{t'} = \frac{1}{\eta}\hat{K}_{t'} + \hat{z}, \tag{11}$$

where $\hat{K}_t$ is aggregate *expert* capital holdings and $\hat{z} = \hat{K}^I/\eta$ is the shock at t'. In the absence of a shock, the only mechanism affecting the user cost is the change in non-expert capital. If experts hold more capital, then non-experts hold less, in which case the user cost increases, because non-experts' marginal productivity rises. However, in the period of the shock t', there is a direct effect on the user cost, due to higher demand from non-experts, as capital gets allocated to the innovative technology; non-experts have higher marginal productivity when using G with less capital.

From the definition of the user cost, we can write $q_t = u_t + \frac{q_{t+1}}{R}$. Linearizing, we have

$$\hat{q}_t = \frac{r}{R}\hat{u}_t + \frac{1}{R}\hat{q}_{t+1} = \frac{r}{R}\sum_{s=0}^{\infty} R^{-t}\hat{u}_{t+s}, \tag{12}$$

where the last line follows from forward iteration. Hence, the change in the asset price at t is proportional to the present value of changes in user costs from t onward.

The experts' budget constraint (5) at $t = 0$ is $u_0 K_0 = aK_* + q_t K_* - RB_*$, where $RB_* = q_* K_*$, because existing debt and capital holdings are at the steady state. Going forward, debt is set according to the collateral constraint (6) so that the experts' budget constraints for $t \geq 1$ simplify to $u_t K_t = aK_{t-1}$. Linearizing the experts' budget constraints yields

$$\hat{K}_0 + \hat{u}_0 = \frac{R}{r}\hat{q}_0, \tag{13}$$

$$\hat{K}_t + \hat{u}_t = \hat{K}_{t-1}, \quad \forall t \geq 1. \tag{14}$$

These equations have an important intuition. Expert capital evolves each period to pay the user cost. For $t \geq 1$, a positive user cost implies a decrease in expert capital. However, at the time of the news, $t = 0$, there is an additional gain in available resources, $\frac{R}{r}\hat{q}_0$, due to the wealth consequences of the increase in the capital price. Because existing debt at $t = 0$ equals the steady-state value of capital, experts' balance sheets improve in the period when agents receive news of the innovation as their capital holdings increase in value without a commensurate increase in their outstanding debts.[5]

[5] Appendix C.4 considers when the repayment varies with q_0, mitigating the wealth gain.

Output and Aggregate Productivity Since the shock is an exogenous demand for capital, the dynamics of capital allocations and prices are independent of the innovative productivity a^I. However, output at the time of the shock depends critically on a^I. In this simple model, aggregate capital is fixed, and thus, fluctuations in output reflect changes in productivity (i.e., changes in capital allocation). When $t \neq t'$, any change in output next period is driven by changes in expert (non-expert) capital holdings:

$$\hat{Y}_{t+1} = (a + c - Ra)\frac{K_*}{Y_*}\hat{K}_t. \tag{15}$$

The percent change in output reflects the productivity difference between experts and non-experts, $a + c - Ra$, times the share of output their capital creates $\frac{K_*}{Y_*}$, times the change in capital $\hat{K}_t$. At $t' + 1$, output is also affected by the capital holdings of the innovative sector at t':

$$\hat{Y}_{t'+1} = (a + c - Ra)\frac{K_*}{Y_*}\hat{K}_{t'} + (a^I - Ra)\frac{K_*}{Y_*}\hat{K}^I. \tag{16}$$

Output changes for two reasons: experts have additional capital $\hat{K}_{t'}$, which increases productivity relative to non-experts by $a + c - Ra$, and the innovative sector has additional capital $\hat{K}^I$, which increases productivity relative to non-experts by $a^I - Ra$. Both terms are weighted by the capital share.

3 Baseline Results

For expositional clarity, our main results consider a one-time impulse shock at $t = t'$. However, we can first prove a general result without imposing any structure on the timing or persistence of the shock.

Proposition 1. *Consider any sequence of shocks regarding when the innovative technology becomes available. In equilibrium,*

$$\sum_{t=0}^{\infty} R^{-t}\hat{K}_t = 0. \tag{17}$$

That is, the present value of the linearized changes in experts' capital holdings $\hat{K}_t$ is zero.

This is a powerful and very general result. For any shocks of the type we propose, any credit-fueled boom must be followed by a bust.[6] More broadly, for any shock that does not directly

[6] This result does not hold if agents' repayments depend on aggregate states. With state-contingent borrowing (equity), the present value of the bust exceeds the boom (see Appendix C.4).

provide experts with more output, experts' budget constraints are given by equations (13) and (14). As we discuss in Section 5, this applies to a shock to financial conditions as well.

Appendix A contains the proof, but we provide the intuition here. Recall from equations (13) and (14) that experts' capital holdings evolve to pay the user costs each period, but experts' receive a wealth gain at $t = 0$ due to the change in the capital price. From equation (12), the change in the capital price is entirely determined by the change in future user costs. Hence, we can write (13) as

$$\hat{K}_0 + \hat{u}_0 = \sum_{t=0}^{\infty} R^{-t} \hat{u}_t \implies \hat{K}_0 = \sum_{t=1}^{\infty} R^{-t} \hat{u}_t.$$

Experts' initial change in capital $\hat{K}_0$ equals the present value of changes in future user costs, but the future changes in capital exactly offset the user costs. Hence, any initial change in capital is offset by future changes in capital.

This result is very different from the main result in Kiyotaki and Moore (1997), in which a productivity shock to experts leads to a persistent boom in their capital holdings. In our model, the future technology shock does not provide experts with more resources with which to buy capital, which is the case in Kiyotaki and Moore (1997). In our setting, experts increase their capital holdings only to the extent that credit markets facilitate borrowing—but experts do not have additional resources in the future. Indeed, Proposition 1 states that the additional credit available at $t = 0$ exactly equals the user costs that experts eventually repay. As a result, if a shock leads to an initial boom, it must eventually be followed by a bust. A credit-fueled boom cannot be sustained without leading to a bust.

Output and Welfare Proposition 1 has important consequences for output and welfare. To illustrate, suppose the technology is available at $t = 1$. From equations (15) and (16), we can write the present value of the linearized changes in output as

$$PV\Delta Y = \underbrace{\underline{\beta} \sum_{t=0}^{\infty} R^{-t} \left(a + c - Ra\right) \frac{K_*}{Y_*} \hat{K}_t}_{Endogenous} + \underbrace{\underline{\beta}^2 (a^I - Ra) \frac{K_*}{Y_*} \hat{K}^I}_{Exogenous}, \tag{18}$$

where $a + c - Ra > 0$ follows from the assumption on agents' productivities, and discounting reflects that capital at t produces output at $t + 1$. Note that there is a direct, exogenous component given directly by the shock $\hat{K}^I$ and an endogenous component that is caused by the reallocation of capital in each period. Thus, it is sufficient to characterize the present value of deviations in capital $\hat{K}_t$

to solve for the present-value change in output. The following result is an immediate corollary of Proposition 1.

Corollary 1. *The present value of the linearized changes in output due to endogenous dynamics in $\hat{K}_t$ is zero.*

Because the technology shock does not affect experts directly, the present value of the linearized changes in total output (endogenous and exogenous) is simply the consequence of the exogenous shock itself.

Welfare considerations are tricky because agents have different discount factors. However, in the limit as $\beta \to \underline{\beta}$, the aggregate welfare consequences of the *endogenous* boom-bust propagation is zero: agents have linear utility, and the present value of the boom in output is exactly equal to the present value of the bust. There are positive welfare consequences directly from the shock, which boosts productivity of at least some capital in that period. Note that since the production function G is concave, the (linearized) deviations in capital would lead to a negative present value in output changes when taking into account the concavity of non-expert production. Since the present value of fluctuations in capital is zero, larger fluctuations lead to a lower present value of output.

Corollary 1 does not mean that a constrained planner would be indifferent to responding to the news shock. On the contrary, the presence of collateral constraints creates pecuniary externalities (see Dávila and Korinek, 2017), and a constrained planner would desire to change the equilibrium, both in the steady state and in response to the shock.[7] Due to concavity in G as we discussed, the initial boom is inefficiently high, and welfare would improve if the initial boom and the following bust were both smaller (particularly if we considered the *non-linear* equilibrium dynamics). The concavity of G implies a present value *loss* of output: a planner would choose to mitigate the response to the shock.

Limiting borrowing by experts in a boom would be welfare improving in order to mitigate the eventual bust. If a regulator were to impose a restriction on leverage so that experts did not fully exhaust their capacity to buy new capital during a boom, then the resulting bust would be smaller. Proposition 1 applies in this case as well. Limiting the boom would not change the present value of the linearized changes in output due to endogenous dynamics in $\hat{K}_t$. However, because of the concavity in G, the present value of the non-linearized changes in output due to endogenous

[7]A steady-state leverage constraint would also have implications for steady-state output. Limiting borrowing would increase the level of capital held by experts, thus increasing output (see Appendix C).

dynamics in $\hat{K}_t$ would increase.

Corollary 2. *Limiting leverage (experts' capital) in a boom decreases the size of both the boom and the bust, which increases the present value of the non-linearized changes in output due to endogenous dynamics. The present value of the linearized changes in capital $\hat{K}_t$ is still zero.*

Going Forward In the rest of our paper, we impose additional structure to fully characterize the dynamics of the economy. In other words, we specify the shocks in greater detail to characterize the size and duration of the boom, which by Proposition 1 will be followed by a bust. We let the economy start in steady state at $t = 0$ and suppose that the innovative technology is available only in the following period ($t' = 1$). Section 4 considers when the shock occurs $N > 1$ periods forward ($t' = N$), implying a greater role for news and anticipation, and also considers persistent shocks. Section 5 discusses the robustness of our results by considering the role of news, expectations, general equilibrium adjustments, and alternative sources of shocks. Appendix D shows that, under mild assumptions, our results also hold when the interest rate fluctuates.

3.1 Dynamics for Shock in One Period, $t' = 1$

We now consider the full general-equilibrium dynamics. After news of the shock has been incorporated, experts' borrowing in future periods equals the value of capital in the next period. In contrast, at $t = 0$, news of the shock can increase the value of capital at $t = 0$ so that it exceeds the debt that needs to be repaid. (Experts' borrowing is inherited from the previous period, which was determined before news of the shock.) We can unequivocally describe the deterministic behavior of capital and asset prices arising due to the change in productivity happening at $t = 1$ as a result of the innovative sector. The economy will experience a boom-bust cycle arising from the initial relaxation of constraints and the subsequent tightening that forces experts to sell capital to non-experts.[8]

Proposition 2. *In response to a news shock regarding the productivity of the innovative sector at $t = 1$, the economy experiences the following deterministic boom-bust dynamics:*

1. *An increase in the capital price at $t = 1$: $\hat{q}_1 = r\gamma\underline{\beta}\hat{z} > 0$,*

2. *A boom at time $t = 0$: $\hat{K}_0 = \underline{\beta}\gamma\hat{z} > 0$ and $\hat{q}_0 = r\underline{\beta}^2\hat{z} > 0$, and*

[8]Unlike Kiyotaki and Moore (1997), we do not see amplification in response to a shock to non-experts' productivity. The increase in capital is of the same order of magnitude at $\hat{z}$, while the increase in the capital price is an order of magnitude smaller. Thus, the model creates boom-bust, but not amplification.

3. A bust going forward: $\hat{K}_t = \gamma^t(\hat{K}_0 - \hat{z}) < 0$ *for all* $t \geq 1$*, with* $\hat{q}_t < 0$ *for all* $t \geq 2$.

The demand for capital from the innovative sector will increase the asset price at $t = 1$, which relaxes collateral constraints at $t = 0$ and increases experts' capital holdings right away. Experts' demand for capital at $t = 0$ increases the asset price and the user cost above the steady-state value, but this means that experts' debt exceeds the sustainable steady-state level. In contrast to Kiyotaki and Moore (1997), experts are not more productive at $t = 1$ as a result of the shock. Experts have higher output because they hold more capital, but their debt burdens are even higher, and since the user cost exceeds the value of their output, experts must sell capital at $t = 1$ in order to repay their debts.

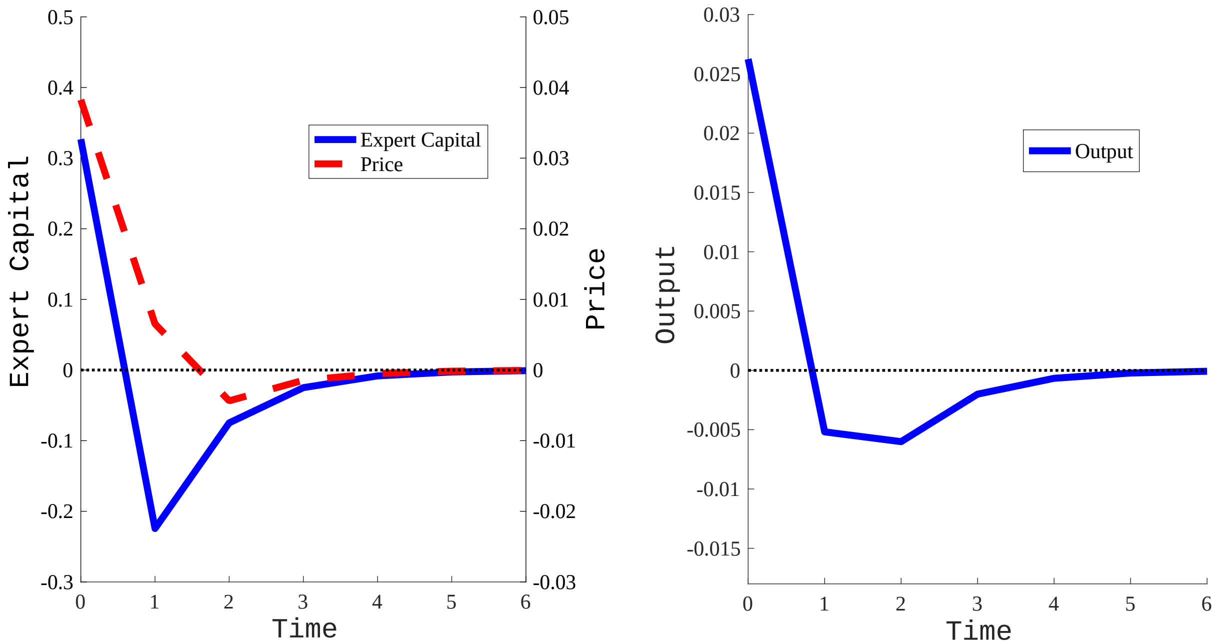
Since experts' debt is higher but their productivity is not, experts are forced to sell capital, pushing their capital holdings below the steady-state level. Once experts' capital is below the steady state, experts slowly rebuild capital as they pay off their debts. The economy converges to the steady state at rate γ, which reflects the elasticity of non-experts' demand.

The following result is an immediate implication of equation (16) and the fact that $\hat{K}_t$ is independent of a^I. Recall that output is subscripted one period forward (i.e., $\hat{Y}_{t+1}$ is produced with $\hat{K}_t$).

Proposition 3. *In response to a news shock at* $t = 0$ *regarding the productivity of the innovative sector at* $t = 1$*, the economy experiences the following deterministic boom-bust dynamics: a boom at time* $t = 0$ *with* $\hat{Y}_1 > 0$ *and a bust going forward after the shock at* $t = 1$ *with* $\hat{Y}_{t+1} < 0$ *for all* $t > 1$*. Furthermore, there exists a maximum productivity* $\bar{a}^I$ *such that the economy experiences a bust at* $t = 1$ *if and only if* $a^I < \bar{a}^I$.

Figure 1 illustrates the results in response to a shock $\hat{z} = 1\%$ at $t = 1$. Panel 1(a) plots the equilibrium dynamics for experts' capital holdings capital prices, and panel 1(b) plots output next period.[9] Capital initially increases and then falls at the time of the shock (in period 2), slowly returning to steady state. The capital price q_t is above steady state for 2 periods before falling below steady state, while output next period falls below steady state even in the time of the shock. (Since output is completely determined by variables in the previous period, we choose to plot output next period as a function of time.) As a result, we have a divergence in output and capital prices when the shock hits: capital prices remain above steady state even though output falls below.

[9]The figures are parameterized to illustrate the key results, not to provide quantifications. We parameterize with $R = 1.02$ and $\eta = 0.50$. For output we parameterize $a = 0.3$, $c = 0.3$, and $a^I = 0.4$; these parameters are relevant for output but not for the evolution of capital, which depends only on R and η. A higher η leads to greater persistence and thus a larger and more prolonged boom and bust.



(a) Expert Capital and Price (b) Output

Figure 1: Changes in expert capital $\hat{K}_t$, capital price $\hat{q}_t$, and output $\hat{Y}_{t+1}$ in response to news at $t = 0$ of an innovative sector at $t = 1$. Source: authors' analysis.

A crucial parameter for the boom-bust dynamics of output, or aggregate productivity, is the productivity of innovative technology, a^I, which determines the severity of the boom or bust at t' only. If a^I is not so large compared to the productivity of the experts $(a + c)$, then the shock leads to output below steady state at t': experts hold less capital than steady state, and even though the innovative technology is marginally more productive than G, aggregate productivity falls because the innovative technology is so much less productive than experts' technology. However, if a^I is sufficiently close to $a+c$ (not necessarily more), aggregate productivity can be above steady state at the time of the shock. The economy will still feature a boom-bust cycle in output, with productivity falling below steady state in the periods after the shock, but the boom will decline more slowly.

3.2 The Critical Role of General Equilibrium Effects

The nature of our reallocative technology shock together with general-equilibrium adjustments in credit markets are crucial for our story. To clearly illustrate the critical role of general-equilibrium adjustments in generating boom-bust cyclical propagation, we now consider two benchmarks. We first eliminate the role of news and show that a contemporaneous innovative technology leads to an exogenous boom without a bust. We then interrogate a deceptive conclusion based on partial-equilibrium reasoning alone. We contrast the mechanisms in the full general-equilibrium case with these benchmarks.

Contemporaneous Shock, $t' = 0$ The role of news working through credit markets is critical for our story. Suppose that the shock occurs contemporaneously: the innovative sector entered at $t = 0$ (no news). In this case, experts' capital holdings would not change even though the asset price would immediately increase.

Proposition 4. *In response to a contemporaneous shock at $t = 0$ regarding the productivity of the innovative sector at $t = 0$:*

1. *An increase in the capital price at $t = 0$: $\hat{q}_0 = r\underline{\beta}\hat{z} > 0$,*
2. *No change in expert capital holdings for all t: $\hat{K}_t = 0$ for all t,*
3. *No change in the capital price for all $t > 0$: $\hat{q}_t = 0$ for $t > 0$, and*
4. *Output increases in $t = 1$ for exogenous reasons (higher non-expert productivity at $t = 0$) and returns to steady state for all $t > 1$.*

A shock without news would lead to a completely transitory expansion. Without news, there is no opportunity for credit markets to fuel an unsustainable boom. News is critical to getting an expansion, as well as a contraction, after the shock. Demand for capital to use in the innovative technology increases the capital price and output in the period of the shock, but there is no capital reallocation, and therefore, there are no persistent effects of the shock.

The economy does not experience boom-bust dynamics if the shock is immediate. The economy immediately returns to steady state, and the only effects of the shock are contemporaneous. In response to this contemporaneous shock, the asset price increases at $t = 0$, increasing experts' value of capital above the value of the debt they need to repay. This increases experts' net worth, but since their increase in net worth is driven purely by an increase in the asset price, there is no reallocation of capital. The value of experts' capital increases, but that does not enable them to buy more capital. Since experts' capital and debt were at steady state, the economy immediately returns to steady state without the innovative technology: $\hat{K}_t = 0$ for all $t \geq 0$, and thus, $\hat{q}_t = 0$ for $t > 0$. Experts do not take on additional debt, and thus, there is no contraction when the shock is gone.

Partial-Equilibrium Reasoning The general-equilibrium costs of reallocation are crucial for a credit boom to expand beyond the sustainable level. It's important to carefully consider these mechanisms because, at face value, the initial boom appears to mitigate rather than cause the boom. Linearizing the experts' budget constraint at $t = 1$, in equilibrium

$$\hat{K}_1 = \gamma \left(\hat{K}_0 - \hat{z} \right).$$

All else equal, a higher $\hat{K}_0$ leads to higher $\hat{K}_1$ and therefore a higher $\hat{K}_t$ for $t > 0$ (i.e., a less severe bust). This partial-equilibrium reasoning suggests that, all else equal, a bigger boom *mitigates* the bust. However, it is not the case that all else is equal.

Proposition 1 implies that a larger boom *must* be followed by a larger bust in equilibrium. The partial-equilibrium conclusion that a larger boom provides more room to cushion the bust is entirely incorrect. In partial equilibrium, a large enough boom would completely offset the bust—but such a boom cannot be sustained in equilibrium. In general equilibrium, the boom is entirely caused by a relaxation of credit conditions, as experts borrow against future user costs; this is the intuition behind Proposition 1. Similarly, we saw in Proposition 4 that a contemporaneous shock implied no bust following the boom. Without news—without a relaxation of credit conditions—experts have no ability to increase their capital holdings precisely because, with this shock, credit fuels the boom.

When we consider the general-equilibrium consequences of the boom, it is clear that the boom is creating the future bust through the effects of increased and unsustainable debt levels. While a sufficiently high initial level of capital (a large boom) would annihilate the future bust, it is not possible in equilibrium for credit markets to fuel such a boom in a sustainable way. The boom that does occur is fueled by unsustainable credit, which is why the bust follows.[10]

The Role of General-Equilibrium Adjustments It is instructive to consider how we can modify the underlying environment to bring the partial- and general-equilibrium results together. First, suppose that $\gamma = 1$ (no curvature in G).[11] Setting $\gamma = 1$ does not completely undo the

[10] Our results relate to the issue of booms that don't turn into busts, as noted by the evidence in Gorton and Ordonez (2020), who consider a theory of information transmission to explain good and bad booms. In our setting, by Proposition 1, any shock that does not give experts more resources in present-value terms will be followed by a bust. If shocks do increase the present value of resources available to experts, then a boom need not be followed by a bust.

[11] Technically, this would have to apply to period $t = 0$ only because if $\gamma = 1$, then $\eta = \infty$ and the shock would have no effect on q_1 because the capital price would be fixed, and so $\hat{z} = \hat{K}^I/\eta = 0$.

general-equilibrium results because in equilibrium, $\hat{K}_s = \gamma^s(\hat{K}_0 - \hat{z}) < 0$ for all $s \geq 1$, which follows from the experts' budget constraint. In other words, even if the initial condition were set to $\hat{K}_0 = \underline{\beta}\hat{z}$, the economy would truly experience a bust going forward because experts need to repay their increase in debt.

However, we can completely annihilate the bust if $\underline{\beta} = 1$, which is a permissible parameterization only in a finite-horizon model. In that case, we have $\hat{K}_0 = \hat{z}$, which would truly imply that $\hat{K}_1 = 0$ in equilibrium. The mechanisms in this case are important to consider. In this case, experts can borrow the full change in future capital prices without changing the price of capital today. Experts would borrow to increase their capital holdings at $t = 0$ by $\hat{z}$, and then at $t = 1$, they would sell the additional capital for use in the innovative technology, returning their capital level to the steady state. Because there are no real rigidities from reallocation ($\gamma = 1$) and the interest rate on debt is zero, experts can use credit to increase their capital holdings and repay their debt, even though they are not more productive in the future. Thus, the economy would experience a boom at $t = 0$ from higher expert capital and also a boom at $t = 1$ from the reallocation toward the innovative technology, and then return to steady state afterward. Because the economy features no real rigidities, the initial booms would not require costly reallocation leading to a bust in following periods.

To summarize, relaxed credit conditions make a boom possible, but the boom is not sustainable in equilibrium. The frictions embedded in the economy constitute the exact reason why the shock at $t = 1$ fuels an unsustainable boom at $t = 0$.

4 Prolonged Anticipation and Persistent Shocks

We now suppose that the innovative technology is available in $N > 1$ periods ($t' = N$), leading to prolonged anticipation of the reallocative technology shock. News about an event further in the future will have distinct consequences for the size of the bust when the reallocation of capital occurs at $t = N$. Finally, we consider a slowly decaying AR(1) shock. The results extend the insights from the previous two analyses using one-time impulse shocks in the future. Appendix B considers shocks occurring further in the future, as well as permanent shocks.

4.1 Dynamics for N-period Forward Shocks, $t' = N$

We now consider the dynamics when agents receive news at time $t = 0$ that an innovative technology will be available at time $t = N$. In contrast to the previous analysis, the initial expansion will slowly decay (at a rate determined by the elasticity η) as experts repay their debts from the initial expansion. However, the reallocation at $t = N$ will have a greater effect on the slump going forward because the boom will have dissipated.

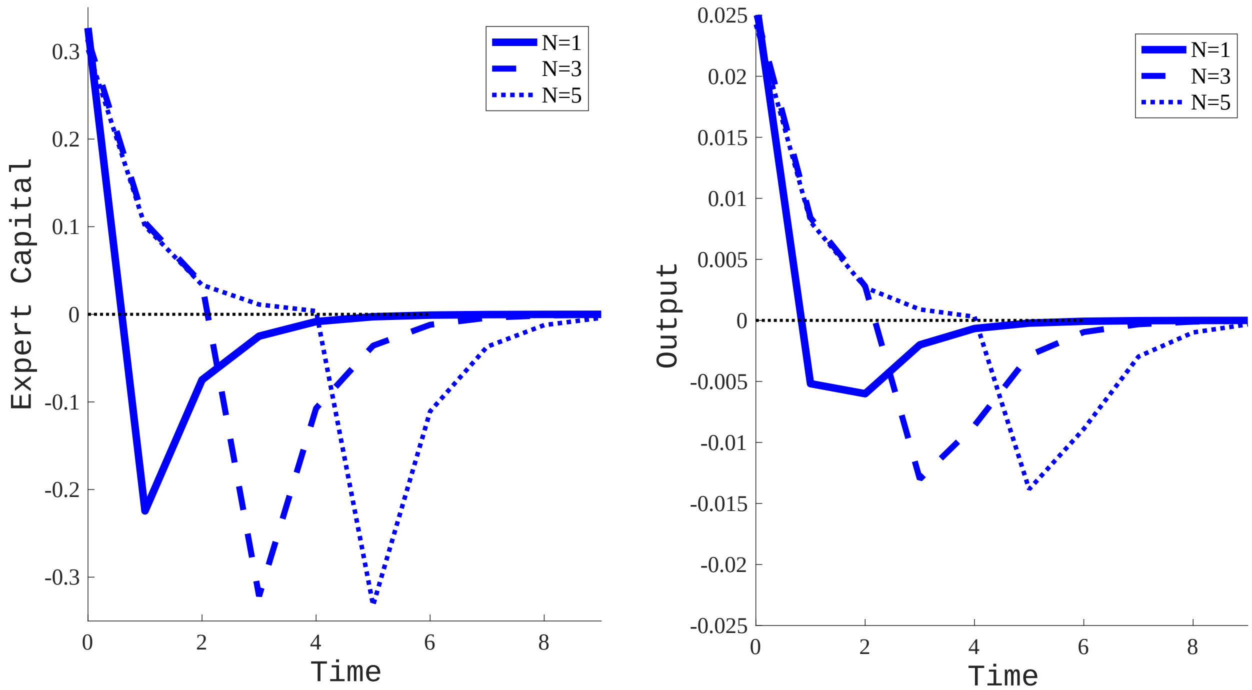
Proposition 5. *In response to a news shock at $t = 0$ regarding the innovative technology at $t = N$, the economy experiences the following boom-bust dynamics:*

1. An increase in the capital price at $t = N$: $\hat{q}_N = r\underline{\beta}\hat{z}\left(\frac{(\underline{\beta}\gamma)^N + \eta - \underline{\beta}\eta}{1 + \eta - \underline{\beta}\eta}\right) > 0$,
2. A boom before $t = N$: $\hat{K}_0 = \hat{z}\underline{\beta}^N\gamma > 0$ and $\hat{K}_t > 0$ for $t < N$, decaying at rate γ, and $\hat{q}_0 = r\underline{\beta}^{N+1}\hat{z} > 0$, with $\hat{q}_t > 0$ for $t < N + 1$, and
3. A bust going forward: $\hat{K}_N = -\hat{z}\gamma(1 - (\underline{\beta}\gamma)^N) < 0$, decaying at rate γ, with $\hat{q}_t < 0$ for all $t \geq N + 1$.

Both $\hat{K}_0$ and $\hat{K}_N$ are decreasing in N, which implies a much larger slump when the innovative sector enters. The initial boom is smaller because, due to the interest rate, the effects of future increases in prices on relaxing collateral constraints get discounted. However, the reallocation $\hat{K}_N$ at $t = N$ becomes more negative because the initial boom decays.

Figure 2 plots experts' capital holdings and output in response to such a shock N periods forward, with $N = 1, 3, 5$. Note that the initial boom gradually decays with greater decay the longer forward is the true shock. Accordingly, the bust is more severe, and the slump more prolonged, when the news is about events further in the future. (Since $\underline{\beta}$ is close to 1, the initial boom is essentially the same across all cases.)

The behavior of the economy in response to news about the future best illustrates Minsky's hypothesis. The boom declines in response to credit tightening: asset prices gradually decline, tightening collateral constraints, and experts are forced to decrease their capital holdings in response to tighter credit. The longer credit tightening persists, the less experts are able to hold on to capital when the innovative sector demands it. Because credit (not additional resources) fueled the boom the boom must gradually dissipate, and the longer the boom lasts, the more expert capital converges back to steady state. As a result, with news, there is a larger reallocation and a deeper, more persistent bust in the period of the shock.



(a) Expert Capital $\hat{K}_t$ (b) Output

Figure 2: Changes in expert capital and output in response to news of an innovative sector at $t = N$, varying N. Source: authors' analysis.

Indeed, in this example, when $N = 5$, the economy has nearly converged back to the steady state by $t = 4$ just before the shock occurs. It may seem surprising that an economy at steady state would experience such a large response in the next period. However, this is precisely what is to be expected. The experts' budget constraint for period $t = N$ is

$$\hat{K}_N = \gamma \left(\hat{K}_{N-1} - \hat{z} \right).$$

Since $\hat{K}_{N-1}$ is close to zero, capital $\hat{K}_N$ will be close to $-\gamma\hat{z}$. An economy closer to steady state at $N - 1$ will therefore experience a larger bust at N.

Recall that from Proposition 1, the present value of the boom equals the present value of the bust. In this case, the economy has experienced a sustained boom for 4 periods, which was financed by the increase in capital prices (wealth gains) at $t = 0$; experts spend the rest of time paying back the user cost. Also, the user cost increases substantially at the time of the shock. Recall that $\hat{u}_{t'} = \frac{1}{\eta}\hat{K}_{t'} + \hat{z}$, implying that experts will need to pay a large user cost (i.e., decrease capital) in the period of the shock.

Because our model features perfect foresight in response to a one-time shock, the counterfactual result from the model is that the boom is immediate and greatest at the time of news. In reality, the economy appears to take time to learn about the news and thus slowly adjust upward to the values plotted in Figure 2. A learning model—as in Blanchard, L'Huillier, and Lorenzoni (2013) or Cao and L'Huillier (2018)—would improve the dynamics of the model in this regard.

4.2 Persistent or Permanent Shocks

In this section, we consider a slowly decaying shock occurring at $t = 1$. In reality, shocks are likely to have a persistent component. Since the model is linearized, the dynamics in response to a persistent shock are merely the sum of the dynamics in response to the individual shocks, and therefore, the response to a decaying shock combines the earlier analyses. Considering a slowly decaying shock strengthens our results, leading to more persistent busts following the boom.

Let's suppose that starting at $t = 1$, the economy experiences an AR(1) decaying shock $\hat{z}_s = \rho^{s-1}$, with $\rho \in (0, 1)$. We have $\hat{z}_1 = 1$, and then the shock decays at rate ρ going forward. Then the initial capital boom is given by

$$\hat{K}_0 = \underline{\beta}\gamma \left(\frac{1}{1 - \underline{\beta}\rho} \right) > 0. \tag{19}$$

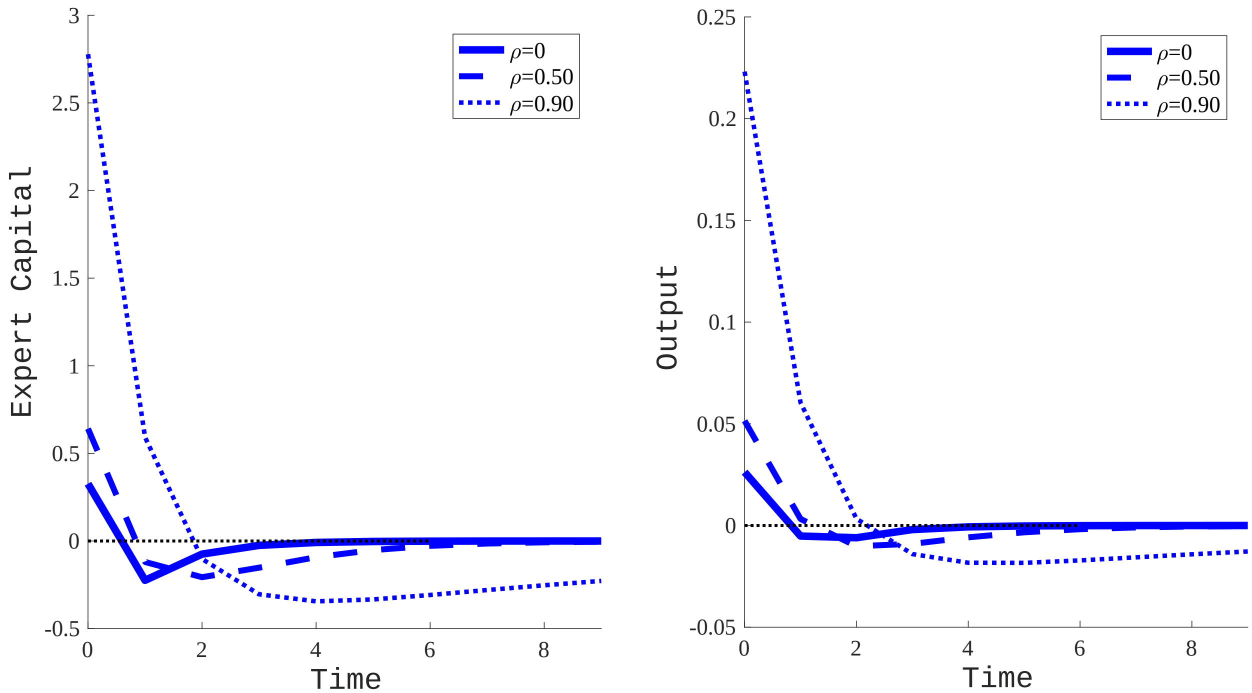
Note that the shock to capital at $t = 0$ exceeds the initial shock ($\hat{K}_0 > 1$) if $\underline{\beta}\gamma > 1 - \underline{\beta}\rho$. When this happens, $\hat{K}_1 > 0$ also. Note that we have for each s

$$\hat{K}_s = \underline{\beta}\gamma^{s+1} \left(\frac{1}{1 - \underline{\beta}\rho} \right) - \gamma \left(\frac{\gamma^s - \rho^s}{\gamma - \rho} \right).$$

Figure 3 plots the dynamics of capital and output for various levels of ρ. The higher ρ is, the larger the initial boom is (since the present value of the shock is larger), and the later the eventual bust is. However, for higher ρ, the bust is more prolonged because the reallocation of capital to the innovative sector lasts longer (Proposition 1 still applies).

5 Discussion and Robustness

The nature of our reallocative technology shock and the general-equilibrium adjustments in credit markets are crucial for our story. We discuss these features in greater detail here.



(a) Expert Capital

(b) Output

Figure 3: Changes in expert capital $\hat{K}_t$, and output $\hat{Y}_{t+1}$ in response to news at $t = 1$ decaying at rate ρ. Source: authors' analysis.

5.1 Credit Frictions and Borrowing Constraints

Given the prominent role that credit markets have in fueling the boom in our story, a reasonable concern is whether the actual problem begins in credit markets directly. Perhaps our proposed shock is a sideshow, so to speak, and what we should actually focus on instead is changes in credit markets. This is not the case. Indeed, our analysis makes it clear that it is the natural behavior of credit markets in propagating the shock, not in shocks to credit markets, that produces the dynamics of the model.

Consider some financial friction that limits borrowing to less than the full value of capital next period. For example, let the borrowing constraint be given by

$$Rb_t = \lambda_t q_{t+1} k_t, \tag{20}$$

where $\lambda_t < 1$. The budget constraint for experts is now

$$\left(q_t - \underline{\beta}\lambda_t q_{t+1}\right) K_t = aK_{t-1} + (1 - \lambda_{t-1}) q_t K_{t-1}. \tag{21}$$

With a constant λ, steady-state values are as follows:

$$q_* = \frac{a}{\lambda(1-\underline{\beta})} = \frac{Ra}{\lambda r}, \quad u_* = \frac{a}{\lambda},$$

where $u_t = q_t - \underline{\beta} q_{t+1}$, as before. As shown in Appendix C, the boom-bust dynamics in response to our suggested news shock go through with slight quantitative differences.

Including a tighter borrowing constraint allows us to emphasize the difference between technology shocks and a shock to the borrowing constraint (i.e., financial shocks). Consider a shock to credit markets directly, which we model as a temporary increase in λ_t. Let $\lambda_0 = \lambda(1+\hat{\lambda})$ with $\hat{\lambda} > 0$, and $\lambda_t = \lambda$ for $t > 0$. Such a shock temporarily increases the flow of credit, reminiscent of a financial liberalization or expansion.

Proposition 6. *In response to a shock regarding the collateral constraint, $\lambda_0 = \lambda(1+\hat{\lambda})$ with $\hat{\lambda} > 0$, and $\lambda_t = \lambda$ for $t > 0$, the economy experiences the following deterministic dynamics:*

1. *A boom in expert capital at time $t = 0$: $\hat{K}_0 = \lambda\hat{\lambda}\left(\frac{R-\sigma-R\sigma/\eta'}{r(R-\lambda)}\right) > 0$,*

2. *A bust in expert capital going forward: $\hat{K}_1 = \frac{\sigma\lambda}{R-\lambda}(\sigma - R)\hat{\lambda} < 0$ and $\hat{K}_t < 0$ for all $t > 1$ returning to steady state at rate σ, and*

3. *No change in capital price at $t = 0$ but depressed prices going forward: $\hat{q}_0 = 0$ and $\hat{q}_t < 0$ for all $t \geq 1$.*

where $\sigma \equiv \left(\frac{1-\underline{\beta}\lambda}{1-\lambda\underline{\beta}+\lambda(1-\underline{\beta})/\eta}\right) = \frac{1}{1+\frac{\lambda r}{(R-\lambda)\eta}} < 1, \frac{\lambda r}{(R-\lambda)} < 1$, $\sigma > \gamma$, and $\sigma \to \gamma$ as $\lambda \to 1$. Furthermore, if agents learn at $t = 0$ that the collateral constraint shock will occur at $t = N$, then the economy experiences no dynamics until the shock occurs: $\hat{K}_t = \hat{q}_t = 0$ for $t < N$, and then dynamics at $t = N$ are given as above with $\hat{q}_N = 0$ and $\hat{K}_N = \lambda\hat{\lambda}\left(\frac{R-\sigma-R\sigma/\eta'}{r(R-\lambda)}\right)$.

Thus, even though experts' capital holdings increase at $t = 0$, the asset price does not change, $\hat{q}_0 = 0$. Experts buy more capital because they can borrow more (the collateral constraint is relaxed). Capital prices fall going forward because $\hat{K}_t < 0$ for $t > 0$. By a similar exercise, the effect of $\hat{\lambda}$ in the future is quite similar, with an important twist. News of a future increase in λ_t has absolutely no effect on equilibrium until the shock occurs. At that point, experts' capital holdings increase but the capital price does not, and then there is a bust (lower expert capital and capital prices) going forward.

The stylized dynamics of a Minsky cycle match dynamics caused by news of a reallocative technology shock but not at all dynamics caused by a financial shock. Asset price booms are an important part of the Minsky narrative, but, perhaps surprisingly, a financial shock does not produce an asset boom at all. Instead, the expansion of debt simply *depresses* future asset prices. Additionally, there is no role for news with a financial shock. The reallocative technology shock matches the Minsky narrative much better than a financial shock does.[12]

5.2 News, Noise, and Reallocation

The shock we consider—news that primarily benefits agents other than the leveraged experts—is critical for our story. As we have noted, the economy does not experience boom-bust dynamics if the shock is immediate. If the innovative sector entered at $t = 0$ (no news), then experts' capital holdings at $t = 0$ would not change even though the asset price would immediately increase.

The boom-bust dynamics we describe are robust to whether or not the information is truly news or just "noise." We have modeled news about future innovative technology as a real technology shock that actually transpires, but all that matters for our story to get moving is positive expectations about future asset prices leading to a credit boom today. Thus, we could just as easily tell our story using a "behavioral shock," in which agents' expectations about the future increase, but perhaps in response to news that does not transpire. To see this, consider when agents receive news at t of a technology shock at $t + 1$ but the shock does not realize.[13] Thus, agents enter the period with additional capital $\hat{K}_t$ and additional debt $\hat{K}_{t+1} + \hat{q}_{t+1}$, derived earlier. We denote the equilibrium prices and capital allocation going forward, once agents learn that the shock does not in fact realize, by $\hat{\hat{q}}$ and $\hat{\hat{K}}$.[14]

Proposition 7. *Suppose the economy receives a news shock at $t = 0$ regarding the productivity of the innovative sector at $t = 1$ of size $\hat{z}$, but at $t = 1$, the shock does not transpire (i.e., the size of the shock is 0, in fact). Then the economy experiences the following deterministic boom-bust dynamics from $t = 1$ onward:*

[12] An alternative potential way to model a shock to financial markets would be to consider a temporary change in the discount rate of non-experts, $\underline{\beta}$. A temporary increase in the discount factor leads to an increase in the capital price, which relaxes borrowing constraints (if it occurs in the future) and increases experts' wealth (since they are leveraged). The equilibrium consequence of such a shock, whether the shock is immediate or in the future, is a persistent boom in expert capital holdings and the asset price. Thus, generating boom-bust dynamics requires a boom and bust in the shocks because a temporary shock to $\underline{\beta}$ does not endogenously generate cycles.

[13] There is also a literature that discusses how bubble assets can serve as collateral in the economy, so that bubbles have real effects but can also leave behind negative consequences (Guerron-Quintana et al., 2022). The behavior of the economy in response to a "noise" shock that does not transpire has similarities in the sense that the increase in the capital price is unfounded by true fundamentals and leads to an unfounded expansion in credit.

[14] For a tractable and general method of handling combinations of shocks of this type, see Auclert et al. (2021), who provide a tractable method of suitably mixing and matching impulse responses using sequence-space Jacobians.

1. *Identical dynamics for experts' capital for all periods:* $\hat{\hat{K}}_t = \hat{K}_t$ *for all* $t > 0$*; accordingly, output dynamics are identical, except at* $t = 1$*, when output suffers (non-experts do not use the innovative technology),*

2. *A price crash at* $t = 1$*:* $\hat{\hat{q}}_1 = -\frac{r}{R\eta}\frac{(1-\underline{\beta}\gamma)\gamma\hat{z}}{1-\underline{\beta}\gamma} = -\frac{r\underline{\beta}}{1+\eta}\hat{z} < 0$*, whereas* $\hat{q}_1 > 0$ *if the shock occurs, and*

3. *Capital prices going forward are identical* $\hat{\hat{q}}_t = \hat{q}_t$ *for all* $t > 1$*.*

An interesting extension to this result is to suppose that news is of a persistent AR(1) shock with persistence $\rho \in (0,1)$. In this case, we have $\hat{K}_0 = \beta\gamma(\frac{1}{1-\beta\rho})$ and $\hat{K}_1 = (\beta\gamma + \beta\rho - 1)(\frac{\gamma}{1-\beta\rho})$, but we have $\hat{\hat{K}}_1 = (\beta\gamma - 1)(\frac{\gamma}{1-\beta\rho}) < \hat{K}_1$ when agents realize that the shock does not occur. In this case, because agents expect the shock to persist, there is a very large initial increase in leverage by experts, and if ρ is sufficiently large, the boom continues into $t = 1$. When the shock fails to realize, there is an even larger deleveraging, leading to a decline in expert capital well below what would have otherwise occurred. In this case, the behavioral nature of the shock leads to an amplified bust relative to the baseline. As a final consideration, if future asset prices are below the behavioral expectation, the economy will likely also feature defaults because collateral constraints will have been set on the expectation of higher collateral values, making full debt enforcement impossible.

An important class of behavioral expectations is diagnostic expectations in which agents' recent experiences determine their beliefs regarding the future (Gennaioli and Shleifer, 2010; Bordalo et al., 2018, 2019a). Bordalo, Gennaioli, Shleifer, and Terry (2019b) use diagnostic expectations to examine the role of expectations in driving Minsky-type credit cycles with predictable returns but also predictable prediction errors. In our model, a tractable way to introduce diagnostic expectations is as follows. Suppose at $t = 0$ the economy experiences a one-time shock $\hat{z}$, but agents expect the shock to continue at a rate ρ going forward. The experience of a good shock at $t = 0$ leads agents to suppose the good times will last. For simplicity, we suppose that at $t = 1$ agents learn the truth that the shock is gone forever. With beliefs formed in this way, the dynamics of expert capital are nearly identical to the case in which agents expect a persistent shock that does not occur: $\hat{K}_0 = \beta\gamma(\frac{\rho}{1-\beta\rho})$ and $\hat{K}_1 = (\beta\gamma + \beta\rho - 1)(\frac{\gamma\rho}{1-\beta\rho})$, but we have $\hat{\hat{K}}_1 = (\beta\gamma - 1)(\frac{\gamma\rho}{1-\beta\rho}) < \hat{K}_1$ when agents realize that the shock does not occur. As before, diagnostic expectations amplify the boom-bust cycle in the main model. In summary, behavioral assumptions would amplify our result.

Hence, our story that news of a positive technology shock to an innovative sector produces boom-bust dynamics is very robust. Whether or not the technology shock realizes, we get the identical dynamics for capital for leveraged investors. Of course, output and the asset price depend on whether the shock occurs or not. But as far as our story about *endogenous* cycles is concerned, once agents get the news, the cyclical properties of expert capital dynamics are already in motion. Whether the shock realizes later matters for some things at $t = 1$, but either way, the economy will experience a boom-bust cycle.

Importantly, that is not the case if news concerns the productivity of experts. Suppose instead that agents learn at $t = 0$ that *experts* will be more productive at $t = 1$. Then it really matters if the shock happens or not. Dynamics in this case are merely the main dynamics in Kiyotaki and Moore (1997), dampened by a factor $\underline{\beta}$ because the shock occurs in the future: the experts' capital holdings increase immediately, as does the capital price. However, the economy experiences a boom-bust cycle only if the shock doesn't occur at $t = 1$: agents expect the boom to continue at $t = 1$ because experts will have lots of output from higher productivity to buy capital and repay their debts. But if productivity is not higher, then experts cannot repay their (higher) debts and are forced to sell capital. In this case, the cycle is not endogenous but the result of good news followed by bad news. In contrast, news about an innovative technology endogenously produces a cycle whether or not it is followed by "bad news" later.

5.3 News and Productivity in the Data

We have so far remained agnostic about mapping our story too closely to particular empirical episodes. In this section we provide two plausible ways of understanding two historical events in light of our story. To do so, we borrow the analysis from (Cao and L'Huillier, 2018) and consider productivity shocks that operate over different time horizons or at different frequencies. Because technological change often occurs at permanent and business-cycle frequencies simultaneously, it can be difficult to tease out the effects of particular shocks in isolation. We also discuss empirical issues related to borrowing constraints.

First, consider the role of information technology (IT) in the 1990s. We interpret news about the potential IT revolution as operating at the level of permanent technological changes, and thus we prefer to consider implied long-run productivity measures to consider this shock. Figure 4 plots smoothed permanent shocks and implied productivity growth from 1985 to 2015 (i.e., a time period centered on 2000).

Implied long-run productivity picks up in the 1990s but falls after 2000. One plausible interpretation is that news about how the internet and information technology more generally would

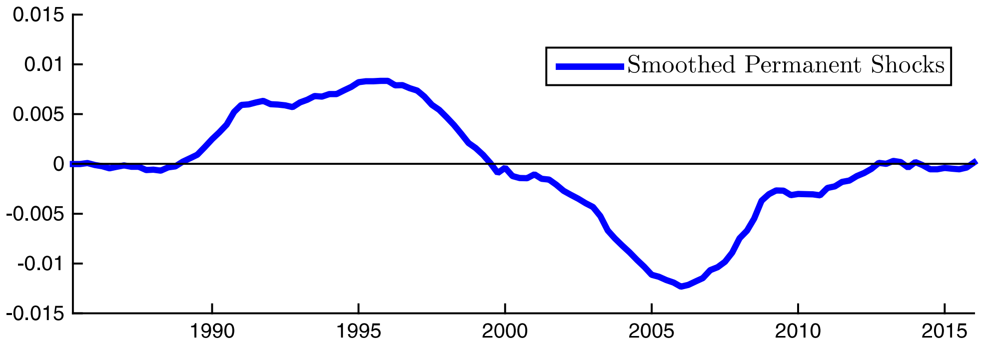


Figure 4: Smoothed permanent shocks and implied productivity growth, Great Recession. Smoothed permanent shocks are estimated using a Kalman smoother on the Great Recession sample. Source: Cao and L'Huillier (2018).

transform the economy led to the booming economy of the 1990s (as evidenced by elevated asset prices). This led to a large reallocation of resources into the IT sector, which was not fundamentally transformative. Hence, implied long-run productivity remained depressed in the 2000s because too many resources were reallocated into this new sector.

Second, consider the shock associated with the housing boom and the expansion of housing credit in the 2000s. We interpret this shock as operating at the business-cycle frequency, not on long-run productivity, and thus we prefer to look at the raw productivity data to consider this shock. Figure 5 plots productivity growth over the same period.

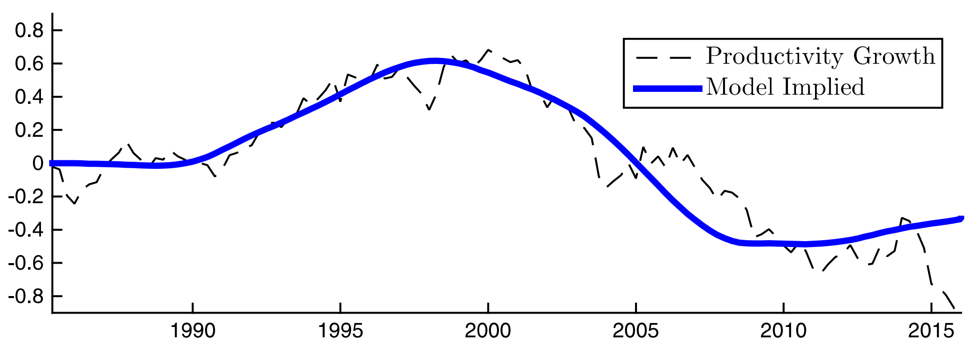


Figure 5: Productivity growth, Great Recession. The line represents productivity growth implied by the smoothed permanent shocks and the dashed black line represents the centered 10-year moving average of the (demeaned) first differences of the logarithm of labor productivity. Source: Cao and L'Huillier (2018).

When considering unfiltered productivity data, a different pattern emerges. In this case, productivity peaked around 2000 and did not fall below trend until about 2005, at which point the housing boom was still in full swing. In the context of our model, the housing boom, caused by the expansion of housing finance and by global capital flows (the cause is not important in the context of our story), led to an expansion in the years prior because credit market constraints relaxed. However, the reallocation of resources into housing, which is not more productive than the rest of the economy, led to depressed productivity growth in the years that followed.

Note that in both cases, asset prices remained high well after productivity declined. In the case of long-run productivity caused by IT, productivity fell below trend prior to the bursting of the NASDAQ. In the case of raw productivity caused by the housing boom, productivity fell prior to the decline in house prices.

Our story emphasizes the role of physical capital as collateral for credit. However, recent empirical work suggests that such asset-based collateral constraints are less relevant in many sectors of the economy, in which case, constraints based on cash flows may be more relevant (Lian and Ma, 2021). Asset-based borrowing constraints are important in real estate markets; thus, the housing boom is a natural candidate for our theory.[15] Another natural application of our theory is not to a two-sector economy but to a small, open economy that receives capital inflows. With this application in mind, our model can be interpreted as describing the surge in capital inflows before a burst. Thus, our paper would describe how technology shocks to the rest of the world could, for example, create unsustainable credit booms for emerging market economies (Boz, 2009)

6 Conclusion

Major boom-bust cycles exhibit large positive productivity shocks followed by sharp, equally large reversals in productivity. We present a model in which news of a future productivity boom in an innovative sector immediately relaxes borrowing constraints, leading to a credit-filled boom. However, the expansion of credit is not sustainable and requires a contraction of credit when the innovative sector is most productive, leading to a slump in productivity going forward. These dynamics are more pronounced when information regards innovations in the far future. The predictable boom-bust cycles produced by reallocative technology shocks match the standard Minsky

[15] Asset-based collateral constraints are important in many financial markets, particularly in securitized markets. To the extent that financial assets of firms subject to cash-flow based constraints become securitized, a dynamic of "multiple leverage cycles" (Fostel and Geanakoplos, 2014) operating at the level of financial assets and the value of the firm could provide an application for our theory. Brianti and Cormun (2022) use a flow-based financial constraint to offer a theory of sentiment-driven fluctuations. In their model, boom-bust dynamics emerge in response to sentiments but not to technology shocks.

narrative in a way that shocks to financial markets directly do not.

Our results have important implications for welfare and policy. We have intentionally kept the model as simple and stripped down as possible. Adding additional features such as nominal rigidities or the zero-lower bound, as other papers do in greater detail (see Rognlie, Shleifer, and Simsek, 2018; Farhi and Werning, 2020), would exacerbate the welfare costs of the bust following the credit expansion, suggesting that the optimal policy is to mitigate the initial expansion to mitigate the size of the bust.

Questions triggered by our analysis include how to test the predictions of the model in the data, and how to interpret particular episodes. Such an exercise is outside of the scope of our contribution, however, but we offer some brief thoughts.

There are three main challenges present for a researcher attempting to uncover the dynamics that our theory describes. First, as discussed in earlier work, the identification of major news shocks is a challenging exercise due to the presence of contemporaneous transitory shocks to productivity (see Beaudry and Portier (2014) for a review of this and other issues). Second, at first sight, the model could give the impression that these dynamics would evolve rapidly. However, work by Cao and L'Huillier (2018) suggests precisely the opposite (i.e., the presence of very slow-moving boom-bust cycles). A case in point is the Great Recession, which can be plausibly interpreted as a medium-term consequence of an original technological shock happening in the 1990s. This shock was caused by the information technology that revolutionized communications and information flows at the time. The "new sector" is embodied in the rapid spread and growth of technology startups, most of which flourished in Silicon Valley. Clearly, the simultaneous general-equilibrium movements of collateralized assets such as housing complicate the analysis, but the model and story offered by Cao and L'Huillier (2018) offer a plausible reading of the macroeconomic unfolding of events. We leave the question of how to tackle these empirical challenges for future work. Finally, the various types of borrowing constraints constitute another important empirical consideration (Lian and Ma, 2021).

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Appendices for Online Publication

A Proofs

Proof of Proposition 1, PV of Endogenous Capital Changes. We prove the result in the baseline economy, and also with borrowing constraints (see Section 5 and Appendix C).

From equation (12), $\hat{q}_0 = \frac{r}{R}\sum_{t=0}^{\infty} R^{-t}\hat{u}_t$. Hence, equation (13) can be written

$$\hat{K}_0 + \hat{u}_0 = \sum_{t=0}^{\infty} R^{-t}\hat{u}_t, \implies \hat{K}_0 = \sum_{t=1}^{\infty} R^{-t}\hat{u}_t.$$

so that the wealth gain exactly equals the PV of future user costs. Using equation (14) for $t \geq 1$, we can write the following present-value equations:

$$\sum_{t=1}^{\infty} R^{-t}\left(\hat{K}_t + \hat{u}_t\right) = \sum_{t=1}^{\infty} R^{-t}\hat{K}_{t-1},$$
$$\sum_{t=1}^{\infty} R^{-t}\hat{K}_t + \sum_{t=1}^{\infty} R^{-t}\hat{u}_t = R^{-1}\sum_{t=0}^{\infty} R^{-t}\hat{K}_t,$$

and so, plugging in for $\hat{K}_0$ and rearranging the summation on the RHS,

$$\sum_{t=1}^{\infty} R^{-t}\hat{K}_t + \hat{K}_0 = R^{-1}\sum_{t=0}^{\infty} R^{-t}\hat{K}_t,$$
$$\sum_{t=0}^{\infty} R^{-t}\hat{K}_t = R^{-1}\sum_{t=0}^{\infty} R^{-t}\hat{K}_t,$$

and that means

$$(1 - R^{-1})\left(\sum_{t=0}^{\infty} R^{-t}\hat{K}_t\right) = 0 \implies \sum_{t=0}^{\infty} R^{-t}\hat{K}_t = 0.$$

Now consider the model with borrowing constraints λ in each period. The linearized budget constraints are now given by

$$\hat{K}_0 + \frac{\lambda r}{R-\lambda}\hat{u}_0 = \frac{\lambda r}{R-\lambda}\frac{R}{r}\hat{q}_0, \tag{22}$$
$$\hat{K}_t + \frac{\lambda r}{R-\lambda}\hat{u}_t = \hat{K}_{t-1}, \forall t \geq 1. \tag{23}$$

Note that if $\lambda = 1$ then $\frac{\lambda r}{R-\lambda} = \frac{r}{R-1} = 1$, and so these are the same equations above. We prove the result in the same way. Plugging in for the capital price $\hat{q}_0$, we can then write the condition at 0 as

$$\hat{K}_0 + \frac{\lambda r}{R-\lambda}\hat{u}_0 = \frac{\lambda r}{R-\lambda}\sum_{t=0}^{\infty} R^{-t}\hat{u}_t, \implies \hat{K}_0 = \frac{\lambda r}{R-\lambda}\sum_{t=1}^{\infty} R^{-t}\hat{u}_t.$$

so that the wealth gain exactly equals the PV of future user costs, modified by the borrowing constraint. Using equations $t \geq 1$, we can write

$$\sum_{t=1}^{\infty} R^{-t} \left(\hat{K}_t + \frac{\lambda r}{R-\lambda} \hat{u}_t \right) = \sum_{t=1}^{\infty} R^{-t} \hat{K}_{t-1},$$

$$\sum_{t=1}^{\infty} R^{-t} \hat{K}_t + \frac{\lambda r}{R-\lambda} \sum_{t=1}^{\infty} R^{-t} \hat{u}_t = R^{-1} \sum_{t=0}^{\infty} R^{-t} \hat{K}_t,$$

and so, plugging in for $\hat{K}_0$ and rearranging the summation on the RHS,

$$\sum^{\infty} R^{-t} \hat{K}_t + \hat{K}_0 = R^{-1} \sum^{\infty} R^{-t} \hat{K}_t,$$

$$\sum_{t=0}^{\infty} R^{-t} \hat{K}_t = R^{-1} \sum_{t=0}^{\infty} R^{-t} \hat{K}_t,$$

and that means

$$\sum_{t=0}^{\infty} R^{-t} \hat{K}_t = 0.$$

□

Proof of Corollary 1, PV of Endogenous Output Changes. The result follows immediately from Proposition 1. We can see this result additionally for the one-period shock at $t = 1$ by explicitly considering the dynamics of capital in Proof of Proposition 2. Since capital converges to steady state at a rate of γ from $t = 1$ on, the present value of (endogenous) output changes from $t = 1$ on is

$$PV\Delta Y_1 = \frac{\hat{K}_1}{1 - \underline{\beta}\gamma}, \tag{24}$$

where this represents a loss since $\hat{K}_1 < 0$ in equilibrium. This means that the total present value of changes from $t = 0$ is

$$PV\Delta Y_0 = \hat{K}_0 + \beta \frac{\hat{K}_1}{1 - \underline{\beta}\gamma}. \tag{25}$$

Using that $\hat{K}_1 = \gamma(\hat{K}_0 - \hat{z})$, we have

$$\begin{aligned} PV\Delta Y_0 =& \hat{K}_0 + \beta \frac{\gamma(\hat{K}_0 - \hat{z})}{1 - \underline{\beta}\gamma}, \\ =& \frac{\hat{K}_0 - \underline{\beta}\gamma\hat{z}}{1 - \underline{\beta}\gamma} = 0, \end{aligned}$$

since $\hat{K}_0 = \underline{\beta}\gamma\hat{z}$ in equilibrium. □

Proof of Corollary 2, Limiting Leverage in Boom. The result follows immediately from Proposition 1, with the following observation. Suppose we exogenously do not allow experts to fully expand their capital holdings at zero. We can model this as if there were a leverage constraint that was tighter than usual for one period: we tighten the constraint by $\hat{\lambda}$ so that experts can borrow only a fraction $\lambda(1-\hat{\lambda})$ of future capital value. Let $\lambda = 1$ be the baseline case (the result immediately generalizes for general λ). Then, by decreasing leverage at zero, we have the following modifications to budget constraints:

$$\hat{K}_0 + \hat{u}_0 = \frac{R}{r}\hat{q}_0 - \frac{1}{r}\hat{\lambda}, \tag{26}$$

$$\hat{K}_1 + \hat{u}_1 = \hat{K}_0 + \frac{R}{r}\hat{\lambda}. \tag{27}$$

Notice that the decrease in borrowing at $t = 0$ leads to an increase in available balance sheet at $t = 1$, but the present value is equal. Thus, the present-value result continues to hold for any $\hat{\lambda}$.

□

Proof of Proposition 2, Baseline Result. At $t = 1$ the user cost is given by $\hat{u}_1 = \frac{1}{\eta}\hat{K}_1 + \hat{z}$, and so we have

$$\frac{1}{\eta}\hat{K}_1 + \hat{z} + \hat{K}_1 = \hat{K}_0, \implies \quad \hat{K}_1 = \gamma\left(\hat{K}_0 - \hat{z}\right), \tag{28}$$

where $\gamma \equiv \frac{\eta}{1+\eta}$ reflects the elasticity of non-expert demand for capital, and $\gamma < 1$. For $t > 1$ the change in the user cost is determined entirely by capital holdings because there is no shock, and so

$$\left(1 + \frac{1}{\eta}\right)\hat{K}_t = \hat{K}_{t-1} \implies \hat{K}_t = \gamma\hat{K}_{t-1}. \tag{29}$$

Hence, for all $s \geq 1$ we have

$$\hat{K}_t = \gamma^t\left(\hat{K}_0 - \hat{z}\right).$$

From (12) we can write the capital price as

$$\hat{q}_0 = \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t + \underline{\beta}\frac{r}{R}\hat{z},$$

where the $\hat{z}$ term reflects that the user cost at $t = 1$ contains the shock. In order to plug in for $\hat{q}_0$, we execute the following manipulations:

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\gamma^t\left(\hat{K}_0-\hat{z}\right)+\frac{r}{R}\frac{1}{\eta}\hat{z}+\underline{\beta}\frac{r}{R}\hat{z},\\
\frac{R}{r}\hat{q}_0 &= \frac{1}{\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\right)\left(\hat{K}_0-\hat{z}\right)+\hat{z}\left(\underline{\beta}+\frac{1}{\eta}\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0-\hat{z}\left(\frac{1}{1-\underline{\beta}\gamma}-\underline{\beta}\eta-1\right).
\end{aligned}$$

Plugging in for $\hat{q}_0$ from the budget constraint, we have

$$\begin{aligned}
(1+\eta)\hat{K}_0 &= \left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0-\hat{z}\left(\frac{\underline{\beta}\gamma}{1-\underline{\beta}\gamma}-\underline{\beta}\eta\right),\\
(1+\eta)\hat{K}_0 &= \left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0+\hat{z}\underline{\beta}\left(\eta-\frac{\gamma}{1-\underline{\beta}\gamma}\right),\\
\left(1+\eta-\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0 &= \hat{z}\underline{\beta}\gamma\left(1+\eta-\frac{1}{1-\underline{\beta}\gamma}\right),\\
\hat{K}_0 &= \hat{z}\underline{\beta}\gamma.
\end{aligned}$$

And so, $\hat{K}_0>0$. Additionally we have $\hat{K}_1=-\hat{z}\gamma(1-\underline{\beta}\gamma)<0$. Thus, using (29), we also have $\hat{K}_t<0$ for all $t>0$.

From the budget constraint at $t=0$, we have that the asset price is given by

$$\frac{R}{r}\hat{q}_0=\left(1+\frac{1}{\eta}\right)\hat{z}\underline{\beta}\gamma \implies \quad \hat{q}_0=r\underline{\beta}^2\hat{z}.$$

Since $\hat{K}_t<0$ for all $s>0$, it follows that $\hat{q}_t<0$ for all $t>0$.

Finally, we can write the capital price at $t=1$ as

$$\hat{q}_1=\frac{r}{R}\left(\sum_{t=0}^{\infty}\underline{\beta}^t\frac{\hat{K}_{t+1}}{\eta}+\hat{z}\right)=\frac{r}{R}\left(\sum_{t=0}^{\infty}(\gamma\underline{\beta})^t\frac{\gamma(\hat{K}_0-\hat{z})}{\eta}+\hat{z}\right),$$

where $\hat{K}_t=\gamma^t(\hat{K}_0-\hat{z})$. Taking the infinite sum, we have

$$\begin{aligned}
\hat{q}_1 &= \frac{r}{R}\left(\frac{\gamma}{\eta}\frac{\hat{K}_0-\hat{z}}{1-\underline{\beta}\gamma}+\hat{z}\right)=\frac{r}{R}\left(\frac{\gamma}{\eta}\frac{(\underline{\beta}\gamma\hat{z})-\hat{z}}{1-\underline{\beta}\gamma}+\hat{z}\right),\\
&= \frac{r}{R}\left(-\frac{\gamma}{\eta}\frac{\hat{z}(1-\underline{\beta}\gamma)}{1-\underline{\beta}\gamma}+\hat{z}\right)=\frac{r}{R}\left(-\frac{\gamma}{\eta}\hat{z}+\hat{z}\right),\\
&= r\underline{\beta}\gamma\hat{z},
\end{aligned}$$

and hence $\hat{q}_1 > 0$. Equivalently, we can manipulate equation (37) by using $\hat{u}_0 = \eta\hat{K}_0$ to write $(1+\eta)\hat{u}_0 = \frac{R}{r}\hat{q}_0$. Plugging into the asset price equation $\hat{q}_0 = \frac{r}{R}\hat{u}_0 + \underline{\beta}\hat{q}_1$ we can write the recursion

$$\hat{q}_0 = \frac{1}{1+\eta}\hat{q}_0 + \underline{\beta}\hat{q}_1 \implies \hat{q}_1 = \frac{\gamma}{\underline{\beta}}\hat{q}_0.$$

Note that again we have $\hat{q}_1 = \frac{\gamma}{\underline{\beta}} r\underline{\beta}^2\hat{z} = r\underline{\beta}\gamma\hat{z}$. □

Proof of Proposition 3, Output boom-bust. The result follows immediately from equations (15)–(16) and Proposition 2. When $t \neq t'$ the change in output is determined entirely by the change in $\hat{K}_t$. In the period of the shock, we have

$$\hat{Y}_{t'+1} = (a+c-Ra)\frac{K_*}{Y_*}\hat{K}_{t'} + (a^I - Ra)\frac{K_*}{Y_*}\hat{K}^I,$$

Output booms in $t'+1$ if

$$(a+c-Ra)\hat{K}_{t'} + (a^I - Ra)\hat{K}^I > 0.$$

Since $\hat{K}_{t'} < 0$, we have a boom in $\hat{Y}_{t'+1}$ if a^I is sufficiently large and otherwise we have a bust in output. □

Proof of Proposition 4, Contemporaneous Shock. Since the shock is contemporaneous, we have $\hat{u}_0 = \frac{1}{\eta}\hat{K}_0 + \hat{z}$ and $\hat{u}_t = \frac{1}{\eta}\hat{K}_t$ for $t > 0$. From (12) we can write the capital price as

$$\hat{q}_0 = \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t + \frac{r}{R}\hat{z},$$

where the $\hat{z}$ term reflects that the user cost at $t = 0$ contains the shock. Linearizing the experts' budget constraint in future periods yields $\hat{u}_{t+1} + \hat{K}_{t+1} = \hat{K}_t$ for $t \geq 0$, and so we have

$$\hat{K}_t = \gamma^t\hat{K}_0. \tag{30}$$

Plugging (30) into the equation for the asset price, we have

$$\begin{aligned}\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\gamma^t\hat{K}_0 + \frac{r}{R}\hat{z},\\ &= \frac{r}{R\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0 + \frac{r}{R}\hat{z}.\end{aligned}$$

Linearizing the experts' budget constraint at $t = 0$ we have $\hat{K}_0 + \hat{u}_0 = \frac{R}{r}\hat{q}_0$, which becomes

$$\left(1 + \frac{1}{\eta}\right)\hat{K}_0 + \hat{z} = \frac{R}{r}\hat{q}_0 \implies \hat{K}_0 = \gamma\left(\frac{R}{r}\hat{q}_0 - \hat{z}\right).$$

Plugging in for $\hat{q}_0$ above, we have

$$\begin{aligned}
\hat{K}_0 &= \gamma\left(\frac{R}{r}\left(\frac{r}{R\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0 + \frac{r}{R}\hat{z}\right) - \hat{z}\right), \\
&= \gamma\left(\frac{1}{\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0 + \hat{z} - \hat{z}\right), \\
&= \frac{\gamma}{\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\right)\hat{K}_0,
\end{aligned}$$

which clearly implies that $\hat{K}_0 = 0$. Together with (30), this implies $\hat{K}_t = 0$ for all $t \geq 0$, and thus $\hat{u}_t = 0$ for $t > 0$. Since the asset price is the present value of the user cost, $\hat{q}_t = 0$ for $t > 0$. Finally, since the capital allocation is unchanged for all t, the only change in output comes from the shock at $t = 0$. □

Proof of Proposition 5, N-forward News. The key equations are the same as before, with the exception of the budget constraint and user cost at $t = N$ instead of at $t = 1$. At time $t = N$, non-experts anticipate a higher marginal productivity of capital, so the user cost is given by

$$\hat{u}_N = \frac{1}{\eta}\hat{K}_N + \hat{z}, \tag{31}$$

hence, we have

$$\hat{K}_N = \gamma\left(\hat{K}_{N-1} - \hat{z}\right). \tag{32}$$

For $0 \leq t < N$,

$$\hat{K}_t = \gamma^t \hat{K}_0, \tag{33}$$

and for $t \geq N$,

$$\hat{K}_t = \gamma^t \hat{K}_0 - \gamma^{t+1-N}\hat{z}. \tag{34}$$

Finally, since the capital price is the discounted sum of future user costs, we have

$$\hat{q}_0 = \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t \hat{K}_t + \underline{\beta}^N \frac{r}{R}\hat{z}. \tag{35}$$

We then plug (33) and (34) into (35) and solve:

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t + \underline{\beta}^N\frac{r}{R}\hat{z}, \\
\frac{R}{r}\hat{q}_0 &= \frac{1}{\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\gamma^t\hat{K}_0 - \frac{1}{\eta}\sum_{t=N}^{\infty}\underline{\beta}^t\gamma^{t+1-N}\hat{z} + \underline{\beta}^N\hat{z}, \\
\frac{R\eta}{r}\hat{q}_0 &= \frac{\hat{K}_0}{1-\underline{\beta}\gamma} + \frac{\hat{z}}{R^N}\left(-\frac{\gamma}{1-\underline{\beta}\gamma} + \eta\right), \frac{R\eta}{r} \\
\hat{q}_0 &= \frac{\hat{K}_0}{1-\underline{\beta}\gamma} - \frac{\hat{z}}{R^{N-1}}\left(\frac{1}{1-\underline{\beta}\gamma} - \frac{\eta}{R} - 1\right).
\end{aligned}$$

We now consider the following lemma.

Lemma 1. *If the capital price at $t = 0$ can be written*

$$\hat{q}_0 = \frac{r}{R\eta}\left(\frac{1}{1-\underline{\beta}\gamma}\hat{K}_0 + X\eta\right) \tag{36}$$

for some X, then in equilibrium $\hat{K}_0 = \frac{X(1-\underline{\beta}\gamma)}{1-\underline{\beta}}$.

Proof. Linearizing the budget constraint at $t = 0$ yields $\hat{K}_0 + \hat{u}_0 = \frac{R}{r}\hat{q}_0$, which becomes

$$\hat{K}_0(1+\eta) = \frac{R\eta}{r}\hat{q}_0 \implies \left(1+\frac{1}{\eta}\right)\hat{K}_0 = \frac{R}{r}\hat{q}_0. \tag{37}$$

Plugging in the proposed capital price and solving for $\hat{K}_0$ yields the solution. □

From Lemma 1 this implies

$$ \tag{38}$$

It then follows from $\frac{R}{r}\hat{q}_0 = \left(1+\frac{1}{\eta}\right)\hat{K}_0$ that

$$\hat{q}_0 = r\underline{\beta}^{N+1}\hat{z}. \tag{39}$$

From (34) we have

$$\hat{K}_N = \gamma^N\hat{K}_0 - \gamma\hat{z} \implies \hat{K}_N = -\gamma\hat{z}\left(1-(\underline{\beta}\gamma)^N\right) < 0. \tag{40}$$

Finally, we can write the capital price at $t = N$ as

$$\hat{q}_N = \frac{r}{R}\left(\sum_{s=0}^{\infty}\underline{\beta}^s\frac{\hat{K}_{s+N}}{\eta} + \hat{z}\right) = \frac{r}{R}\left(\sum_{s=0}^{\infty}(\underline{\beta}\gamma)^s\frac{\hat{K}_N}{\eta} + \hat{z}\right).$$

We then use $\hat{K}_N = -\gamma\hat{z}\left(1 - \left(\underline{\beta}\gamma\right)^N\right)$. Taking the infinite sum, we have

$$\begin{aligned}
\hat{q}_N &= \frac{r}{R}\left(-\frac{\gamma}{\eta}\frac{\hat{z}\left(1-\left(\underline{\beta}\gamma\right)^N\right)}{1-\underline{\beta}\gamma} + \hat{z}\right) = r\underline{\beta}\hat{z}\left(-\frac{1}{1+\eta}\frac{\left(1-\left(\underline{\beta}\gamma\right)^N\right)}{1-\underline{\beta}\gamma} + 1\right), \\
&= r\underline{\beta}\hat{z}\left(\frac{\left(\underline{\beta}\gamma\right)^N - 1}{1+\eta-\underline{\beta}\eta} + 1\right) = r\underline{\beta}\hat{z}\left(\frac{\left(\underline{\beta}\gamma\right)^N - 1 + 1 + \eta - \underline{\beta}\eta}{1+\eta-\underline{\beta}\eta}\right), \\
&= r\underline{\beta}\hat{z}\left(\frac{\left(\underline{\beta}\gamma\right)^N + \eta - \underline{\beta}\eta}{1+\eta-\underline{\beta}\eta}\right) > 0.
\end{aligned}$$

□

Proof of Proposition 7, Behavioral Shocks. Note that the linearized budget constraint at $t = 1$ becomes

$$\hat{\hat{u}}_1 + \hat{\hat{K}}_1 = \hat{K}_0 + \frac{R}{r}\left(\hat{\hat{q}}_1 - \hat{q}_1\right),$$

reflecting that capital and debt were predetermined. It's useful to rewrite this as

$$\left(1 + \frac{1}{\eta}\right)\hat{\hat{K}}_1 = \frac{R}{r}\hat{\hat{q}}_1 + Z, \tag{41}$$

where $Z \equiv \hat{K}_0 - \frac{R}{r}\hat{q}_1 = \underline{\beta}\gamma\hat{z} - \frac{R}{r}r\underline{\beta}\gamma\hat{z} = -\gamma(1-\underline{\beta})\hat{z}$. It is as if experts face a negative productivity shock. They have more capital than steady state, $\hat{K}_0 > 0$, but also more debt, $\hat{q}_1 > 0$, and the additional debt weighs on available funds by more than the additional output from higher capital.

We can write the capital price, which is the discounted value of future user costs, as

$$\hat{\hat{q}}_1 = \frac{r}{R\eta}\frac{\hat{\hat{K}}_1}{1-\underline{\beta}\gamma}.$$

Plugging into the budget constraint, we therefore have

$$\begin{aligned}
\left(1+\frac{1}{\eta}\right)\hat{\hat{K}}_1 &= \frac{1}{\eta}\frac{\hat{\hat{K}}_1}{1-\underline{\beta}\gamma}+Z,\\
(1+\eta)\left(1-\underline{\beta}\gamma\right)\hat{\hat{K}}_1 &= \hat{\hat{K}}_1+\eta(1-\underline{\beta}\gamma)Z,\\
\hat{\hat{K}}_1 &= \frac{\eta(1-\underline{\beta}\gamma)}{(1+\eta)\left(1-\underline{\beta}\gamma\right)-1}Z,\\
\hat{\hat{K}}_1 &= \frac{\eta(1-\underline{\beta}\gamma)}{1+\eta-\underline{\beta}\eta-1}Z,\\
\hat{\hat{K}}_1 &= \frac{\eta(1-\underline{\beta}\gamma)}{\eta(1-\underline{\beta})}Z,\\
\hat{\hat{K}}_1 &= -\frac{(1-\underline{\beta}\gamma)}{(1-\underline{\beta})}\gamma(1-\underline{\beta})\hat{z},\\
\hat{\hat{K}}_1 &= -(1-\underline{\beta}\gamma)\gamma\hat{z}=\hat{K}_1.
\end{aligned}$$

Hence, the experts' capital holdings in the new equilibrium are exactly as they would have been. The asset price, however, is lower:

$$\hat{\hat{q}}_1=-\frac{r}{R\eta}\frac{(1-\underline{\beta}\gamma)\gamma\hat{z}}{1-\underline{\beta}\gamma}=-\frac{r\underline{\beta}}{1+\eta}\hat{z}<0.$$

Recall that $\hat{q}_1>0$.

Finally, suppose that at $t=0$ agents learn that experts will have additional productivity Δ at $t=1$ (i.e., consider news about *expert* productivity). We can write the asset price at $t=1$ as a function of capital at $t=1$ as

$$\hat{q}_1=\frac{r}{R\eta}\left(\frac{1}{1-\beta\gamma}\right)\hat{K}_1.$$

Plugging in for the value of capital at $t=1$ we have

$$\hat{q}_1=\frac{r}{R\eta}\left(\frac{1}{1-\beta\gamma}\right)\left(\gamma\Delta+\gamma\hat{K}_0\right)=\frac{r}{R\eta}\left(\frac{\gamma}{1-\beta\gamma}\right)\left(\Delta+\hat{K}_0\right).$$

From the equation for the asset price at $t=0$, we have

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\hat{K}_0+\frac{r}{R\eta}\left(\frac{\beta\gamma}{1-\beta\gamma}\right)\left(\Delta+\hat{K}_0\right)\\
&= \frac{r}{R\eta}\left(\frac{1}{1-\beta\gamma}\right)\hat{K}_0+\frac{r}{R\eta}\left(\frac{\beta\gamma}{1-\beta\gamma}\right)\Delta.
\end{aligned}$$

Plugging in the budget constraint at $t = 0$, we have

$$\hat{K}_0 = \left(\frac{1}{r(1+\eta)}\right)\Delta.$$

Plugging this value into the budget constraint equation to get the asset price at $t = 0$, we have

$$\hat{q}_0 = \left(\frac{\beta}{\eta}\right)\Delta,$$

and then we have

$$\hat{q}_1 = \frac{R}{1+\frac{1}{\eta}}\left(\frac{\beta}{\eta}\right)\Delta = \frac{\Delta}{1+\eta}.$$

Finally, we have

$$\hat{K}_1 = \gamma\Delta + \gamma\left(\frac{1}{r(1+\eta)}\right)\Delta = \gamma\left(1+\frac{1}{r(1+\eta)}\right)\Delta.$$

Plugging $\hat{q}_1$ and $\hat{K}_0$ into equation (41) it is clear that we end up with $\hat{\hat{K}}_1 < 0 < \hat{K}_1$ implying different capital dynamics. $\square$

B Persistent Shocks

We first consider persistent decaying AR(1) shocks and then a permanent shock.

B.1 Persistent Shock Beginning at $t = 1$

Let's suppose that starting at $t = 1$ the economy experiences an AR(1) decaying shock $\hat{z}_t = \rho^{t-1}$, with $\rho \in (0, 1)$. We have $\hat{z}_1 = 1$ and then the shock decays at rate ρ going forward. Accordingly, for $s > 0$ we have the user cost

$$\hat{u}_t = \frac{1}{\eta}\hat{K}_t + \hat{z}_t.$$

Capital dynamics are as follows. Linearizing budget constraints for $s > 0$ we have

$$\hat{K}_{t+1} = \gamma\left(\hat{K}_t - \hat{z}_{t+1}\right).$$

We solve the model as before, plugging these conditions into the two key equations at $t = 0$. The budget constraint at is given by

$$\hat{K}_0 = \gamma\frac{R}{r}\hat{q}_0,$$

and the asset price is

$$\hat{q}_0 = \frac{r}{R}\sum_{t=0}^{\infty}\underline{\beta}^t \hat{u}_t = \frac{r}{R}\sum_{t=0}^{\infty}\underline{\beta}^t\left(\frac{1}{\eta}\hat{K}_t + \hat{z}_t\right),$$

keeping in mind that $\hat{z}_0 = 0$. Iterating forward the equation for capital dynamics,

$$\hat{K}_t = \gamma^t \hat{K}_0 - \sum_{i=1}^{t}\gamma^{t+1-i}\hat{z}_i = \gamma^t\hat{K}_0 - \sum_{i=1}^{t}\gamma^{t+1-i}\rho^{i-1} = \gamma^t\hat{K}_0 - \gamma^t\sum_{i=1}^{t}(\rho/\gamma)^{i-1},$$

and summing yields

$$\hat{K}_t = \gamma^t\hat{K}_0 - \gamma^t\frac{1-\left(\frac{\rho}{\gamma}\right)^t}{1-\frac{\rho}{\gamma}} = \gamma^t\hat{K}_t - \gamma\left(\frac{\gamma^t-\rho^t}{\gamma-\rho}\right).$$

Plugging into the equation for the asset price, we have

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R}\left(\sum_{t=0}^{\infty}\underline{\beta}^t\left(\frac{1}{\eta}\hat{K}_t\right) + \sum_{t=1}^{\infty}\underline{\beta}^t\hat{z}_t\right),\\
&= \frac{r}{R}\left(\sum_{t=0}^{\infty}\underline{\beta}^t\frac{1}{\eta}\left(\gamma^t\hat{K}_0 - \gamma\left(\frac{\gamma^t-\rho^t}{\gamma-\rho}\right)\right) + \sum_{t=1}^{\infty}\underline{\beta}^t\rho^{t-1}\right),\\
&= \frac{r}{R}\left(\frac{1}{\eta}\frac{1}{1-\underline{\beta}\gamma}\hat{K}_0\right) + \frac{r}{R}\left(\frac{\underline{\beta}}{1-\underline{\beta}\rho}\right)\left(-\frac{r}{R}\frac{1}{\eta}\frac{\gamma}{\gamma-\rho}\quad\sum_{t=0}^{\infty}(\underline{\beta}\gamma)^t - (\underline{\beta}\rho)^t\right),\\
&= \frac{r}{R}\left(\frac{1}{\eta}\frac{1}{1-\underline{\beta}\gamma}\hat{K}_0\right) + \frac{r}{R}\left(\frac{\underline{\beta}}{1-\underline{\beta}\rho}\right) - \frac{r}{R}\frac{1}{\eta}\frac{\gamma}{\gamma-\rho}\left(\frac{1}{1-\underline{\beta}\gamma} - \frac{1}{1-\underline{\beta}\rho}\right).
\end{aligned}$$

From Lemma 1 with $X = \frac{\underline{\beta}\gamma(1-\underline{\beta})}{(1-\underline{\beta}\gamma)(1-\underline{\beta}\rho)}$ we have

$$\hat{K}_0 = \underline{\beta}\gamma\left(\frac{1}{1-\underline{\beta}\rho}\right) > 0.$$

B.2 Persistent Shock Beginning N Periods Forward, $t' = N$

Now suppose the shock starts at $t = N$, $\hat{z}_t = \rho^{t-N}$ for $t \geq N$ and $\hat{z}_t = 0$ for $t < N$. Accordingly, for $t > 0$ we have the user cost

$$\hat{u}_t = \frac{1}{\eta}\hat{K}_t + \hat{z}_t.$$

Capital dynamics are as follows. Linearizing budget constraints for $t > 0$ we have

$$\hat{K}_{t+1} = \gamma\left(\hat{K}_t - \hat{z}_{t+1}\right),$$

keeping in mind that the shock is zero for $s < N$.

We solve the model as before, plugging these conditions into the two key equations at $t = 0$. The budget constraint is given by

$$\hat{K}_0 = \gamma \frac{R}{r} \hat{q}_0,$$

and the asset price is

$$\hat{q}_0 = \frac{r}{R} \sum_{t=0}^{\infty} \underline{\beta}^t \hat{u}_t = \frac{r}{R} \sum_{t=0}^{\infty} \underline{\beta}^t \left(\frac{1}{\eta} \hat{K}_t + \hat{z}_t \right),$$

keeping in mind that $\hat{z}_t = 0$ for $t < N$. Iterating forward the equation for capital dynamics, for $0 \leq t < N$, $\hat{K}_t = \gamma^t \hat{K}_0$, and then also,

$$\hat{K}_{N+t} = \gamma^{t+N} \hat{K}_0 - \sum_{i=0}^{t} \gamma^{t+1-i} \hat{z}_i = \gamma^{t+N} \hat{K}_0 - \sum_{i=0}^{t} \gamma^{t+1-i} \rho^i = \gamma^{t+N} \hat{K}_0 - \gamma^{t+1} \sum_{i=0}^{t} (\rho/\gamma)^i.$$

Summing yields

$$\hat{K}_{N+t} = \gamma^{t+N} \hat{K}_0 - \gamma^{t+1} \frac{1 - \left(\frac{\rho}{\gamma}\right)^{t+1}}{1 - \frac{\rho}{\gamma}} = \gamma^{t+N} \hat{K}_0 - \gamma \left(\frac{\gamma^{t+1} - \rho^{t+1}}{\gamma - \rho} \right),$$

implying for $s \geq N$ we can write

$$\hat{K}_t = \gamma^t \hat{K}_0 - \gamma \left(\frac{\gamma^{t+1-N} - \rho^{t+1-N}}{\gamma - \rho} \right),$$

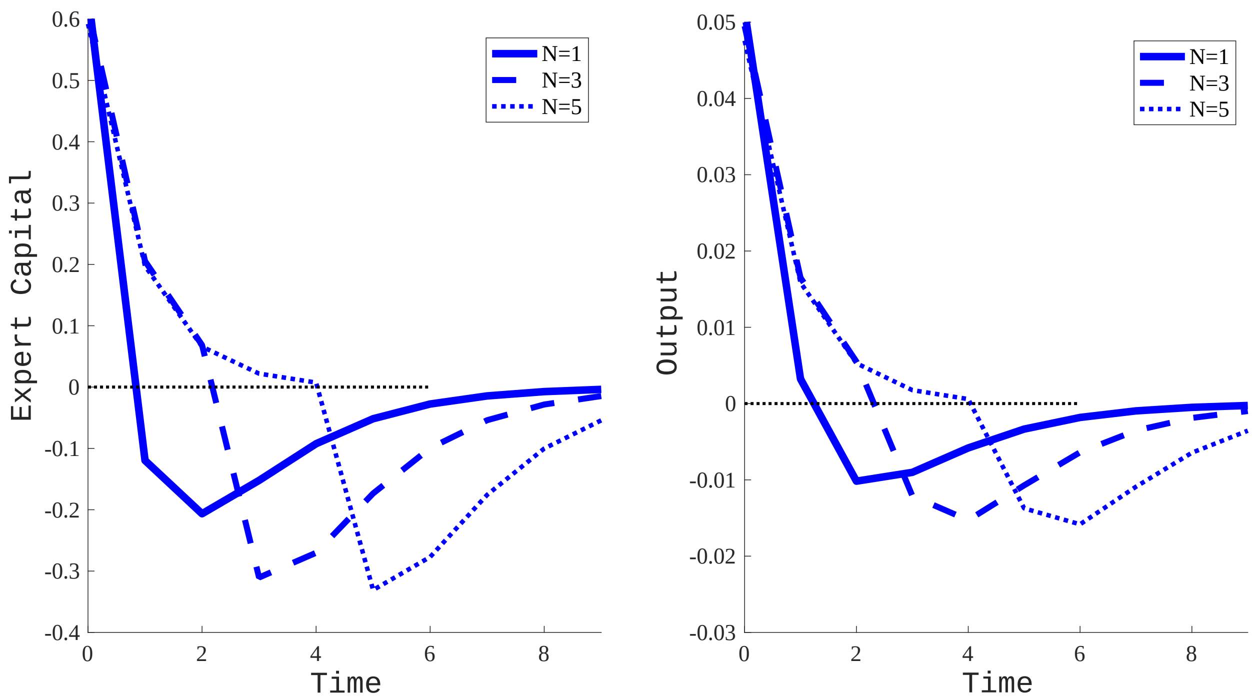
Plugging into the equation for the asset price, starting the shock at $t = N$, we have

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R} \left(\sum_{t=0}^{\infty} \underline{\beta}^t \left(\frac{1}{\eta} \hat{K}_t \right) + \sum_{t=N}^{\infty} \underline{\beta}^t \hat{z}_t \right), \\
&= \frac{r}{R} \left(\sum_{t=0}^{\infty} \underline{\beta}^t \frac{1}{\eta} \gamma^t \hat{K}_0 + \sum_{t=N}^{\infty} \underline{\beta}^t \left(\rho^{t-N} - \frac{\gamma}{\eta} \left(\frac{\gamma^{t+1-N} - \rho^{t+1-N}}{\gamma - \rho} \right) \right) \right), \\
&= \frac{r}{R} \left(\frac{1}{\eta} \frac{1}{1 - \underline{\beta}\gamma} \hat{K}_0 \right) + \frac{r}{R} \left(\frac{\underline{\beta}^N}{1 - \underline{\beta}\rho} \right) - \frac{r}{R} \frac{1}{\eta} \frac{\gamma}{\gamma - \rho} \left(\sum_{t=N}^{\infty} \gamma^{1-N} (\underline{\beta}\gamma)^t - \rho^{1-N} (\underline{\beta}\rho)^t \right), \\
&= \frac{r}{R} \left(\frac{1}{\eta} \frac{1}{1 - \underline{\beta}\gamma} \hat{K}_0 \right) + \frac{r}{R} \left(\frac{\underline{\beta}^N}{1 - \underline{\beta}\rho} \right) - \frac{r}{R} \frac{1}{\eta} \frac{\gamma}{\gamma - \rho} \left(\frac{\underline{\beta}^N \gamma}{1 - \underline{\beta}\gamma} - \frac{\underline{\beta}^N \rho}{1 - \underline{\beta}\rho} \right).
\end{aligned}$$

Note that we now have $X = \frac{\underline{\beta}^N \gamma (1-\underline{\beta})}{(1-\underline{\beta}\gamma)(1-\underline{\beta}\rho)}$. From Lemma 1 we have

$$\hat{K}_0 = \underline{\beta}^N \gamma \left(\frac{1}{1-\underline{\beta}\rho} \right) > 0.$$

Figures 6 and 7 plot the dynamics of capital and output, varying $N = 1, 3, 5$ for $\rho = 0.5$ and $\rho = 0.9$.



(a) Expert Capital $\hat{K}_t$

(b) Output

Figure 6: Changes in expert capital and output in response to news at $t = N$, varying N, decaying at rate $\rho = 0.5$. Source: authors' analysis.

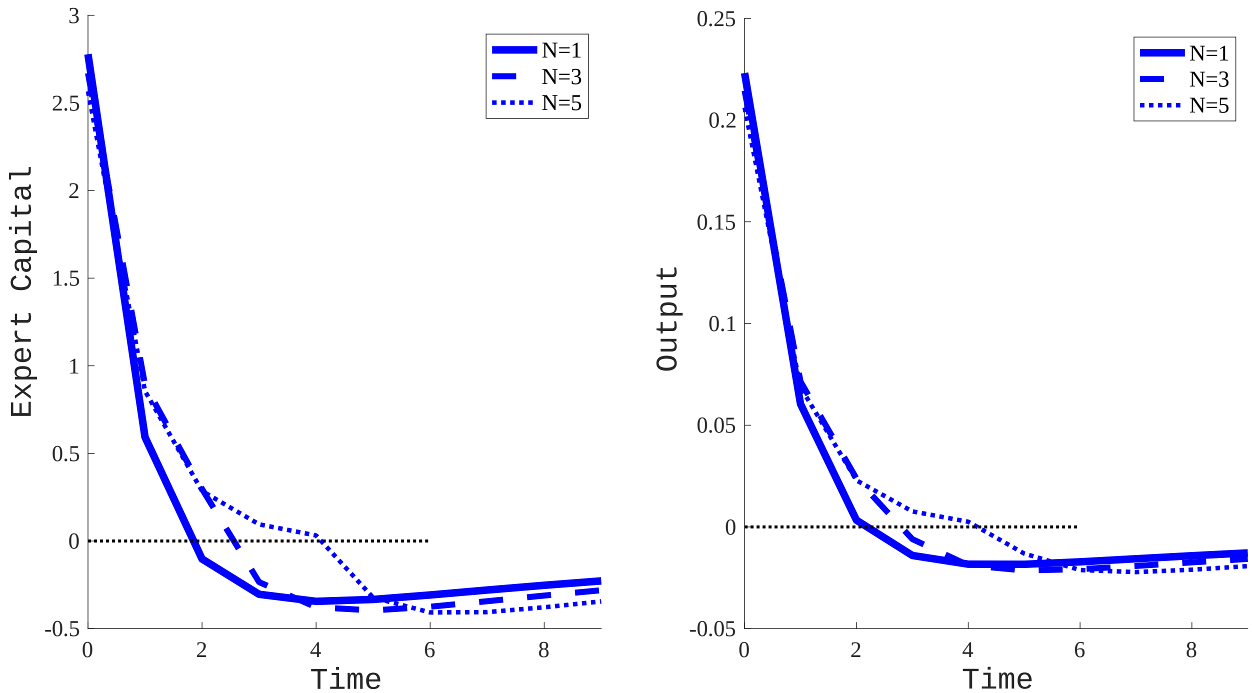
B.3 Permanent Shock

Suppose that the shock $\hat{z}$ occurs in every period after $t = 1$. The key equations are

$$\hat{K}_0 = \gamma \frac{R}{r} \hat{q}_0, \quad \hat{q}_0 = \frac{r}{R} \sum_{t=0}^{\infty} \underline{\beta}^t \hat{u}_t, \quad \hat{K}_{t+1} = \gamma(\hat{K}_t - \hat{z}). \tag{42}$$

Note that the last equation implies that

$$\hat{K}_N = \gamma^N \hat{K}_0 - \hat{z} \sum_{t=1}^{N} \gamma^t = \gamma^N \hat{K}_0 - \hat{z} \frac{\gamma - \gamma^{N+1}}{1-\gamma}. \tag{43}$$



(a) Expert Capital $\hat{K}_t$ (b) Output

Figure 7: Changes in expert capital and output in response to news at $t = N$, varying N, decaying at rate $\rho = 0.9$. Source: authors' analysis.

For $t > 0$ we have

$$\hat{u}_t = \frac{1}{\eta}\hat{K}_t + \hat{z},$$

and so the asset price can be written

$$\begin{aligned}\hat{q}_0 &= \frac{r}{R}\left(\sum_{t=0}^{\infty}\underline{\beta}^t\left(\frac{1}{\eta}\hat{K}_t\right) + \sum_{t=1}^{\infty}\underline{\beta}^t\hat{z}\right), \\ &= \frac{r}{R}\left(\sum_{t=0}^{\infty}\underline{\beta}^t\left(\frac{1}{\eta}\gamma^t\hat{K}_0\right) + \sum_{t=1}^{\infty}\underline{\beta}^t\left(-\hat{z}\frac{1}{\eta}\frac{\gamma-\gamma^{t+1}}{1-\gamma} + \hat{z}\right)\right).\end{aligned}$$

From Lemma 1 we have

$$\hat{K}_0 = \hat{z}\frac{\underline{\beta}\gamma}{1-\underline{\beta}} = \hat{z}\frac{\gamma}{r} > 0.$$

Note that asymptotically $\hat{K}_t \to -\hat{z}\frac{\gamma}{1-\gamma} = -\eta\hat{z}$. We converge back to the original steady-state price q_*, but first the price rises and experts hold more capital because collateral constraints are relaxed. But the price converges back to the steady state, and experts hold less capital, consistent with non-experts' increased productivity.

C Borrowing Constraints

This section considers two relevant modifications of the collateral constraint. First, we consider when agents can borrow only a fraction of the value of future capital ("tight" borrowing constraints). Second, we consider when agents can borrow using state-contingent payments that can be made contingent on aggregate variables ("equity").

C.1 Tight Borrowing Constraints

Let the borrowing constraint be given by

$$Rb_t = \lambda_t q_{t+1} k_t, \tag{44}$$

where $\lambda_t < 1$. The budget constraint for experts is now

$$\left(q_t - \underline{\beta}\lambda_t q_{t+1}\right) K_t = aK_{t-1} + (1 - \lambda_{t-1}) q_t K_{t-1}. \tag{45}$$

With a constant $\lambda \geq \frac{a}{a+c}$, steady-state values are as follows:

$$q_* = \frac{a}{\lambda(1 - \underline{\beta})} = \frac{Ra}{\lambda r}, \quad u_* = \frac{a}{\lambda},$$

where $u_t = q_t - \underline{\beta} q_{t+1}$ as before. The collateral constraint is binding as long as $\lambda \geq \frac{a}{a+c}$. Note that $u_* = \frac{a}{\lambda}$, which is higher than the user cost in the baseline model when experts can borrow the full value of capital. Since the non-experts' marginal cost cannot exceed $a + c$ in equilibrium, this equilibrium regime holds so long as $\lambda \geq \frac{a}{a+c}$. Note that a tighter λ leads to a more efficient allocation of capital.

We first reconsider the main results in the paper, which are quantitatively dampened if $\lambda < 1$ but otherwise the same, and then we consider shocks to λ_t. We refer to shocks to λ_t as "financial shocks." Our main findings are that the consequences of financial shocks are quite distinct from the consequences of technology shocks.

C.2 Technology Shocks

We first consider a technology shock $\hat{z}$ as before.

Proposition 8. *In response to a news shock at $t = 0$ regarding the productivity of the innovative sector at $t = 1$, the economy experiences the following deterministic boom-bust dynamics:*

1. *A larger increase in capital prices at $t = 1$: $\hat{q}_1 = r\sigma\underline{\beta}\hat{z} > r\gamma\underline{\beta}\hat{z}$,*

2. *A dampened boom at time $t = 0$: $\hat{K}_0 = \underline{\beta}\left(\frac{\eta}{1+(\frac{R-\lambda}{r\lambda})\eta}\right)\hat{z} < \underline{\beta}\gamma\hat{z}$, and $\hat{q}_0 = r\underline{\beta}^2\hat{z} > 0$ (same), and*

3. *A dampened but prolonged bust going forward: $\hat{K}_1 = -\sigma\left(\frac{r\lambda}{R-\lambda}\right)\left(1-\underline{\beta}\sigma\right)\hat{z} > -\gamma(1-\underline{\beta}\gamma)\hat{z}$, and $\hat{K}_s = \sigma^s\left(\hat{K}_{t+s-1} - \hat{z}\left(\frac{\lambda r}{R-\lambda}\right)\right) < 0$ for all $s \geq 1$, and $\hat{q}_{s+1} < 0$ for all $s \geq 1$,*

where $\sigma \equiv \left(\frac{1-\underline{\beta}\lambda}{1-\lambda\underline{\beta}+\lambda(1-\underline{\beta})/\eta}\right) = \frac{1}{1+\frac{\lambda r}{(R-\lambda)\eta}} < 1, \frac{\lambda r}{(R-\lambda)} < 1$, $\sigma > \gamma$, and $\sigma \to \gamma$ as $\lambda \to 1$.

The tighter borrowing constraint has two consequences for dynamics. First, the initial response is dampened because experts are less leveraged, and thus credit markets have less of a role in propagating shocks. The result extends analogously when considering news N periods forward, multiplying the initial capital deviations by $\underline{\beta}^N$ as in the main model. Second, deviations from steady state are more persistent ($\sigma > \gamma$) and so it takes longer to recover from the bust. However, the bust is not so severe.

Proof of Proposition 8. We first log-linearize the budget constraint at $t = s$ when there is no technology shock. In this case, debt is set with perfect foresight and we have

$$\hat{K}_s = \sigma\hat{K}_{s-1}, \tag{46}$$

where $\sigma \equiv \left(\frac{1-\underline{\beta}\lambda}{1-\lambda\underline{\beta}+\lambda(1-\underline{\beta})/\eta}\right) = \frac{1}{1+\frac{\lambda r}{(R-\lambda)\eta}} < 1$. We can define $\eta' \equiv \frac{(R-\lambda)\eta}{r\lambda}$ and then we have $\sigma = \frac{1}{1+\frac{1}{\eta'}} = \frac{\eta'}{1+\eta'}$, analogous to the definition of γ. Note that $\sigma \to \gamma$ as $\lambda \to 1$ and that $\sigma > \gamma$ since $\frac{\lambda r}{(R-\lambda)} < 1$.

In the period with the technology shock we instead would have

$$\begin{aligned}
&\lambda(1-\underline{\beta})\left(\hat{K}_s/\eta + \hat{z}\right) + \hat{K}_s(1-\lambda\underline{\beta}) = \hat{K}_{s-1}(\lambda(1-\underline{\beta}) + 1 - \lambda), \\
&\hat{K}_s(1-\lambda\underline{\beta} + \lambda(1-\underline{\beta})/\eta) = \hat{K}_{s-1}(1-\underline{\beta}\lambda) - \lambda(1-\underline{\beta})\hat{z}, \\
&\hat{K}_s = \hat{K}_{s-1}\left(\frac{1-\underline{\beta}\lambda}{1-\lambda\underline{\beta}+\lambda(1-\underline{\beta})/\eta}\right) - \hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}+\lambda(1-\underline{\beta})/\eta}\right),
\end{aligned}$$

which we can write as

$$\hat{K}_s = \sigma\left(\hat{K}_{s-1} - \hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right) = \sigma\left(\hat{K}_{s-1} - \hat{z}\left(\frac{\lambda r}{R-\lambda}\right)\right). \tag{47}$$

Since $\frac{\lambda r}{R-\lambda} < 1$, it is as if the shock enters in a smaller way compared to the baseline model (i.e., with $\lambda = 1$).

Finally, we log-linearize the budget constraint at $t = 0$. We can write the budget constraint as

$$\hat{K}_0\left(1 - \underline{\beta}\lambda + \lambda(1-\underline{\beta})/\eta\right) = \lambda\hat{q}_0, \tag{48}$$

which we can write as

$$\hat{K}_0 = \sigma\frac{\lambda}{1-\underline{\beta}\lambda}\hat{q}_0, \tag{49}$$

or equivalently,

$$\hat{K}_0\left(1 + \frac{(R-\lambda)\eta}{r\lambda}\right) = \hat{K}_0\left(1+\eta'\right) = \frac{R\eta}{r}\hat{q}_0.$$

With the shock occurring at $t = 1$, for all $t \geq 1$ we have

$$\hat{K}_t = \sigma^t\left(\hat{K}_{t-1} - \hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right) = \sigma^t\left(\hat{K}_{t-1} - \hat{z}\left(\frac{\lambda r}{R-\lambda}\right)\right).$$

From (12) we can write the capital price as

$$\hat{q}_0 = \frac{1-\underline{\beta}}{\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t + \underline{\beta}(1-\underline{\beta})\hat{z},$$

where the $\hat{z}$ term reflects that the user cost at $t = 1$ contains the shock. In order to plug in for $\hat{q}_0$, we execute the following manipulations:

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\sigma^t\left(\hat{K}_0 - \hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right) + \frac{r}{R}\frac{1}{\eta}\hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right) + \underline{\beta}\frac{r}{R}\hat{z},\\
\frac{R}{r}\hat{q}_0 &= \frac{1}{\eta}\left(\frac{1}{1-\underline{\beta}\sigma}\right)\left(\hat{K}_0 - \hat{z}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right) + \hat{z}\left(\underline{\beta} + \frac{1}{\eta}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{z}\left(\frac{1}{1-\underline{\beta}\sigma}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right) - \underline{\beta}\eta - \left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right)\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{z}\underline{\beta}\left(\frac{\sigma}{1-\underline{\beta}\sigma}\left(\frac{\lambda(1-\underline{\beta})}{1-\lambda\underline{\beta}}\right) - \eta\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{z}\underline{\beta}\left(\frac{\sigma}{1-\underline{\beta}\sigma}\left(\frac{r\lambda}{R-\lambda}\right) - \eta\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{z}\left(\frac{r\lambda}{R-\lambda}\right)\underline{\beta}\left(\frac{\sigma}{1-\underline{\beta}\sigma} - \eta\left(\frac{R-\lambda}{r\lambda}\right)\right).
\end{aligned}$$

Note that we can write the budget constraint in equation (49) as

$$\hat{K}_0\left(1+\eta'\right) = \frac{R\eta}{r}\hat{q}_0.$$

Hence we can write

$$\hat{K}_0\left(1+\eta'\right) = \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{z}\left(\frac{r\lambda}{R-\lambda}\right)\underline{\beta}\left(\frac{\sigma}{1-\underline{\beta}\sigma} - \eta\left(\frac{R-\lambda}{r\lambda}\right)\right),$$

which is identical to the result from earlier, with η' replacing η, σ replacing γ, and $\hat{z}$ multiplied by $\left(\frac{r\lambda}{R-\lambda}\right)$. Since $\sigma = \frac{1}{1+1/\eta'}$, we can therefore solve out to get

$$\hat{K}_0 = \underline{\beta}\sigma\hat{z}\left(\frac{r\lambda}{R-\lambda}\right) = \underline{\beta}\left(\frac{\eta}{1+\left(\frac{R-\lambda}{r\lambda}\right)\eta}\right)\hat{z} < \underline{\beta}\gamma\hat{z}, \tag{50}$$

where the final inequality follows because $\frac{R-\lambda}{r\lambda} > 1$ and $\gamma = \frac{\eta}{1+\eta}$. From the budget constraint we have

$$\frac{R\eta}{r}\hat{q}_0 = \hat{K}_0\left(1+\eta'\right) \implies \hat{q}_0 = r\underline{\beta}^2\hat{z}.$$

Plugging $\hat{K}_0\left(1+\eta'\right) = \frac{R\eta}{r}\hat{q}_0$ into the asset price equation $\hat{q}_0 = \frac{r}{R}\hat{u}_0 + \underline{\beta}\hat{q}_1$, we can write the recursion

$$\hat{q}_0 = \frac{1}{1+\eta'}\hat{q}_0 + \underline{\beta}\hat{q}_1 \implies \hat{q}_1 = \frac{\sigma}{\underline{\beta}}\hat{q}_0.$$

Note that we have $\hat{q}_1 = \frac{\sigma}{\underline{\beta}} r \underline{\beta}^2 \hat{z} = r\underline{\beta}\sigma\hat{z}$. Additionally, we have

$$\hat{K}_1 = -\sigma \left(\frac{r\lambda}{R-\lambda} \right) \left(1 - \underline{\beta}\sigma\right) \hat{z} > -\gamma(1 - \underline{\beta}\gamma)\hat{z}, \tag{51}$$

which is closer to zero than we get when $\lambda = 1$. □

C.3 Proof of Proposition 6, Financial Shocks

Proof of Proposition 6. Log-linearizing the budget constraint at $t = 0$, we have

$$q_* K_* (\hat{q}_0 + \hat{K}_0) - \underline{\beta}\lambda q_* K_* (\hat{q}_1 + \hat{K}_0 + \hat{\lambda}_0) = 1 q_* \hat{q}_0 K_*,$$

where the RHS reflects that debt equals $\lambda q_* K_*$ and capital is predetermined. Rearranging and collecting terms we have

$$\begin{aligned}
(\hat{q}_0 + \hat{K}_0) - \underline{\beta}\lambda(\hat{q}_1 + \hat{K}_0 + \hat{\lambda}_0) =& \hat{q}_0, \\
\lambda\hat{q}_0 - \underline{\beta}\lambda\hat{q}_1 + \hat{K}_0(1 - \underline{\beta}\lambda) =& \lambda\hat{q}_0 + \underline{\beta}\lambda\hat{\lambda}_0, \\
\hat{K}_0 \left(1 - \underline{\beta}\lambda + \lambda(1 - \underline{\beta})/\eta\right) =& \lambda\hat{q}_0 + \underline{\beta}\lambda\hat{\lambda}_0,
\end{aligned}$$

which we can write as

$$\hat{K}_0 \left(1 + \eta'\right) = \frac{R\eta}{r}\hat{q}_0 + \frac{\eta}{r}\hat{\lambda}_0. \tag{52}$$

Next, consider the budget constraint at $t = 1$. In this case, debt is set with perfect foresight and we have

$$\begin{aligned}
q_* K_* (\hat{q}_1 + \hat{K}_1) - \underline{\beta}\lambda q_* K_* (\hat{q}_2 + \hat{K}_1) =& a K_* \hat{K}_0 + q_* K_* (\hat{q}_1 + \hat{K}_0) - \lambda q_* K_* (\hat{q}_1 + \hat{K}_0 + \hat{\lambda}), \\
(\hat{q}_1 + \hat{K}_1) - \underline{\beta}\lambda(\hat{q}_2 + \hat{K}_1) =& \lambda(1 - \underline{\beta})\hat{K}_0 + (1 - \lambda)(\hat{q}_1 + \hat{K}_0) - \lambda\hat{\lambda}, \\
\lambda\hat{q}_1 - \underline{\beta}\lambda\hat{q}_2 + \hat{K}_1(1 - \lambda\underline{\beta}) =& \lambda(1 - \underline{\beta})\hat{K}_0 + (1 - \lambda)\hat{K}_0 - \lambda\hat{\lambda}, \\
\lambda(1 - \underline{\beta})\hat{K}_1/\eta + \hat{K}_1(1 - \lambda\underline{\beta}) =& \hat{K}_0(\lambda(1 - \underline{\beta}) + 1 - \lambda) - \lambda\hat{\lambda}, \\
\hat{K}_1(1 - \lambda\underline{\beta} + \lambda(1 - \underline{\beta})/\eta) =& \hat{K}_0(1 - \underline{\beta}\lambda) - \lambda\hat{\lambda},
\end{aligned}$$

which we can write as

$$\hat{K}_1 = \hat{K}_0 \left(\frac{1 - \underline{\beta}\lambda}{1 - \lambda\underline{\beta} + \lambda(1 - \underline{\beta})/\eta} \right) - \frac{\lambda}{1 - \underline{\beta}\lambda} \left(\frac{1 - \underline{\beta}\lambda}{1 - \lambda\underline{\beta} + \lambda(1 - \underline{\beta})/\eta} \right) \hat{\lambda},$$

or equivalently

$$\hat{K}_1 = \sigma\left(\hat{K}_0 - \frac{\lambda}{1-\underline{\beta}\lambda}\hat{\lambda}\right) = \sigma\left(\hat{K}_0 - \frac{R\lambda}{R-\lambda}\hat{\lambda}\right). \tag{53}$$

Then equation (46) holds in every period thereafter, $\hat{K}_t = \sigma\hat{K}_{t-1} = \sigma^t\left(\hat{K}_0 - \frac{\lambda}{1-\underline{\beta}\lambda}\hat{\lambda}\right)$.

From (12) we can write the capital price as $\hat{q}_0 = \frac{r}{R\eta}\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t$. Then we have

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\hat{K}_0 + \frac{r}{R\eta}\sum_{t=1}^{\infty}\underline{\beta}^t\sigma^t\left(\hat{K}_0 - \frac{\lambda}{1-\underline{\beta}\lambda}\hat{\lambda}\right),\\
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_0 - \hat{\lambda}\left(\frac{\underline{\beta}\sigma}{1-\underline{\beta}\sigma}\right)\left(\frac{\lambda}{1-\underline{\beta}\lambda}\right).
\end{aligned}$$

Plugging into the budget constraint we have

$$\begin{aligned}
(1+\eta')\hat{K}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{K}_t - \hat{\lambda}\left(\frac{\underline{\beta}\sigma}{1-\underline{\beta}\sigma}\right)\left(\frac{\lambda}{1-\underline{\beta}\lambda}\right) + \frac{\eta}{r}\hat{\lambda}_0,\\
\hat{K}_0\left((1+\eta')(1-\underline{\beta}\sigma)-1\right) &= \hat{\lambda}\left(\frac{\eta}{r}(1-\underline{\beta}\sigma) - (\underline{\beta}\sigma)\left(\frac{\lambda}{1-\underline{\beta}\lambda}\right)\right),\\
\hat{K}_0\left(\eta'(1-\underline{\beta})\right) &= \hat{\lambda}\left(\frac{\eta}{r}(1-\beta\sigma) - \left(\frac{\underline{\beta}\sigma\lambda}{1-\underline{\beta}\lambda}\right)\right),\\
\hat{K}_0\left(\eta'(1-\underline{\beta})\right) &= \hat{\lambda}\left(\frac{\eta'\lambda}{R-\lambda}(1-\beta\sigma) - \left(\frac{\sigma\lambda}{R-\lambda}\right)\right),\\
\hat{K}_0\left(\eta'(1-\underline{\beta})\right) &= \hat{\lambda}\left(\frac{\eta'\lambda(1-\beta\sigma)-\lambda\sigma}{R-\lambda}\right),\\
\hat{K}_0 &= \hat{\lambda}\left(\frac{\eta'\lambda(1-\beta\sigma)-\lambda\sigma}{\eta'(1-\underline{\beta})(R-\lambda)}\right),\\
\hat{K}_0 &= \lambda\hat{\lambda}\left(\frac{1-\beta\sigma-\sigma/\eta'}{(1-\underline{\beta})(R-\lambda)}\right),\\
\hat{K}_0 &= \lambda\hat{\lambda}\left(\frac{R-\sigma-R\sigma/\eta'}{r(R-\lambda)}\right),\\
\hat{K}_0 &= \hat{\lambda}\left(\frac{\sigma\lambda}{R-\lambda}\right).
\end{aligned}$$

Note that this implies that

$$\hat{K}_1 = \sigma\left(\frac{\sigma\lambda}{R-\lambda}\hat{\lambda} - \frac{R\lambda}{R-\lambda}\hat{\lambda}\right) = \frac{\sigma\lambda}{R-\lambda}(\sigma-R)\hat{\lambda} < 0,$$

where the inequality follows because $\sigma < 1 < R$. Thus, we see a boom-bust in capital and thus in output.

Plugging into the capital equation above we have

$$
\begin{aligned}
\frac{R\eta}{r}\hat{q}_0 &= \left(\frac{1}{1-\underline{\beta}\sigma}\right)\hat{\lambda}\left(\frac{\sigma\lambda}{R-\lambda}\right) - \hat{\lambda}\left(\frac{\underline{\beta}\sigma}{1-\underline{\beta}\sigma}\right)\left(\frac{\lambda}{1-\underline{\beta}\lambda}\right), \\
&= \hat{\lambda}\left(\frac{1}{1-\underline{\beta}\sigma}\right)\left(\frac{\sigma\lambda}{R-\lambda} - \underline{\beta}\sigma\left(\frac{R\lambda}{R-\lambda}\right)\right), \\
&= \lambda\hat{\lambda}\left(\frac{1}{1-\underline{\beta}\sigma}\right)\left(\frac{\sigma}{R-\lambda} - \frac{\sigma}{R-\lambda}\right) = 0, \\
&\implies \hat{q}_0 = 0.
\end{aligned}
$$

Financial Shocks and News Now suppose that the financial shock occurs in period $t = N$ and agents learn of the shock at $t = 0$. First, linearizing the budget constraints with news implies

$$
\hat{K}_t = \begin{cases} \sigma^t \hat{K}_0 & 0 < t < N \\ \sigma^t \hat{K}_0 + \frac{\sigma\lambda\hat{z}}{R-\lambda} & t = N \\ \sigma^t \hat{K}_0 + \sigma^{t-N}\frac{\lambda\hat{z}}{R-\lambda}(\sigma - R) & t > N \end{cases}
$$

Therefore, we are able to calculate $\hat{q}_0$

$$
\begin{aligned}
\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty} R^{-t}\hat{K}_t, \\
\hat{q}_0 &= \frac{r}{R\eta}\left(\sum_{t=0}^{\infty} R^{-t}\sigma^t\hat{K}_0 + \sum_{t=N}^{\infty} R^{-t}\sigma^{t-N+1}\frac{\lambda\hat{z}}{R-\lambda} - \sum_{t=N+1}^{\infty} R^{-t}\sigma^{t-N}\frac{\lambda R\hat{z}}{R-\lambda}\right), \\
\hat{q}_0 &= \frac{r}{R\eta}\sum_{t=0}^{\infty} R^{-t}\sigma^t\hat{K}_0.
\end{aligned}
$$

Plugging in the budget constraint (49) at $t = 0$ we have

$$
\begin{aligned}
\hat{K}_0\left(\frac{\lambda}{\eta} + \frac{R-\lambda}{r}\right)\frac{r}{\lambda R} &= \frac{r}{R\eta}\sum_{t=0}^{\infty} R^{-t}\sigma^t\hat{K}_0, \\
\hat{K}_0\left(1 + \frac{\eta(R-\lambda)}{\lambda r}\right) &= \frac{\hat{K}_0}{1-\frac{\sigma}{R}} \implies \hat{K}_0 = 0.
\end{aligned}
$$

For a shock occurring at time $t = N$, then for $t < N$, $\hat{K}_t = 0$, and

$$
\hat{K}_N = \frac{\sigma\lambda\hat{z}}{R-\lambda} > 0, \quad \hat{K}_{N+1} = \frac{\sigma\lambda\hat{z}}{R-\lambda}(\sigma - R) < 0, \quad \hat{K}_{N+t} = \frac{\sigma^t\lambda\hat{z}}{R-\lambda}(\sigma - R) < 0. \tag{54}
$$

For $t \leq N$, $\hat{q}_t = 0$, and for $t > N$, $\hat{q}_t < 0$. $\square$

C.4 State-contingent Contracts

The baseline model is deterministic and so there is no need for state-contingent borrowing (equity). Indeed, there is no distinction between debt and equity finance in the baseline model and the model features a one-time, unexpected shock. In principle, when shocks are expected ex-ante, agents may condition payments on aggregate variables. We consider how such behavior would affect our main results.

We now suppose that experts can issue contingent contracts so that payments depend on the aggregate state. As is common in the literature, we continue to suppose that market incompleteness limits the fraction of claims that can be equity. Let a fraction $\phi \in [0,1]$ be debt (non-contingent) claims and let $1-\phi$ be contingent.

The contingent repayment has consequences at the time of the shock when prices adjust. As noted in the main text, debt is predetermined, and so the level of repayment at $t=0$ is RB_*. Suppose instead that the level of repayment adjusts to $q_t k_*$. Then the wealth gain for experts from the increase in capital prices will be a fraction ϕ of what it was before, because a fraction $1-\phi$ of their borrowing will be repaid equal to the new value of capital. After the shock, there is no more uncertainty.

The experts' budget constraints are now modified to

$$\hat{K}_0 + \hat{u}_0 = \phi \frac{R}{r} \hat{q}_0, \tag{55}$$

$$\hat{K}_t + \hat{u}_t = \hat{K}_{t-1}, \quad \forall t \geq 1. \tag{56}$$

The constraint at $t=0$ is modified to reflect that experts now enjoy only a fraction ϕ of the wealth gain compared to the benchmark model. The constraints for $t \geq 1$ are unchanged. We can therefore write

$$\hat{K}_0 + \hat{u}_0 = \phi \sum_{t=0}^{\infty} R^{-t} \hat{u}_t, \implies \hat{K}_0 = \phi \sum_{t=1}^{\infty} R^{-t} \hat{u}_t.$$

Several results are immediately clear.

First, for $\phi < 1$, Proposition 1 no longer holds. The additional resources at $t=0$ equal only a fraction of the present value of future user costs. Thus,

$$\sum_{t=0}^{\infty} R^{-t} \hat{K}_t < 0,$$

which means that, with equity issuance, the present value of the bust would exceed the present value of the boom.

This also means that the boom would be smaller and the bust would be more pronounced. Recall that $\hat{K}_1 = \gamma(\hat{K}_0 - \hat{z})$, and thus a smaller $\hat{K}_0$ implies a deeper bust.

As a stark example, consider when $\phi = 0$ so that all borrowing would be state-contingent. In that case, experts would enjoy no wealth gains when their capital values increase. Then the budget constraint would be

$$\hat{K}_0 + \hat{u}_0 = 0,$$

and since $\hat{u}_0 = \hat{K}_0/\eta$ this requires $\hat{K}_0 = 0$ in equilibrium. However, $\hat{K}_1 = \gamma(\hat{K}_0 - \hat{z})$ implies a bust at the time of the shock, $\hat{K}_1 = -\gamma\hat{z}$, because we have $\hat{K}_1 + \hat{u}_1 = 0$ and $\hat{u}_1 = \hat{K}_1/\eta + \hat{z}$. Thus, we would have no endogenous boom at $t = 0$, an exogenous boom due to the technology shock in the innovative sector, and a bust in periods following due to the capital reallocation. The present value of the bust would therefore be

$$-\sum_{t=1}^{\infty} R^{-t}\gamma^t \hat{z} = -\frac{1}{1-\underline{\beta}\gamma}\hat{z}.$$

D Model with Concavity and Changing Interest Rates

In our baseline model we consider linear utility so that the interest rate is constant. In this section we show that as long as a positive increase in demand for capital raises the capital price, meaning that the interest rate doesn't change by too much, then considering endogenous changes in interest rates is a quantitative question, not a qualitative concern. Our qualitative results go through as long as asset prices rise in response to demand.

We first consider a setup with concavity following Cordoba and Ripoll (2004). Let agents have concave utility and let the experts production function be given by F which is concave. Then in steady state we have

$$\begin{aligned}
&R = 1/\underline{\beta}, \\
&\beta F'(K_t) = \underline{\beta} G'(\bar{K} - K_t), \\
&u_* = \underline{\beta} G'(\bar{K} - K_t), \\
&q_* = \frac{u_*}{1-\underline{\beta}}.
\end{aligned}$$

In addition, consumption is pinned down by the usual Euler equations, with potential inequality when the collateral constraint binds. Cordoba and Ripoll (2004) provide parameter restrictions to guarantee determinacy and saddle-path stability.

Crucially, in this setting the interest rate changes in response to shocks. We can derive our results with this property in mind and then specify conditions on the behavior of the interest rate in equilibrium that maintains our results.

Proposition 9. *Suppose that when experts increase their demand for capital, the user cost and the asset price increase. Then in a model with endogenous interest rates, a news shock next period leads to boom-bust dynamics in output and asset prices, as in the baseline model.*

Let R_t be the real interest rate and let R be the steady-state rate, which is still pinned down by preferences in steady state because consumption is constant and so risk aversion doesn't change the steady-state rate. From the asset-pricing equation, we have

$$\hat{q}_t = \frac{r}{R}\hat{u}_t + \frac{1}{R}\left(\hat{q}_{t+1} - \hat{R}_t\right) = \frac{r}{R}\hat{u}_t - \frac{1}{R}\hat{R}_t + \frac{1}{R}\hat{q}_{t+1}.$$

Note that when $\hat{K}_t > 0$ then $\hat{u}_t > 0$. Now suppose a demand for capital $\hat{K}_t > 0$ raises the interest rate (i.e., $\hat{R}_t = \varepsilon \hat{K}_t$ for some ε). As long as the interest rate does not increase by too much, then the asset price will rise at t.

The user cost is more complicated now because $G' = u_t R_t$, and so in the absence of the shock we have

$$\frac{1}{\eta}\hat{K}_t = \hat{u}_t + \hat{R}_t \implies \hat{u}_t = \left(\frac{1}{\eta} - \varepsilon\right)\hat{K}_t = \left(\frac{1 - \varepsilon\eta}{\eta}\right)\hat{K}_t.$$

Now consider what happens when non-experts hold less capital and experts hold more. Because experts hold more capital, by assumption the interest rate increases. The higher interest rate makes future output less valuable, and so the user cost increases by less than it would otherwise. Put differently, non-experts' marginal product has gone way up, but that does not increase the user cost by as much as before because the higher interest rate discounts future output. As long as ε is not so large, an increase in demand for capital by experts will increase the user cost. This means that the evolution of capital converges at a rate of $\varsigma = \frac{1}{1+\frac{1-\varepsilon\eta}{\eta}}$, not $\gamma = \frac{1}{1+\frac{1}{\eta}}$ with $\varsigma > \gamma$.

In this case, when $\hat{u}_t = \frac{1-\varepsilon\eta}{\eta}\hat{K}_t$, we can write the asset price equation as

$$\begin{aligned}
\hat{q}_t &= \frac{r}{R}\frac{1-\varepsilon\eta}{\eta}\hat{K}_t - \frac{1}{R}\varepsilon\hat{K}_t + \frac{1}{R}\hat{q}_{t+1},\\
&= \left(\frac{r}{R}\frac{1-\varepsilon\eta}{\eta} - \frac{1}{R}\varepsilon\right)\hat{K}_t + \frac{1}{R}\hat{q}_{t+1},\\
&= \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\hat{K}_t + \frac{1}{R}\hat{q}_{t+1},
\end{aligned}$$

and using $\hat{u}_1 = \frac{1-\varepsilon\eta}{\eta}\hat{K}_1 + \hat{z}$, we have

$$\begin{aligned}
\hat{q}_1 &= \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta}\hat{K}_1 + \hat{z}\right) - \frac{1}{R}\varepsilon\hat{K}_t + \frac{1}{R}\hat{q}_2,\\
&= \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\hat{K}_1 + \frac{r}{R}\hat{z} + \frac{1}{R}\hat{q}_2.
\end{aligned}$$

Note that this means we can write the asset price from period $t > 1$ forward as

$$\hat{q}_t = \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t,$$

and

$$\hat{q}_0 = \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\sum_{t=0}^{\infty}\underline{\beta}^t\hat{K}_t + \underline{\beta}\frac{r}{R}\hat{z},$$

where $\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)$ replaces $\frac{1}{\eta}$ in the original formula. Note that we can write

$$\begin{aligned}
\hat{q}_0 &= \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\sum_{t=0}^{\infty}\underline{\beta}^t\varsigma^t\left(\hat{K}_0 - \hat{z}\right) + \frac{r}{R}\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\hat{z} + \underline{\beta}\frac{r}{R}\hat{z},\\
\frac{R}{r}\hat{q}_0 &= \left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\left(\hat{K}_0 - \hat{z}\right) + \hat{z}\left(\underline{\beta} + \frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right),\\
\frac{R}{r}\hat{q}_0 &= \left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\hat{K}_0 - \hat{z}\left(\left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)\frac{1}{1-\underline{\beta}\varsigma} - \underline{\beta} - \frac{1-\varepsilon\eta}{\eta} + \frac{\varepsilon}{r}\right).
\end{aligned}$$

Let $C = \left(\frac{1-\varepsilon\eta}{\eta} - \frac{\varepsilon}{r}\right)$. Then we have

$$\begin{aligned}
\frac{R}{r}\hat{q}_0 &= C\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\hat{K}_0 - \hat{z}\left(C\frac{1}{1-\underline{\beta}\varsigma} - \underline{\beta} - C\right), \\
&= C\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\hat{K}_0 - \hat{z}\left(\frac{\underline{\beta}\varsigma C}{1-\underline{\beta}\varsigma} - \underline{\beta}\right), \\
&= C\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\hat{K}_0 + \hat{z}\underline{\beta}\left(1 - \frac{\varsigma C}{1-\underline{\beta}\varsigma}\right).
\end{aligned}$$

The budget constraint at $t = 0$ is

$$\hat{K}_0 = \varsigma\frac{R}{r}\hat{q}_0,$$

and hence

$$\begin{aligned}
\hat{K}_0 =& \varsigma C\left(\frac{1}{1-\underline{\beta}\varsigma}\right)\hat{K}_0 + \hat{z}\underline{\beta}\varsigma\left(1 - \frac{\varsigma C}{1-\underline{\beta}\varsigma}\right), \\
\hat{K}_0\left(1 - \frac{\varsigma C}{1-\underline{\beta}\varsigma}\right) =& \hat{z}\underline{\beta}\varsigma\left(1 - \frac{\varsigma C}{1-\underline{\beta}\varsigma}\right), \\
\hat{K}_0 =& \hat{z}\underline{\beta}\varsigma,
\end{aligned}$$

which is the same value we get in equilibrium when the interest rate is constant with ς replacing γ. Note also from the budget constraint that the asset price is therefore

$$\hat{q}_0 = r\underline{\beta}^2\hat{z},$$

which is the same as we get in the baseline model.

Thus, since the dynamics of $\hat{K}_t$ are the same with ς replacing γ, we get all the same dynamics going forward. What is important is that $\varsigma \leq 1$ so that the boom is followed by a bust, and this condition holds so long as the user cost increases at $t = 0$.

The assumption that $\frac{1}{\eta} - \frac{\varepsilon}{r} > 0$ is only required so that asset prices behave as desired in the following periods. On the assumption that any demand for capital would increase asset prices—including the interest-rate term then this is merely a quantitative change to the equation. We still get $\hat{K}_0 > 0$ and since the dynamics of $\hat{K}_t$ are the same (the expert budget constraint does not depend independently on R_t once the user cost is defined), we get all the same dynamics going forward. Endogenous changes in interest rates don't affect the initial equilibrium (up to the change in the elasticity) because the present value of changes in capital is zero, and thus the present value of changes in the interest rate is zero.