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Abstract

We study how introducing a central bank digital currency (CBDC) would affect the stability of the banking system. We present a model that captures a concern commonly raised in policy discussions: the option to hold CBDC can increase the incentive for depositors to run on weak banks. Our model highlights two countervailing effects. First, banks do less maturity transformation when depositors have access to CBDC, which leaves them less exposed to runs. Second, monitoring the flow of funds into CBDC allows policymakers to identify and resolve weak banks sooner, which also decreases depositors' incentive to run. Our results suggest that a well-designed CBDC may decrease rather than increase financial fragility.

Keywords: CBDC, digital currency, financial stability, bank runs

JEL Codes: E43, E58, G21

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1 Introduction

There is growing interest in the possibility of central banks issuing an electronic form of currency that would be widely available to firms and households. The potential benefits and risks of such a central bank digital currency (CBDC) have been discussed by academics, market participants, and policymakers, and have been detailed in reports issued by central banks and other agencies. One of the primary risks raised in these discussions is that a CBDC might increase the fragility of the financial system. For example, a recent report by the European Central Bank (2020, p. 17) states "in crisis situations, when savers have less confidence in the whole banking sector, liquid assets might be shifted very rapidly from commercial bank deposits to the digital euro." Similarly, a report from the Federal Reserve (2022, p. 17) worries that "CBDC could make runs on financial firms more likely or more severe."[1] The logic of these arguments seems compelling: having access to a new safe, convenient form of money can make it more attractive for short-term creditors to pull funds out of banks and other financial institutions in periods of financial stress. Some observers, however, are optimistic that this concern can be mitigated through appropriate design choices.[2] Despite its clear importance, there has been relatively little formal analysis of this issue, and the effect of a CBDC on financial stability remains an open question.

We highlight two points that have been largely overlooked in this debate. First, the availability of CBDC will change the financial arrangements private agents make in normal times. We construct a model in which banks perform maturity transformation to insure depositors against idiosyncratic liquidity risk. The resulting maturity mismatch can create financial fragility in the form of runs by depositors following an adverse shock. We show that introducing a CBDC decreases the amount of maturity transformation performed by banks in the constrained-efficient allocation. Intuitively, having access to CBDC makes experiencing a liquidity shock less costly for depositors in our model, which leads banks to provide less insurance against this risk. When banks perform less maturity transformation, they are less exposed to the possibility of a run. In this way, the adjustments in private financial arrangements in response to a CBDC may tend to stabilize rather than destabilize the financial system.

Our second point is that observing the flow of funds into a CBDC can give policymakers more information about the state of the financial system and, in particular, about depositors' confidence in their banks. In periods of financial stress, banks and other financial

[1] This same concern is expressed in reports from the Bank for International Settlements (2018), Bank of England (2020), Danmarks Nationalbank (2017), Norges Bank (2018), and Sveriges Riksbank (2018).

[2] See, for example, Bindseil (2020), Kumhof and Noone (2021) and Bordo and Levin (2019).

intermediaries have private information about both the quality of their assets and their liquidity position, that is, the willingness of their depositors and other short-term creditors to continue to provide funding. A bank in a weak position will often have an incentive to hide this fact from regulators, at least for a while, to avoid triggering supervisory actions. The fact that this information remains hidden delays policymakers' response to an incipient financial crisis, making the crisis more severe. We show how observing the flow of funds into a CBDC can allow policymakers to infer when a run by a bank's depositors is underway more quickly and to place troubled banks into resolution sooner. When depositors anticipate this faster policy reaction, their incentive to join the run decreases. In other words, by allowing a quicker policy reaction to a crisis, this *information effect* is another channel through which CBDC may tend to improve rather than worsen financial stability.

To understand how this information effect operates, suppose a bank's depositors learn the quality of its assets has declined. If they wish to withdraw funding from the bank, they can currently shift their funds to another bank or other liquid assets (for example, government bonds). Regulators might not immediately observe these withdrawals and, even if they do, might have difficultly distinguishing them from the regular inflows and outflows generated by a bank's client transactions. Once a CBDC is introduced, policymakers have a new source of information: the flow of funds into the digital currency. The design features of this currency, including ease of access and any fees or interest paid on balances, will determine how attractive it is to potential users both in normal times and during a run. Policymakers can then evaluate whether or not the inflows into a CBDC are consistent with a bank's depositors maintaining confidence. We show that policymakers can infer when a run is underway more quickly by observing inflows into the CBDC than by simply observing withdrawals from the bank. In effect, observing inflows into the CBDC allows policymakers to make inferences about *why* depositors are withdrawing, which provides additional information beyond *whether* they are withdrawing.

We base our analysis on a model in the tradition of Diamond and Dybvig (1983). As is standard in this literature, private agents have an incentive to pool their resources in a bank to insure against idiosyncratic liquidity risk.[3] In our setting, this risk arises because some agents will be relocated before the investment technology has matured, as in Champ et al. (1996). Relocated agents must withdraw from their bank, and they earn an idiosyncratic return on goods they carry to their new location. Because depositors' relocation status is private information, banks give depositors the choice of when to withdraw. We study

[3] The *bank* in this type of model represents a variety of financial intermediation arrangements in practice where creditors have short-term claims against an intermediary holding longer-term assets. See Yorulmazer (2014) for a discussion of several such arrangements outside of the commercial banking system. Similarly, the depositors in our model represent all short-term creditors of such arrangements.

a situation in which some banks have suffered a loss in the value of their assets, but other banks have not. Depositors privately observe the value of their bank's assets at the beginning of the early period, before making their withdrawal decisions. Our focus is on whether the banking system is fragile in the sense that, in equilibrium, depositors with no immediate liquidity need withdraw from weak banks.

There is a government that can place weak banks into a resolution process and can bail out the depositors in those banks at the cost of providing less of a public good. However, the government only observes the value of a bank's assets at the end of the early period. Depending on the government's fiscal capacity, the resolution process may impose some losses (or "haircuts") on the remaining depositors. The government cannot commit to the details of a bailout policy in advance, which implies it will end up providing larger bailouts to banks in worse financial condition, as in Keister (2016). Anticipating this reaction, weak banks have no incentive to voluntarily restrict withdrawals or impose losses on their depositors before being placed in resolution. Instead, weak banks will continue operating as normal for as long as possible to avoid revealing their status to the government, as in Keister and Mitkov (2021). Depositors in a weak bank may be incentivized to withdraw during this period if they anticipate haircuts will be imposed on the remaining depositors during the resolution process. This incentive is the source of financial fragility in our model.

We introduce a CBDC into this environment as an alternative way for depositors to transfer funds across locations and periods. Because relocated depositors earn idiosyncratic returns on resources they hold directly, some will choose to use the CBDC and others will not. The fraction of relocated agents who use CBDC in normal times will depend on its design characteristics, which determine its effective return. In addition to serving this productive role, the CBDC can also be used by non-relocated depositors who choose to run on their bank. We first abstract from the information effect and show that introducing CBDC in this framework has an ambiguous impact on financial fragility. The effect highlighted in policy discussions is present in our model: because the CBDC offers a better way to hold resources outside the banking system, it increases the incentive for depositors in a weak bank to run. At the same time, however, the fact that depositors will have the option to use CBDC if they are relocated implies they have a lower demand for liquidity insurance. The banking contract optimally adjusts, therefore, to offer smaller payments for depositors who withdraw early and larger payments for depositors who wait. These changes decrease the incentive for depositors in a weak bank to run and tend to become stronger as the CBDC is designed to serve payment needs better. We use examples to illustrate how parameter values determine which of these two effects dominates, that is, whether introducing CBDC increases or decreases financial fragility when we abstract from the information effect.

We then turn our focus to CBDC as a new source of information for policymakers. One important determinant of depositors' incentive to run is how quickly weak banks are placed in resolution. Withdrawals made before resolution deplete a bank's resources and lead to larger haircuts for the remaining depositors. The later resolution occurs, therefore, the stronger the incentive becomes for depositors in a weak bank to run. The latest a weak bank will be placed in resolution is at the end of the early period, when the government observes the value of all banks' assets. However, resolution may occur earlier if the government can infer which banks are weak by observing two flows in real time: the withdrawals from each bank and the flow of funds from each bank into the CBDC. The government can infer a run is underway if the number of withdrawals from a bank goes above the normal level, in which case the bank in question must be weak and can be placed into resolution. With the introduction of CBDC, the government can also infer a bank is weak if the flows from the bank into CBDC go above the normal level. We show that this latter process is faster: policymakers can identify a run more quickly by monitoring flows into CBDC than by monitoring withdrawals. Weak banks can then be resolved sooner, which leads to smaller haircuts for the remaining depositors and decreases the incentive to run. In this way, the information effect associated with CBDC tends to decrease financial fragility.

This result can also be understood by thinking about the strategic interaction in depositors' withdrawal decisions. When there is no CBDC, these decisions are strategic complements for the standard reasons: early withdrawals by other depositors lead to larger haircuts for the remaining depositors, which increases the incentive for an individual depositor to withdraw early. Introducing a CBDC weakens this complementarity because early withdrawals also generate information that leads to a faster policy response, and this faster response tends to decrease the haircuts imposed on the remaining depositors. We show that, in some cases, the information effect is strong enough to dominate, meaning the net effect of early withdrawals by other depositors is to decrease the incentive for an individual depositor to withdraw early. In other words, introducing a CBDC can make depositors' withdrawal decisions *strategic substitutes* and thereby eliminate the multiplicity of equilibrium that typically appears in this class of models. In these cases, introducing a CBDC clearly improves stability in the banking system.

Related literature. A growing literature is developing formal economic models that can be used to study how the introduction of CBDC may affect banks, the broader financial system, and macroeconomic outcomes. Most of this literature focuses on the impact of CBDC in normal (non-crisis) periods; notable contributions include Barrdear and Kumhof

(2022), Chiu et al. (2021), Keister and Sanches (2022), and Williamson (2022b).[4] However, while the financial stability implications of CBDC have been discussed extensively in policy circles, fewer models focusing on this issue have been developed to date.

Two recent papers share our focus on how the introduction of CBDC would change the fragility of the banking system. Williamson (2022a) studies a model in which currency and bank deposits are used in decentralized exchange, in the tradition of Lagos and Wright (2005). Some banks experience a loss that will make deposits in those banks worthless, but depositors do not observe whether their own bank is weak and a run, if it occurs, will affect all banks. CBDC is more attractive than physical currency in this environment because it can be used in a more situations. This fact makes withdrawing from the bank more attractive when CBDC is introduced, increasing fragility. Williamson (2022a) emphasizes, however, that this fact also makes a banking panic less costly because agents can trade using CBDC even if bank deposits no longer circulate. As a result, introducing a CBDC can raise welfare even if fragility increases.

Kim and Kwon (2022) use a version of the model in Champ et al. (1996) to study how the introduction of CBDC would affect fragility through its impact on bank's asset portfolios. In their model, the introduction of CBDC leads to a decrease in bank deposits and raises the equilibrium real interest rate. This higher interest rate leads banks to allocate a smaller fraction of their portfolio to liquid assets and a larger fraction to loans. This portfolio shift, in turn, increases the probability that banks will have insufficient liquid assets to cover the fundamental demand for early withdrawals, an event that Champ et al. (1996) liken to historical banking panics. Kim and Kwon (2022) also show that this result can be reversed if the central bank deposits the funds raised by issuing CBDC into the banking system, as suggested by Brunnermeier and Niepelt (2019).

Our approach differs from these papers in several respects, two of which deserve emphasis. First, our model focuses on how changes in the *liabilities* side of banks' balance sheets affect their susceptibility to depositor runs. We identify a channel through which the introduction of CBDC leads banks to do less maturity transformation, which at least partially offsets the incentive for depositors to run into CBDC. Second, policymakers in our model endogenously *respond* to a banking crisis, which allows us to study how CBDC affects the timing of this response. The information effect we identify, which we believe is novel to both the formal literature and the policy debate, is another channel through which CBDC may help stabilize the banking system.

[4] See also Agur et al. (2022), Andolfatto (2021), Anhert et al. (2022), Davoodalhosseini (2022), Dong and Xiao (2021), Fernández-Villaverde et al. (2021), Monnet et al. (2021), Piazzesi and Schneider (2022) and Schilling et al. (2020).

Outline. In the next section, we present our baseline environment and analyze equilibrium when there is no CBDC. In Section 3, we introduce CBDC but assume policymakers make no inferences based on the flow of funds into CBDC, that is, we abstract from the information effect. We show that, in this case, introducing a CBDC may either increase or decrease financial fragility, depending on which of two competing effects dominates. We then introduce the information effect in Section 4 and show how this effect always tends to reduce fragility. We also highlight a tradeoff polcymakers face in choosing the design features that determine how attractive a CBDC is to users. Finally, in Section 5, we offer some concluding remarks.

2 A baseline model with no CBDC

Our baseline model is a version of the classic framework of Diamond and Dybvig (1983) that combines elements from Champ et al. (1996) and Keister and Mitkov (2021), among others. In this section, we present this baseline model and provide an analysis of equilibrium when there is no central bank digital currency.

2.1 The environment

There are two time periods, indexed by $t \in \{1, 2\}$, and a single private consumption good in each period. There is also a public good that can be produced at $t = 1$.

Depositors and banks. There is a continuum of depositors, indexed by $i \in [0, 1]$, in each of a continuum of locations, indexed by $j \in [0, 1]$. A depositor is initially endowed with one unit of the consumption good and desires private consumption only at $t = 2$. Preferences are given by

$$u(c_i^j) + v(g) = \frac{\left(c_i^j\right)^{1-\gamma}}{1-\gamma} + \delta\, \frac{g^{1-\gamma}}{1-\gamma}, \tag{1}$$

where c_i^j denotes private consumption of depositor i in location j and g denotes the level of the public good, which is common across locations. The parameter δ measures the relative importance of public consumption in depositors' preferences, and the coefficient of relative risk aversion satisfies $\gamma > 1$. Before period $t = 1$, depositors in each location pool their endowments to form a *bank*. The banking technology allows each depositor to contact their bank and withdraw goods in one of the two periods. Goods that are not withdrawn from the bank at $t = 1$ earn a gross return $R > 1$ between periods.

At the beginning of $t = 1$, a fraction $n > 0$ of banks lose a fraction $\sigma > 0$ of their assets, with each bank being equally likely to experience this loss. The parameters n and σ are common knowledge, but whether the bank in a particular location is "sound" or "weak" is observed only by the depositors in that location. Next, a fraction π of depositors in each location learn that they will be relocated to another location at the end of the period. Depositors cannot contact their bank after relocating and, therefore, must withdraw at $t = 1$. Relocating depositors earn an idiosyncratic return ρ_i on goods they carry with them to their new location between $t = 1$ and $t = 2$. This return is distributed according to a continuous cumulative distribution function F on the interval $\left[\underline{\rho}, \bar{\rho}\right]$, with $\bar{\rho} \leq R$. Non-relocating depositors earn a return of ρ^N between periods on any goods they withdraw from their bank at $t = 1$.

A depositor's risk of being relocated plays a similar role in our setting to the liquidity-preference shocks in Diamond and Dybvig (1983). Instead of withdrawing goods to consume immediately, however, relocated depositors withdraw from their bank to consume in the next period, as in Champ et al. (1996). We interpret the idiosyncratic return ρ_i as measuring how well existing payment methods serve the needs of each depositor. When we introduce CBDC in the next section, this approach will imply the CBDC is attractive to some withdrawing depositors, but not to others.

Both a depositor's relocation status and her idiosyncratic return on storage ρ_i are private information. After the relocation shocks are realized, each depositor chooses whether to withdraw from their bank at $t = 1$ or $t = 2$. Depositors withdrawing at $t = 1$ then arrive at their bank one at a time and must be served as they withdraw, as in Wallace (1988). At $t = 2$, the remaining resources in a bank are paid out to the bank's remaining depositors.

Government. The government plays two roles in our baseline model. First, it is endowed with $\tau > 0$ units of the private good at $t = 1$ and has a technology for converting these units one-for-one into the public good. We refer to τ as the government's *fiscal capacity*. Resources that are not converted into the public good can be transferred to weak banks. The government will choose these *bailout payments* without commitment, acting to maximize the sum of all depositors' utilities while taking past actions as given.

The government can also place weak banks into a resolution process. In this process, the government dictates how a bank's remaining resources, including any bailout payment, are allocated among its remaining depositors. However, the government observes bank-specific states with a delay; it knows at the beginning of $t = 1$ that some banks have experienced losses, but does not initially know which ones. We assume the government is able to directly observe which banks are weak at the end of period 1. However, it may be able to infer that a bank is weak more quickly by observing withdrawal behavior. This inference process varies across policy regimes and is discussed in detail below.

Once a bank is either observed or inferred to be weak, it is placed in resolution. If a run is underway and some non-movers have withdrawn early, we assume the run stops once the bank is in resolution. The government lacks commitment in the resolution process and will choose the allocation of the remaining resources to maximize the *ex post* sum of depositors' utilities. Our assumption that the government directly dictates this allocation simplifies the notation and exposition of the model, but is not necessary. In practice, the same allocation might be implemented in a variety of ways, including allowing banks to choose the allocation privately, perhaps as part of a living will. Because any bailout payments have already been made at this point, incentives are no longer distorted and private and public incentives are aligned. Any mechanism that allocates the remaining resources efficiently within a weak bank will lead to the same outcome as our approach.

2.2 The banking contract

A sound bank has one unit of the consumption good per depositor at $t = 1$. Suppose for the moment that only the fraction π of depositors who are being relocated withdraw at $t = 1$. Because the idiosyncratic storage returns of these depositors are private information, incentive compatibility requires that the bank in location j give a common amount x_1^j to each of them. Efficiency requires that the bank also give a common amount x_2^j to each of the fraction $1 - \pi$ of non-movers who withdraw at $t = 2$. The constrained efficient allocation of the resources in this bank then solves

$$\max_{\{x_1^j, x_2^j\}} \quad \pi \int_{\underline{\rho}}^{\bar{\rho}} u\left(\rho_i x_1^j\right) dF\left(\rho_i\right) + (1-\pi)\, u\left(x_2^j\right) \tag{2}$$

subject to

$$\pi x_1^j + (1-\pi) \frac{x_2^j}{R} \leq 1. \tag{3}$$

Let (x_1^*, x_2^*) denote the solution to this problem, which modifies the standard Diamond-Dybvig allocation problem to include the idiosyncratic return ρ_i on storage. The first-order condition

$$\int_{\underline{\rho}}^{\bar{\rho}} \rho_i u'\left(\rho_i x_1^*\right) dF\left(\rho_i\right) = Ru'\left(x_2^*\right) \tag{4}$$

characterizes this solution, which satisfies $1 < x_1^* < x_2^* < R$. We assume the return non-movers earn on resources held outside the banking system, ρ^N, is small enough that this allocation is strictly incentive compatible, that is,

$$\rho^N x_1^* < x_2^* \qquad \text{or} \qquad \rho^N < \frac{x_2^*}{x_1^*} \equiv \bar{\rho}^N. \tag{5}$$

We restrict the government's fiscal capacity to be small enough that the marginal value of public consumption is weakly higher than marginal value of private consumption in this constrained-efficient allocation,

$$v'(\tau) \geq \int_{\underline{\rho}}^{\bar{\rho}} \rho_i u'\left(\rho_i x_1^*\right) dF\left(\rho_i\right). \tag{6}$$

This condition ensures the government will not want to provide bailouts to sound banks. Let $\bar{\tau}$ denote the upper bound on fiscal capacity, that is, the value of τ for which equation (6) holds with equality.

We assume all banks offer their depositors a contract based on this constrained-efficient allocation. Because relocation status is private information, this contract gives depositors the option to withdraw at either $t = 1$ or $t = 2$. Depositors who withdraw at $t = 1$ are each given x_1^* unless the bank has been placed in resolution. We assume throughout the analysis that depositors do not run on sound banks, which implies all movers will receive x_1^* at $t = 1$ and all non-movers will receive x_2^* at $t = 2$. In other words, this deposit contract implements the constrained efficient allocation and, hence, is an optimal arrangement for depositors in a sound bank. We describe below how the anticipation of being bailed out makes this arrangement optimal for depositors in a weak bank as well, since choosing a different payment schedule would immediately reveal the bank's status to regulators.

While a weak bank initially follows this deposit contract, it will eventually be placed in resolution and the government will determine the payments made to its remaining depositors. While the banking contract is summarized by the promised payments (x_1^*, x_2^*), therefore, the payout a depositor receives from a weak bank will depend on the resolution process as well as on the withdrawal decisions of other depositors and her position in the withdrawal order. In the next subsection, we derive this payout and its implications for withdrawal behavior.

2.3 Resolution and withdrawal decisions

When a weak bank is placed in resolution, the government determines how its remaining resources are allocated. This allocation depends on how many depositors have already withdrawn and on how many of the remaining depositors are movers and still need to withdraw at $t = 1$. We begin by presenting a general specification of the resolution problem that applies to environments both without and with CBDC. We then specialize to the case without a CBDC and analyze the equilibrium withdrawal behavior of depositors. We focus throughout on symmetric equilibria, where the withdrawal behavior of depositors is the same in all weak banks.

Bailouts and resolution. Let θ denote the fraction of depositors who withdraw before the government is able to either observe or infer which banks are weak. Once these n weak banks are placed in resolution, the government will choose a bailout payment $\hat{b}$ to give to each of them, then use its remaining fiscal capacity to provide the public good. Within each weak bank, the government will give a payment $\hat{x}_1$ to each of the remaining movers at $t = 1$ and a payment $\hat{x}_2$ to each of the remaining non-movers at $t = 2$. Letting $\hat{\pi}$ denote the fraction of the remaining depositors who are movers,[5] these payments will be chosen to maximize

$$\max_{\{\hat{x}_1, \hat{x}_2, \hat{b}\}} \quad n(1-\theta)\left(\hat{\pi}\int_{\underline{\rho}}^{\bar{\rho}} u(\rho_i \hat{x}_1) dF(\rho_i) + (1-\hat{\pi})\, u\left(\hat{x}_2\right)\right) + v(\tau - n\hat{b}) \tag{7}$$

subject to the resource constraint

$$(1-\theta)\left(\hat{\pi}\hat{x}_1 + (1-\hat{\pi})\frac{\hat{x}_2}{R}\right) \leq 1 - \sigma - \theta x_1^* + \hat{b}. \tag{8}$$

The first term in the objective function is the sum of utilities of the $1-\theta$ depositors who remain in the n weak banks. The fraction $\hat{\pi}$ of these depositors who are movers each receive $\hat{x}_1$ at $t = 1$ and earn the idiosyncratic return ρ_i, while the remaining $1 - \hat{\pi}$ non-movers each consume the $\hat{x}_2$ they receive at $t = 2$. The final term in the objective function is the utility all depositors in the economy receive from the public good, which is produced using the government's fiscal capacity τ minus the bailout payments made to n banks of $\hat{b}$ each. The resource constraint in equation (8) says that the payments to each weak bank's remaining depositors come from the bank's remaining funds – the initial endowment of 1 minus the loss σ and the payment x_1^* made for each of the first θ withdrawals – plus the bailout payment $\hat{b}$. We assume $\sigma < 1 - \pi x_1^*$, which implies the loss in a weak bank is never so large that the bank runs completely out of resources before being placed in resolution.

When there is no CBDC, the government is able to determine which banks are weak after a fraction π of depositors have withdrawn. If there is no run on weak banks, the government directly observes their losses at the end of period 1, at which point the π movers have all withdrawn and the remaining depositors are all non-movers. If a fraction $\alpha > 0$ of non-movers run and attempt to withdraw at $t = 1$, the government will observe that a run is underway – and infer that the affected banks are weak – as soon as withdrawals go above π. In this case, some of the first π withdrawals will have been made by non-movers, and some of the remaining $1 - \pi$ depositors will be movers who still need to withdraw at $t = 1$.

The fraction of the remaining depositors who are movers when weak banks are placed in

[5] Note that our focus on symmetric equilibria implies $\hat{\pi}$ is the same in all weak banks.

resolution is

$$\hat{\pi}(\alpha) \equiv \frac{\alpha\pi}{\pi + \alpha(1-\pi)}. \tag{9}$$

When there is no CBDC, the resolution problem is given by equations (7) and (8) with $\theta = \pi$ and $\hat{\pi}$ given by equation (9). Let

$$\left(\hat{x}_1^N(\alpha), \hat{x}_2^N(\alpha), \hat{b}^N(\alpha)\right) \tag{10}$$

denote the solution to this resolution problem, where the N superscript indicates we are in the policy regime with no CBDC. This solution is characterized by the resource constraint in equation (8) together with the first-order conditions

$$\int_{\underline{\rho}}^{\bar{\rho}} \rho_i u'\left(\rho_i \hat{x}_1^N(\alpha)\right) dF(\rho_i) = Ru'\left(\hat{x}_2^N(\alpha)\right) = v'\left(\tau - n\hat{b}^N(\alpha)\right). \tag{11}$$

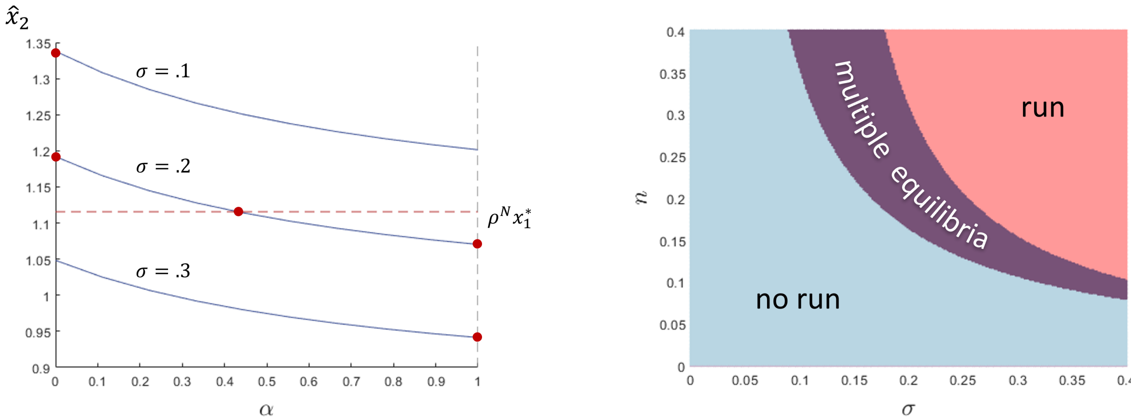
The withdrawal game. Depositors in a weak bank anticipate that their bank will be placed in resolution after π withdrawals. Movers will always withdraw at $t = 1$, since doing so is their only way to consume. For non-movers, the banking contract and the solution to the resolution problem together determine the payoffs of a game in which they each choose a withdrawal strategy.

If a non-mover chooses to withdraw at $t = 1$ and arrives before the bank is placed in resolution, she will receive x_1^* and store it at return ρ^N. If she arrives after the bank is placed in resolution or if she chooses to wait, she will receive $\hat{x}_2^N$ from the resolution allocation in equation (10) at $t = 2$.[6] Each non-mover chooses a strategy $\alpha_i^j \in [0,1]$ that corresponds to the probability of withdrawing at $t = 1$. If $\rho_N x_1^* < \hat{x}_2^N$, the best choice is to set $\alpha_i^j = 0$ and wait until $t = 2$ to withdraw. If $\rho_N x_1^* > \hat{x}_2^N$, the best choice is to run on the bank by setting $\alpha_i^j = 1$. We allow for mixed strategies, with $0 < \alpha_i^j < 1$, for reasons that will become clear below. We focus on symmetric equilibria of the withdrawal game, in which all depositors in weak banks choose the same value of α_i^j; we denote this common value by α. Such equilibria are characterized by a scalar $\alpha^N \in [0,1]$ satisfying

$$\alpha^N \begin{Bmatrix} = 0 \\ \in [0,1] \\ = 1 \end{Bmatrix} \quad \text{if} \quad \hat{x}_2^N\left(\alpha^N\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \rho_N x_1^*. \tag{12}$$

In particular, an equilibrium exists in which all non-movers in weak banks wait until $t = 2$

[6] Note that the restriction on ρ^N in equation (5) implies implies the allocation in resolution is strictly incentive compatible and, since we assume the run stops, a non-mover will prefer to withdraw at $t = 2$.



(a) Payoffs with $n = 0.2$

(b) Fragility

Figure 1: Equilibrium with no CBDC
Source: Authors' analysis

to withdraw if $\hat{x}_2^N(0) \geq \rho_N x_1^*$. An equilibrium exists in which all non-movers run on weak banks and attempt to withdraw at $t = 1$ if $\hat{x}_2^N(1) \leq \rho_N x_1^*$.

2.4 Fragility

Figure 1 depicts payoffs and equilibrium in the withdrawal game for varying levels of losses in the banking system.[7] Panel (a) compares the payoffs available to a non-mover in a weak bank as a function of the fraction α of other non-movers who withdraw early. The red dots in the graph correspond to equilibria for different values of the loss σ. When σ is small, the payoff at $t = 2$ in resolution is larger than the payoff from withdrawing at $t = 1$ for all values of α. Withdrawing at $t = 2$ is a dominant strategy in this case, which implies $\alpha^N = 0$ is the unique equilibrium and no bank runs occur. When the loss σ is large, the payment $\hat{x}_2$ is smaller than the payoff from withdrawing at $t = 1$ for all values of α and withdrawing early is a dominant strategy. In this case, the unique equilibrium has $\alpha^N = 1$ and runs occur on all weak banks. For the middle value of σ in the figure, both of these equilibria exist, as does a mixed-strategy equilibrium with $0 < \alpha^N < 1$. The fact that the $\hat{x}_2$ curves are decreasing in α reflects the usual strategic complementarity in withdrawal decisions: as more depositors withdraw early, the bank is in worse shape when resolution comes and the payments available to depositors who wait become smaller.

Panel (b) of Figure 1 shows how the equilibria of the withdrawal game change as the parameters σ (the loss in each weak bank) and n (the fraction of banks that are weak) vary. The following two propositions give conditions under which the patterns illustrated in this

[7] The parameter values used in this figure are $R = 2$, $\pi = 0.5$, $\gamma = 6$, $\delta = 10^{-6}$, $\tau = 0.13$, $\bar{\rho} = 1.8$, $\underline{\rho} = 0.9$, and $\rho^N = 0.9$.

figure hold in general. First, assuming fiscal capacity is above a minimum value $\underline{\tau}$, only the no-run equilibrium exists when the fraction of affected banks n is small enough.

Proposition 1. *With no CBDC, there exists $\underline{\tau} < \bar{\tau}$ such that $\tau > \underline{\tau}$ implies $\alpha^N = 0$ is the unique equilibrium of the withdrawal game for all σ when n is sufficiently small.*

When there are few weak banks (and τ is not too small), the bailouts chosen by the government will be sufficiently generous to ensure a non-mover has no incentive to withdraw early regardless of what she expects other non-movers to do.

Our second result shows that, if the return earned by non-movers is above a minimum value ρ_N, a bank run necessarily occurs at all weak banks when the loss σ and fraction of weak banks n are large.

Proposition 2. *With no CBDC, there exists $\underline{\rho}^N < \bar{\rho}^N$ such that $\rho^N > \underline{\rho}^N$ implies $\alpha^N = 1$ is the unique equilibrium of the withdrawal game when σ and n are sufficiently large.*

Significant losses at a sufficiently large fraction of banks will strain the government's resources and result in a relatively small bailout payment for each weak bank. This small bailout, in turn, implies that a large part of the bank's losses will fall on its remaining depositors when the bank is placed in resolution. As long as the return ρ^N is not too small, a non-mover will then be incentivized to withdraw before the bank is placed in resolution, even if she thinks other non-movers will wait to withdraw. In this case, withdrawing early is a dominant strategy for non-movers and the only equilibrium of the withdrawal game has a run on all weak banks. In between these two cases, there is a region where both the bank run and the no-run equilibrium exist, as illustrated in Figure 1.

For the remainder of the analysis, we assume both τ and ρ^N lie in the regions identified by these propositions, which implies the banking system is fragile for some pairs (σ, n) but not others. We analyze the financial-stability effects of introducing a CBDC below by studying how it alters the sets of (σ, n) for which each type of equilibrium exists. Before addressing this question, however, we discuss some of the key assumptions in our baseline model.

2.5 Discussion

Studying how the introduction of a CBDC affects financial fragility requires starting from a framework in which banks may or may not be fragile, depending on parameter values and the incentives they create for agents in the model. Many papers follow Diamond and Dybvig (1983) in assuming that banks must pay a promised amount to depositors until they are completely out of resources. In such settings, a run will always exhaust the resources of an illiquid bank at $t = 1$, implying it is always fragile. We follow Wallace (1990), Green and

Lin (2003), Peck and Shell (2003), Ennis and Keister (2009, 2010) and others in assuming policymakers *react* once they learn the relevant features of the situation. This reaction preserves resources in the banking system and may give depositors an incentive to stay invested. This approach allows us to study how the introduction of a CBDC would change the policy reaction to a crisis and, hence, withdrawal incentives and financial fragility. In the following paragraphs, we briefly discuss some key features of our model and their roles in our analysis.

Fixed fiscal capacity. We assume the government's fiscal capacity is fixed at the time our model begins and moderate in size. If τ were very large, the government would choose *ex post* to fully compensate banks for their losses, including any losses associated with depositor runs. In that case, financial fragility would never arise. Conversely, if τ were very small, banks would very often be susceptible to self-fulfilling runs. One can think of τ as reflecting tax revenue that was raised in the past, as in Keister (2016). If the *ex ante* probability of the loss event is sufficiently small, the optimal tax rate would put the government's fiscal capacity in this moderate region. Our assumption that fiscal capacity is fixed simplifies the analysis while allowing us to focus on this relevant region.

No runs on sound banks. A sound bank could, in principle, be subject to the standard type of self-fulfilling run by its depositors. However, it is well known that this type of run can be prevented by deposit contracts that sufficiently limit early payouts by, for example, suspending payments or imposing withdrawal fees after a pre-specified threshold of early withdrawals. Following Keister and Mitkov (2019), we could assume that banks can use such contracts and these contracts are enforced whenever the bank is not in resolution. The availability of these contracts would then imply that runs never occur on sound banks in equilibrium. Our assumption of no runs on sound banks leads to this same outcome without the need for the additional notation associated with such contracts.

No looting. Given that weak banks anticipate being bailed out, they have an incentive to pay out as much as possible to withdrawing depositors before being placed in resolution. We assume regulators can prevent banks from paying out more than x_1^* to a withdrawing depositor at $t = 1$, since doing so would be inefficient for either type of bank. Similar results can be obtained in a version of the model where depositors fully "loot" a bank before it is placed in resolution, but at the cost of additional model complexity. We also assume depositors cannot loot a sound bank, which implies there will never be an incentive to bail out a sound bank or place it in resolution.

No bail-ins. When weak banks are in the fragile region, their depositors might want the bank to impose fees on early withdrawals to prevent a run. However, applying this type of voluntary "bail in" is costly because it decreases the bailout payment the bank will receive from the government once it is placed in resolution. The incentive distortion created by bailouts thus helps justify our assumption that weak banks continue to allow depositors to withdraw as if things were normal.[8] Keister and Mitkov (2021) study what they call the *bail-in game*, where each bank decides whether to impose withdrawal fees taking the actions of other banks as given. They show the outcome of this game often has no voluntary bail-ins, but not always. To simplify our analysis, we assume voluntary bail ins are not allowed, which corresponds to the case in Keister and Mitkov (2021) where voluntary bail-ins are zero in equilibrium. It may be interesting to extend our analysis to include the bail-in game and to study how introducing a CBDC can affect equilibrium bail-ins.

Keister and Mitkov (2021) also show how a policy of requiring all banks to impose withdrawal fees can raise welfare when policymakers cannot determine the size of the losses in each bank. Such a policy would be useful in the present model for some configurations of parameter values. Adding a mandatory, system-wide bail-in would complicate the notation and change payoffs in our model, but would not alter our two fundamental points. Introducing a CBDC into an environment with mandatory bail-ins would still (i) change the equilibrium banking contract and (ii) provide information that potentially allows policymakers to place weak banks into resolution (and to remove the mandatory bail-in at strong banks) more quickly. We use the simpler model with no bail-ins to illustrate these points in the sections below.

3 Introducing CBDC

We now introduce a central bank that can issue digital currency, which is an alternative way for depositors to transfer funds from $t = 1$ to $t = 2$. We ask how the introduction of CBDC changes the fragility of weak banks *holding fixed* the point θ at which they are placed in resolution. We defer to the next section the question of how the information generated by a CBDC changes the timing of the policy reaction to a bank run. We first show that – absent information effects – introducing a CBDC has two competing effects on fragility. First, the ability to store funds that have been withdrawn from a bank in the new CBDC gives non-movers in weak banks a stronger incentive to withdraw early. This effect, on its own, would make weak banks more fragile. At the same time, however, the availability of CBDC in normal times leads banks to do less maturity transformation, which on its own

[8] The withdrawals made from a weak bank before it is placed in resolution can be interpreted as partially coming from informed creditors or bank insiders. Several recent papers have documented the prevalence of withdrawals by informed creditors before regulators intervene in a failing bank; see, for example, Acharya et al. (2011), Henderson et al. (2015), Iyer et al. (2016).

would make weak banks more stable. We illustrate how parameter values determine which of these two effects dominates.

3.1 The central bank

The central bank operates in all locations in the model.[9] We assume the central bank has access to a storage technology with return $\rho^C \in \left[\underline{\rho}, \bar{\rho}\right]$. The central bank accepts deposits of goods at any time in period 1 and pays the return ρ^C on these deposits at $t = 2$. We interpret these deposits as CBDC. Importantly, depositors can withdraw these funds in any location at t = 2, which implies that CBDC provides movers with an alternative to transporting goods to their new location. Note that, in a symmetric allocation, the central bank does not need to transport goods across locations, as the withdrawals of movers arriving in a location can be paid using the deposits of movers who exited that location.

Whether CBDC is attractive to an individual mover depends on whether her idiosyncratic return ρ_i is higher or lower than ρ^C. We interpret these values as capturing how well different payment methods meet an individuals' needs. A mover with a high value of ρ_i is well served by existing payment methods and will have no incentive to use CBDC, while a mover with a low value of ρ_i will find CBDC preferable to existing payment methods. We think of the return ρ^C as capturing a range of design features that will determine how attractive CBDC is to users in practice, including privacy features, ease of access, caps and other limits on its use, offline capabilities, and any fees or interest paid on balances. In Section 4.4, we discuss some tradeoffs that arise when policymakers can determine ρ^C through such design choices. For now, however, we treat ρ^C as fixed and study the implications of introducing a CBDC with given design features.

Non-movers are also able to deposit funds in the CBDC. Because relocation status is private information, the central bank is initially unable to tell if the funds being placed into CBDC are coming only from movers or from a mix of movers and non-movers. In the next section, we show how policymakers can make inferences based on the amount of funds flowing into the CBDC. For now, however, we assume that a weak bank is placed into resolution at the end of $t = 1$ or when the fraction of early withdrawals exceeds π, whichever comes first.

[9] There is no central bank in Champ et al. (1996), but they study a policy regime in which private banknotes can be traded across locations. Our interpretation here is similar to Antinolfi et al. (2001), who study lender-of-last-resort policy by assuming a central bank operates across locations.

3.2 Revised payoffs and withdrawal incentives

Revised banking contract The availability of the CBDC technology changes the constrained efficient allocation of resources within a sound bank. In particular, a fraction $F\left(\rho^C\right)$ of movers will have an idiosyncratic return on storage ρ_i lower than ρ^C and will deposit goods with the central bank after withdrawing at $t = 1$. The fact that these movers now receive a higher return changes the marginal value of resources paid out at $t = 1$. The constrained efficient allocation now maximizes expected utility in equation (2), but with the distribution function F replaced by

$$F^C\left(\rho_i;\rho^C\right) \equiv \left\{\begin{matrix} 0 \\ F(\rho_i) \end{matrix}\right\} \quad \text{for} \quad \rho_i \left\{\begin{matrix} < \\ \geq \end{matrix}\right\} \rho^C. \tag{13}$$

The resource constraint in equation (3) is unchanged. Let $\left(x_1^*(\rho^C), x_2^*(\rho^C)\right)$ denote the solution to this modified problem, which is characterized by the first-order condition

$$\rho^C u'\left(\rho^C x_1^*\right) F^C\left(\rho^C;\rho^C\right) + \int_{\rho^C}^{\bar{\rho}} \rho_i u'\left(\rho_i x_1^*\right) dF^C\left(\rho_i;\rho^C\right) = Ru'\left(x_2^*\right). \tag{14}$$

Note that if $\rho^C = \underline{\rho}$, no one will use the CBDC and the constrained efficient allocation is unchanged. When ρ^C is greater than $\underline{\rho}$, some movers will use CBDC to receive higher consumption. Our next result shows how (x_1^*, x_2^*) varies with ρ^C in this case.

Proposition 3. *When CBDC is introduced, $x_1^*\left(\rho^C\right)$ is strictly decreasing and $x_2^*\left(\rho^C\right)$ is strictly increasing for $\rho^C \in \left(\underline{\rho}, \bar{\rho}\right)$.*

Recall that fragility arises in the Diamond-Dybvig framework because the efficient allocation provides liquidity insurance, that is, sets $x_1^* > 1$ as a way of sharing the benefits of long-term investment with depositors who need to withdraw early. When CBDC allows some of these depositors to earn a higher return after they withdraw, there is less need for the banking system to provide this liquidity insurance. As a result, Proposition 3 shows that the constrained efficient allocation gives a smaller payment $x_1^*(\rho^C)$ to movers and a larger payment $x_2^*(\rho^C)$ to non-movers, who thus share some of the benefits of CBDC.

We continue to assume that banks offer deposit contracts based on the constrained-efficient allocation, meaning that depositors who withdraw at $t = 1$ are each given $x_1^*(\rho^C)$ unless the bank has been placed into resolution. As a standard, $x_1^*(\rho^C)$ can be interpreted as a bank's short term liabilities per depositor. Proposition 3 thus shows that the introduction of CBDC will lead banks to issue fewer short term liabilities or, equivalently, to do less maturity transformation.

Bailouts and resolution. We assume in this section that a weak bank is still placed in resolution after a fraction π of depositors have withdrawn at t = 1. The availability of the CBDC technology changes the optimal resolution of a weak bank at this point. The government's objective function in choosing bailout payments and resolution plans is again given by equation (7), using $\theta = \pi$ and $\hat{\pi}$ from equation (9), but now with the distribution function F replaced by F^C from equation (13). Let

$$\left(\hat{x}_1^C\left(\alpha;\rho^C\right), \hat{x}_2^C\left(\alpha;\rho^C\right), \hat{b}^C\left(\alpha;\rho^C\right)\right) \tag{15}$$

denote the solution to the resolution problem in this case, where the C superscript indicates that the environment now includes a CBDC. This solution is characterized by the resource constraint in equation (8) together with the first-order conditions

$$\rho^C u'\left(\rho^C \hat{x}_1^C\right) F^C\left(\rho^C;\rho^C\right) + \int_{\rho^C}^{\bar{\rho}} \rho_i u'\left(\rho_i \hat{x}_1^C\right) dF(\rho_i;\rho^C) = Ru'\left(\hat{x}_2^C\right) = v'\left(\tau - n\hat{b}^C\right). \tag{16}$$

Following the logic of Proposition 3, it is straightforward to show that $\rho^C > \underline{\rho}$ will lead the government to decrease $\hat{x}_1^C$ and increase $\hat{x}_2^C$, all else equal. In other words, the resolution process will tend to provide smaller payments to the remaining movers, some of whom will choose to use CBDC, and larger payments to non-movers who wait to withdraw at t = 2.

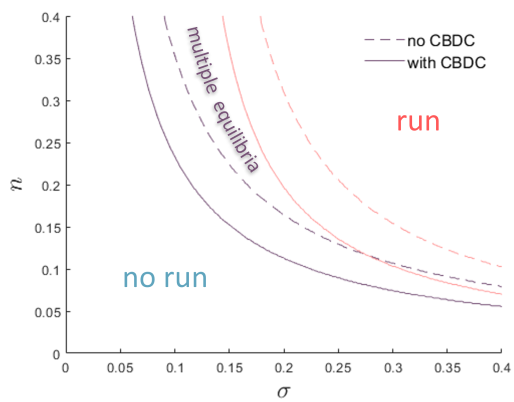
The withdrawal game. A non-mover who withdraws early will prefer depositing in CBDC over storing the funds directly if $\rho^C > \rho^N$, and they will not use CBDC if this inequality is reversed. As before, the revised banking contract and the solution to the revised resolution problem determine the payoffs of the withdrawal game. We continue to focus on symmetric equilibria, which are now characterized by a scalar $\alpha^C \in [0,1]$ satisfying

$$\alpha^C \begin{Bmatrix} = 0 \\ \in [0,1] \\ = 1 \end{Bmatrix} \quad \text{if} \quad \hat{x}_2^C\left(\alpha^C, \rho^C\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \max\{\rho^C, \rho^N\}\, x_1^*\left(\rho^C\right). \tag{17}$$

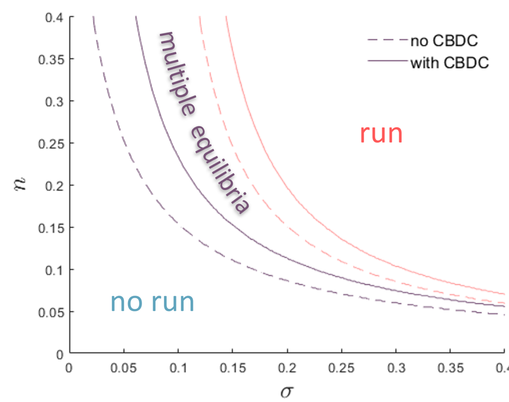
3.3 Competing effects on fragility

The three places where ρ^C appears in equation (17) represent the three ways in which the introduction of CBDC can affect the fragility of weak banks. Two of these effects have already been discussed. First, $\rho^C > \underline{\rho}$ decreases x_1^* in the constrained-efficient allocation, which by itself would decrease the incentive for a non-mover to withdraw early, making the no-run equilibrium more likely and the run equilibrium less likely to exist. Second, $\rho^C > \underline{\rho}$ increases $\hat{x}_2^C$ in the resolution process, holding all else equal, which would have the same effects. However, the third effect points in the opposite direction: $\rho^C > \rho^N$ raises the return to non-movers of withdrawing at t = 1. This effect captures the concern expressed by policymakers that runs into CBDC may destabilize the financial system.

Figure 2 demonstrates that introducing a CBDC can either increase or decrease fragility, depending on parameter values. One key factor in determining which effect dominates is the difference between the returns non-movers earn on the goods they store across periods, ρ^N, and the return on CBDC, ρ^C. Suppose we normalize $\rho^C = 1$. Panel (a) presents the fragility diagram for an economy with CBDC using the same parameter values as in Figure 1. The dashed lines correspond to the boundaries of the regions in Figure 1, where there is no CBDC, and the solid lines show these boundaries when CBDC is introduced. In this case, introducing CBDC clearly increases the fragility of weak banks: the set of (σ, n) for which the run equilibrium exists expands and the set for which the no-run equilibrium exists shrinks.



(a) $\rho^N = 0.9$



(b) $\rho^N = 1$

Figure 2: CBDC may increase or decrease fragility
Source: Authors' analysis

Panel (b) of Figure 2 presents the fragility diagram for a different economy where non-movers are better able to store goods across periods on their own; that is, the return ρ^N is higher in economy (b) than in economy (a). All other parameter values are the same, which implies the constrained efficient allocation (x_1^*, x_2^*) and the banking contract are the same in both economies.[10] However, comparing the dashed curves in the two panels shows that economy (b) is more fragile than economy (a) before CBDC is introduced because the better storage technology ρ^N makes withdrawing early less costly for non-movers. When CBDC

[10] Recall that non-movers only withdraw at $t = 2$ in the efficient allocation, so the value of ρ^N plays no role in determining that allocation.

is introduced, non-movers in both economies can store goods at the same return $\rho^C = 1$, which implies all payoffs are the same in both economies and, hence, they are equally fragile. (In other words, the solid lines are the same in both panels of the figure). Comparing the dashed and solid lines in panel (b) demonstrates that introducing CBDC *reduces* fragility in this economy. That is, the set of (σ, n) for which the run equilibrium exists shrinks and the set for which the no run equilibrium exists expands.

The intuition for these patterns is easy to see. In both cases, introducing CBDC changes the banking contract and the resolution process by the same amount. The difference between the two cases is how much better non-movers find CBDC than directly storing goods. For the economy in panel (a), where $\rho^N = 0.9$, CBDC is significantly better, and the direct effect of CBDC on non-movers' withdrawal incentives dominates the indirect effects. For the economy in panel (b), $\rho^N = 1$ and CBDC is equally effective as storing goods directly for non-movers. In this case, the direct effect disappears and the indirect effects dominate.

Some observers claim that the destabilizing effects of a CBDC could be minimized through appropriate design choices. Suppose, for example, policymakers could place a cap on individuals' CBDC holdings that would not be binding in normal times but would prevent individuals from shifting significant amounts into the CBDC in periods of financial stress. A design that allows CBDC to be useful in normal times would correspond to having ρ^C significantly higher than $\underline{\rho}$ in our model, which would lead to significant changes in the banking contract and resolution process. To the extent that the design limits the usefulness of CBDC as a store of value, it would correspond to having a small difference between ρ^N and ρ^C, as for the economy in panel (b) of Figure 2. Our results in this section thus support the idea that such a design, if feasible, would not increase and may even decrease financial fragility.

However, the analysis in this section and in most of the policy discussion of CBDC leaves out an important point. If introducing a CBDC increases fragility, it is because non-movers find it attractive to withdraw from their banks and deposit in CBDC. Policymakers will observe this flow of funds into the CBDC and can use the information to improve the resolution process. We now turn to the study of this information effect.

4 The information effect

In the economy with no CBDC in Section 2, the government can either observe or infer which banks are weak after a fraction π of depositors have withdrawn. In Section 3, we introduced CBDC but assumed the government continued to place weak banks in resolution after π withdrawals. In this section, we show how observing the flow of funds into CBDC provides the government with additional information that allows it to infer more quickly when a run is underway. This quicker inference can allow the government to resolve weak banks sooner, which has two beneficial effects. First, quicker resolution implies fewer early withdrawals are made before the run stops and, therefore, the consumption levels of the remaining depositors are higher. In addition, these higher consumption levels diminish the incentive for non-movers to withdraw early, which improves financial stability.

4.1 Inferring a run through CBDC

To begin, it is useful to review why the policy reaction occurs after π withdrawals when there is no CBDC. If there is no run, only the fraction π of depositors who are movers withdraw at $t = 1$, and the government observes each bank's status at the end of the period. If depositors do run on weak banks, more withdrawals will occur before the government directly observes each bank's status. However, the government sees these withdrawals as they are being made from each bank. Once withdrawals at a bank go above π, the government will infer that a run is underway and, since runs only occur on weak banks, the bank must be weak and should be placed in resolution.

When CBDC is introduced, we assume the government observes the flow of funds into the CBDC from each bank as they occur. In this way, the government gains additional information in real time within the period. If a bank is not experiencing a run, only the π depositors who are being relocated withdraw at $t = 1$. Those movers with $\rho_i < \rho^C$ will now deposit in CBDC, which implies total conversions to CBDC from a bank that is not experiencing a run will be $\pi F\left(\rho^C\right)$.

Now suppose a fraction $\alpha > 0$ of non-movers run on the bank and attempt to withdraw at $t = 1$. If $\rho^C > \rho^N$, these non-movers will all convert to CBDC.[11] The measure of depositors who desire to withdraw and convert to CBDC is now given by

$$\pi F\left(\rho^C\right) + \alpha(1-\pi) \;>\; \pi F\left(\rho^C\right) \text{ for any } \;\alpha > 0.$$

Define θ to be the total measure of depositors who have withdrawn from a bank when the conversions to CBDC by that bank's depositors reaches $\pi F\left(\rho^C\right)$, the amount that would occur when there is no run. We then have

$$\theta\left(\alpha, \rho^C\right) = \frac{\left(\pi + \alpha(1-\pi)\right) F(\rho^C)}{\pi F(\rho^C) + \alpha(1-\pi)}\pi \quad \leq \; \pi. \tag{18}$$

[11] If $\rho^C < \rho^N$ held, non-movers who withdrew at $t = 1$ would not use CBDC. In this case, there would be no information effect and equilibrium outcomes would be identical to those in regime C above. For the remainder of the analysis, we assume $\rho^C \geq \rho^N$ holds. In addition, to simplify notation, we assume all non-movers use CBDC in the knife-edge case where these two returns are equal.

Taking as given the withdrawal strategies of non-movers, $\theta\left(\alpha, \rho^C\right)$ measures the speed with which the government is able to identify weak banks and place them in resolution. Once withdrawals at a given bank exceed $\theta\left(\alpha, \rho^C\right)$, deposits into CBDC from that bank will exceed $\pi F\left(\rho^C\right)$ and the government will know a run is underway.

Note that setting $\alpha = 0$ in equation (18) yields $\theta = \pi$; if there is no run, the government will not be able to learn which banks are weak until the end of $t = 1$, as before. For $\alpha > 0$, however, it is straightforward to show that θ is strictly decreasing in α: as more non-movers withdraw early and convert to CBDC, fewer total withdrawals occur before the central bank is able to infer a run is underway. In other words, a larger run will be detected more quickly. We show below that this feature makes withdrawing early less attractive and can switch the withdrawal decisions of depositors from stategic complements to strategic substitutes.

Discussion. Because the only uncertainty in our model is idiosyncratic, it is important in our setting that the government be able to observe the flow into CBDC originating from each bank. In practice, such tracing may not be straightforward, especially if weak banks could take actions to mask the eventual destination of funds being withdrawn (by routing transfers through third parties, for example). The effects we identify here will be present as long as the government can extract some information about the flow of funds from each bank into CBDC, even if this information is imperfect. In addition, as part of our concluding remarks in Section 5, we describe an extension of the model where there is aggregate uncertainty and the government does not know the total number of weak banks. In that case, monitoring aggregate inflows into CBDC, regardless of their origin, would generate an information effect similar in spirit to the idiosyncratic one we identify here.

4.2 How faster resolution affects withdrawal incentives

The constrained-efficient allocation $\left(x_1^*(\rho^C), x_2^*(\rho^C)\right)$ and the banking contract are the same as in the previous section. However, the change in timing of the government's intervention changes the resolution problem and non-movers' withdrawal incentives.

Bailouts and resolution. When the government intervenes after θ withdrawals, the fraction of the remaining depositors in weak banks who are movers is given by

$$\hat{\pi}\left(\alpha, \rho^C\right) = \frac{\pi}{1 - \theta\left(\alpha, \rho^c\right)} \frac{\pi + \alpha(1-\pi) - \theta\left(\alpha, \rho^c\right)}{\pi + \alpha(1-\pi)}. \tag{19}$$

The resolution problem is now given by equations (7) - (8) with θ from equation (18), $\hat{\pi}$ from equation (19) and the distribution function F^C from equation (13). Let

$$\left(\hat{x}_1^I\left(\alpha;\rho^C\right), \hat{x}_2^I\left(\alpha;\rho^C\right), \hat{b}^I\left(\alpha;\rho^C\right)\right) \tag{20}$$

denote the solution to the problem in this case, where the I superscript indicates that resolution now responds to the information generated by the CBDC. It bears emphasizing that the only differences in this optimization problem from the resolution problem in Section 3 are the timing of the intervention, θ, and the type-composition of the remaining depositors, $\hat{\pi}$. The solution is again characterized by the resource constraint in equation (8) and the first-order conditions in equation (16).

When there is no run ($\alpha = 0$), the government again identifies weak banks after π withdrawals and the resolution process is exactly the same as in the previous section. When $\alpha > 0$, however, equation (18) shows $\theta < \pi$ and fewer withdrawals occur before weak banks are placed into resolution. At this point, fewer resources have been paid out and weak banks have more resources per remaining depositor. This fact increases the payments made to remaining depositors and thus decreases the incentive for non-movers to run on the bank.

The withdrawal game. The revised banking contract and the solution to the revised resolution problem determine the payoffs of the game played by non-movers. A symmetric equilibrium is $\alpha^I \in [0,1]$ satisfying

$$\alpha^I \left\{\begin{array}{c} = 0 \\ \in [0,1] \\ = 1 \end{array}\right\} \quad \text{if} \quad \hat{x}_2^I\left(\alpha^I, \rho^C\right) \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} \rho^C\, x_1^*\left(\rho^C\right). \tag{21}$$

The only difference between equations (17) and (21) is in the payoff $\hat{x}_2$ that a non-mover will receive as part of the resolution process if she waits until $t = 2$ to withdraw.

4.3 Information reduces fragility

Our next result shows that the information effect does not change the set of parameter values for which the no-run equilibrium exists. When all non-movers wait to withdraw ($\alpha = 0$), the flow of funds into the CBDC does not reveal any information about which banks are weak. The government places weak banks into resolution only at the end of $t = 1$, after all movers have withdrawn. The payoffs of the withdrawal game are then exactly the same as in the previous section, where the information effect was assumed to be absent, as are the conditions under which the no-run equilibrium exists.

Proposition 4. *$\alpha^I = 0$ is an equilibrium if and only if $\alpha^C = 0$ is an equilibrium.*

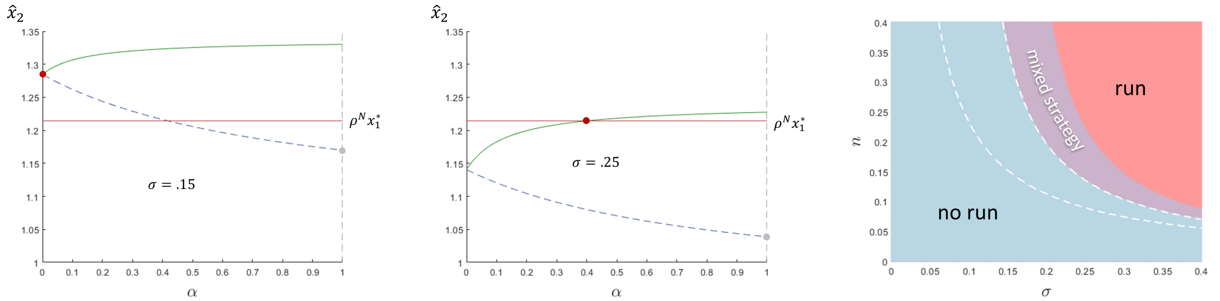
The effect of information on fragility appears when we ask whether there exists an equilibrium where non-movers run on weak banks. When there is a run ($\alpha = 1$), equation (18) shows $\theta < \pi$ holds and weak banks are placed in resolution sooner. As a result, a non-mover who deviates and waits to withdraw receives a higher payoff. This higher payoff, in turn, shrinks the set of parameter values for which the bank run equilibrium exists.

Proposition 5. *The set of (σ, n) for which $\alpha^I = 1$ is an equilibrium is strictly smaller than the set for which $\alpha^C = 1$ is an equilibrium.*

One way of understanding the intuition behind these two results is to observe that the information effect decreases the strategic complementarity in non-movers' withdrawal decisions. When the resolution timing is fixed at π withdrawals, our model exhibits the usual strategic complementarity in the Diamond-Dybvig framework. In our notation, early withdrawals by other non-movers increase α, which decreases the amount $\hat{x}_2^C(\alpha)$ received by a non-mover who waits until $t = 2$ to withdraw (as depicted in panel (a) of Figure 1), giving an individual non-mover a stronger incentive to withdraw early. When the resolution timing is given by equation (18), in contrast, a countervailing effect arises. Early withdrawals by other non-movers now also decrease θ, meaning they allow the government to more quickly infer which banks are weak and place them in resolution. This second effect raises $\hat{x}_2^I(\alpha)$ and thus strengthens the incentive for a non-mover to wait and withdraw at $t = 2$.

The effects of these changes are illustrated in Figure 3. Panel (a) shows the payoffs of a non-mover who waits to withdraw as a function of α, as in panel (a) of Figure 1. The dashed curve is the payoff in regime C, where the information effect is absent. This curve is decreasing, reflecting the usual strategic complementarity in withdrawal decisions, and both the no-run and the run equilibrium exist. When the information effect is introduced, $\hat{x}_2^I$ takes the same value at $\alpha = 0$, but lies above $\hat{x}_2^C$ for all $\alpha > 0$. In this example, $\hat{x}_2^I$ has become an increasing function of α, meaning the withdrawal decisions of non-movers are now strategic substitutes. As a result, the run equilibrium no longer exists and the unique equilibrium of the withdrawal game has all non-movers waiting to withdraw at $t = 2$.

Panel (b) of Figure 3 depicts a situation in which $\hat{x}_2^C$ lies below $\rho^C x_1^* \left(\rho^C\right)$ for all α and, therefore, only the run equilibrium exists under regime C. When the information effect is introduced, $\hat{x}_2^I$ again becomes an increasing function of α and, as in panel (a), $\alpha = 1$ is no longer an equilibrium. In this case, the unique symmetric equilibrium is in mixed strategies, with $0 < \alpha^I < 1$. The outcome in this equilibrium is a form of partial bank run in which a fraction α^I of non-movers attempt to withdraw early and the remaining fraction $(1 - \alpha^I)$ choose to wait.



(a) Eliminating the run equilibrium

(b) Unique equilibrium in mixed strategies

(c) Fragility with information

Figure 3: The information effect
Source: Authors' analysis

Panel (c) of Figure 3 presents the fragility diagram once the information effect is taken into account, using the same parameter values as in panel (a) of Figure 2. Recall that, in this case, a CBDC increases fragility when the information effect is neglected. Panel (c) shows how the information effect can reverse this result. In line with Proposition 4, the set of (σ, n) for which the no-run equilibrium exists is the same as that shown for regime C in Figure 2. In line with Proposition 5, the set of (σ, n) for which the full-run equilibrium exists is strictly smaller. In this example, the information effect has eliminated the overlap of these two regions, so there is now a unique symmetric equilibrium for all (σ, n). In between the regions where this equilibrium has $\alpha^I = 0$ and where it has $\alpha^I = 1$, there is now a region where the equilibrium is in mixed strategies, with $0 < \alpha^I < 1$.

In summary, the information effect generated by introducing a CBDC in our model always reduces the incentive for depositors to participate in a run on their bank. This decreased incentive to run can affect equilibrium outcomes in one of three ways, depending on the current situation. If there are multiple equilibria with no CBDC, introducing a CBDC may eliminate the run equilibrium and leave 'no bank run' ($\alpha^I = 0$) as the unique equilibrium. If only the run equilibrium currently exists, introducing a CBDC may lead to a unique, mixed-strategy equilibrium ($0 < \alpha^I < 1$) with a smaller run on weak banks. Finally, even if a full bank run ($\alpha^I = 1$) occurs in equilibrium, introducing a CBDC will lead to quicker resolution of weak banks, which mitigates the effect of the run.

4.4 Policy tradeoffs

So far, we have taken the return ρ^C that depositors earn on CBDC as a fixed parameter. In practice, we think of this return as representing a variety of design features that determine how well the CBDC meets the needs of households and firms. Such features include, for example, the form in which CBDC is held and transferred, any restrictions on holdings, offline capabilities, and any fees or interest payments on CBDC. Many of these features are choices a central bank can make when designing a CBDC. In other words, the central bank will be able to influence the value of ρ^C, at least within some bounds, which raises the question of how it should be set. How attractive should the CBDC be made to potential users? In the absence of financial stability concerns, the answer in our model would be clear: ρ^C should be set has high as possible. Doing so would maximize the usage of the CBDC by movers, for whom CBDC represents a more efficient way of transferring funds across locations. However, a tradeoff emerges when financial stability concerns are taken into account.

To highlight this tradeoff, consider first an extreme case where $\rho^N < \underline{\rho}$. That is, suppose non-movers receive a lower return on funds held outside the banking system than all movers, and suppose further that policymakers were to set ρ^C strictly between ρ^N and $\underline{\rho}$. Then movers who withdraw at $t = 1$ will never convert to CBDC, while non-movers who withdraw will always use CBDC instead of their own storage technology. *Any* flow of funds into CBDC would then indicate that a run is underway and would identify the depositors' bank as weak. Formally, $\rho^C < \underline{\rho}$ implies $F\left(\rho^C\right) = 0$, which by equation (18) implies $\theta = 0$ for all α. In this extreme case, a run would be detected before any measurable fraction of depositors withdraw. The fact that weak banks would be immediately placed in resolution, in turn, implies the payment $\hat{x}_2$ received by non-movers at $t = 2$ will be relatively large. In other words, quick resolution minimizes the incentive for non-movers to run in the first place. In this way, designing a CBDC so that it is never used in normal times allows it to be an effective financial stability tool.

Now suppose policymakers increase ρ^C above $\underline{\rho}$. Doing so will give some movers a more efficient way of transferring consumption across locations. As described above, the banking contract will then adjust in a way that benefits all depositors. However, the fact that movers now use CBDC in normal times implies that it takes longer for policymakers to infer when a run is underway. Formally, equation (18) shows that, for any $\alpha > 0$, $\theta\left(\alpha, \rho^C\right)$ is strictly increasing in ρ^C on $\left(\underline{\rho}, \bar{\rho}\right)$. The more non-movers use a CBDC, the more withdrawals that occur before policymakers are able to identify weak banks. This delay in resolving weak banks implies lower payments $\hat{x}_2$ to non-movers who wait until $t = 2$ to withdraw, which increases their incentive to join the run. If ρ^C is set to $\bar{\rho}$, so that all movers choose to use CBDC, equation (18) shows $\theta\left(\alpha, \rho^c\right) = \pi$ for all α, meaning the information effect completely disappears.

Figure 4 shows how increasing the attractiveness of the CBDC affects financial stability. As ρ^C increases, the set of values of the loss σ for which the no-run equilibrium exists becomes smaller and the set for which the run equilibrium exists becomes larger. In addition, as the information effect becomes less important, the strategic complementarity in non-movers' withdrawal decisions returns. When ρ^C is small, there is a range of values for σ where the unique equilibrium is in mixed strategies and a partial run occurs, as in panel (b) of Figure 3. For larger values of ρ^C, however, multiple equilibria reappear for some values of σ.

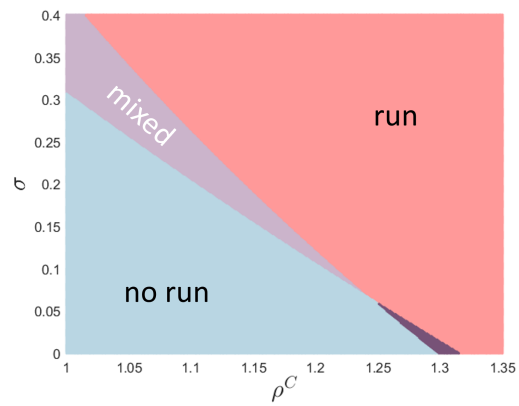


Figure 4: Fragility as ρ^C varies
Source: Authors' analysis

These results illustrate a tension between designing a CBDC to promote efficiency in normal times and designing it to promote financial stability. If digital currency is superior to existing payment technologies in some respects, then efficiency considerations would argue for a design that will be widely used. However, widespread use of the CBDC masks the flows that would identify an incipient run on some banks, slowing the policy response to a crisis. As Figure 4 shows, this muting of the information effect makes a CBDC more likely to increase financial fragility.

This tension might be eased if the CBDC could be designed to have differential appeal to movers and non-movers, or if the central bank could issue two distinct types of CBDC. Suppose, for example, the central bank could introduce one type of CBDC that is redeemable only in a different location from where the deposit was made, and a second type of CBDC that is redeemable only in the same location. If non-movers are unable to travel to another location, then use of each type of CBDC would be restricted to a single type of depositor. In practice, this type of separation between transactions and store-of-value users might be achieved by placing balance limits on the first type of CBDC and perhaps fees on balances not spent in a certain timeframe. The second type of CBDC might have unlimited balances but high transaction fees or perhaps not offer payment services. However the separation is achieved, the optimal policy would be to make the transactions CBDC as attractive as possible, maximizing the value it creates, and to intervene immediately whenever any funds are placed in the store-of-value CBDC, which serves only as a financial stability tool.

It may be difficult, however, to cleanly separate depositors by motive in this way. In our model, if non-movers can choose to move to another location, they would be able to use the first type of CBDC and the policy tradeoff re-emerges. In practice, individuals and firms wishing to hold large CBDC balances may be able to open multiple accounts and automatically move funds across accounts to circumvent balance limits or inactivity fees. More generally, it may be difficult to design a CBDC that is effective as a payments instrument without also being an attractive store of value.[12] To the extent that these two roles cannot be cleanly separated, our analysis highlights an important tradeoff between efficiency and financial stability that policymakers will face in designing a CBDC.

5 Concluding Remarks

The possibility of central banks issuing digital currency has raised fundamental questions about the role of money and its relation to the banking and financial system. Research is beginning to provide insights into some of these questions, but much remains to be learned. One concern that has been raised repeatedly in policy discussions is that the option to hold CBDC may be particularly attractive during times of financial turmoil and, as a result, could increase the likelihood of runs on banks and other financial intermediaries. Some commentators have suggested that this risk can be mitigated by imposing caps, graduated fees, or other restrictions on CBDC holdings during periods of financial stress. Others have even suggested that holding CBDC should be made risky to discourage excessive use.[13] Whether such policies would be credible in times of financial stress remains to be seen. Even if they are credible, however, policies that limit the use or attractiveness of CBDC risk losing many of its potential benefits as well.

Our analysis suggests that other factors may mitigate the financial stability concerns associated with CBDC, without the need to restrict its use. Our model is designed to capture the commonly-expressed concern: all else equal, the ability to hold CBDC increases the incentive for depositors in a weak bank to withdraw. However, the introduction of CBDC creates two other key changes. First, it reduces depositors' need for liquidity insurance and, therefore, leads banks to do less maturity transformation. Second, inflows into the CBDC give policymakers real-time information that can be used to improve the policy response to a crisis. Both of these changes decrease depositors' incentive to withdraw from weak banks.

[12] Keister and Sanches (2022) discuss the possibility of issuing two types of CBDC in different context, where digital currency may compete with physical currency, bank deposits, or both. They also discuss the difficulty of designing a type of CBDC that can only be used for a single purpose.

[13] Birch, D.G.W. "Lost your electronic cash? Tough luck." *Forbes*, March 14, 2022.

In some cases, the net effect of introducing a CBDC in our model is to improve rather than undermine financial stability.

While our results are derived in the context of a particular model, the main lessons seem likely to apply much more broadly. For example, our model assumes the government knows the aggregate state of the economy, including what fraction of banks have experienced losses, from the start. In reality, policymakers have much less detailed information at the onset of a crisis, which increases the scope for them to learn by observing usage of the CBDC. Consider an alternative environment with two aggregate states, one in which the fraction of banks that have experienced a loss is close to zero and another in which this fraction is much larger. In such a setting, learning the aggregate state would help the government take appropriate actions even before learning which individual banks are weak. In this case, observing the *aggregate* flow of funds into the CBDC would help policymakers learn the aggregate state more quickly, even if they cannot observe the flows originating from each bank as we assume here. If policymakers can use this information to impose withdrawal fees or otherwise "bail in" depositors at all banks (as in Keister and Mitkov, 2021), the information gleaned from CBDC usage would decrease financial fragility in much the same way as we have shown here.

More generally, the information effect of CBDC is likely to be relevant in many settings where policymakers' reaction to a run on the financial system is endogenous. Even relatively coarse information about inflows into CBDC can affect policymakers' beliefs about the state of the financial system and about the motivations behind depositors' actions. If changes in these beliefs lead policymakers to act sooner, they will also alter depositors' withdrawal incentives and, therefore, the likelihood of a run on one or more banks. We believe this mechanism will prove important in settings beyond the one we study here and hope our analysis provides a starting point for future work that studies the stability and information effects of CBDC.

Appendix: Proofs

Proposition 1. *With no CBDC, there exists $\underline{\tau} < \bar{\tau}$ such that $\tau > \underline{\tau}$ implies $\alpha^N = 0$ is the unique equilibrium of the withdrawal game for all σ when n is sufficiently small.*

Proof. As a first step, we show that the payoff for the remaining non-movers in a weak bank that has been placed in resolution, $\hat{x}_2^N$, is strictly decreasing in the size of the run, α. Using the utility function in equation (1), the first-order conditions in equation (11) can be written as

$$\mathbb{E}\left[\rho_i^{1-\gamma}\right]\left(\hat{x}_1^N\right)^{-\gamma} = R\left(\hat{x}_2^N\right)^{-\gamma} = \delta\left(\tau - n\hat{b}^N\right)^{-\gamma},$$

where

$$\mathbb{E}\left[\rho_i^{1-\gamma}\right] \equiv \int_{\underline{\rho}}^{\bar{\rho}} \rho_i^{1-\gamma} dF(\rho_i).$$

Combining these conditions with the resource constraint in equation (8) and solving for $\hat{x}_2^N$ yields

$$\hat{x}_2^N = \left(\frac{R}{A}\right)^{\frac{1}{\gamma}}\left(1 - \sigma - \theta x_1^* + \frac{\tau}{n}\right), \tag{22}$$

where

$$A \equiv \left((1-\theta)\left(\hat{\pi}\mathbb{E}\left[\rho_i^{1-\gamma}\right]^{\frac{1}{\gamma}} + (1-\hat{\pi})R^{\frac{1-\gamma}{\gamma}}\right) + \frac{1}{n}\delta^{\frac{1}{\gamma}}\right)^{\gamma}. \tag{23}$$

Note that A depends on the size of the run α through the fraction of remaining depositors who are movers, $\hat{\pi}$. Equation (9) shows that $\hat{\pi}$ is strictly increasing in α. Because $\rho_i \leq R$ for all i, with strict inequality for some i, and $\gamma > 1$, A is strictly increasing in $\hat{\pi}$ and, therefore, in α. It then follows from equation (22) that $\hat{x}_2^N$ is strictly decreasing in α.

To establish that $\alpha^N = 0$ is the unique equilibrium of the withdrawal game, therefore, it is sufficient to show that a non-mover will strictly prefer to wait to withdraw even when all other non-movers attempt to withdraw early, that is,

$$\hat{x}_2^N(\alpha = 1) > \rho^N x_1^*.$$

Using equations (22) and (23) together with the fact that $\hat{\pi}(1) = \pi$, we can write this condition as

$$\frac{R^{\frac{1}{\gamma}}}{(1-\pi)\left(\pi\mathbb{E}\left[\rho_i^{1-\gamma}\right]^{\frac{1}{\gamma}} + (1-\pi)R^{\frac{1-\gamma}{\gamma}}\right) + \frac{1}{n}\delta^{\frac{1}{\gamma}}}\left(1 - \sigma - \pi x_1^* + \frac{\tau}{n}\right) > \rho^N x_1^*.$$

The left-hand side of this inequality is strictly decreasing in σ, so it suffices for the inequality to hold at the maximum loss, $\sigma = 1 - \pi x_1^*$. We can write the resulting inequality as

$$R^{\frac{1}{\gamma}} \left(\frac{\tau}{n}\right) > \left((1-\pi) \left(\pi \mathbb{E} \left[\rho_i^{1-\gamma}\right]^{\frac{1}{\gamma}} + (1-\pi) R^{\frac{1-\gamma}{\gamma}} \right) + \frac{1}{n} \delta^{\frac{1}{\gamma}} \right) \rho^N x_1^*$$

or

$$R^{\frac{1}{\gamma}} \tau > \left(n(1-\pi) \left(\pi \mathbb{E} \left[\rho_i^{1-\gamma}\right]^{\frac{1}{\gamma}} + (1-\pi) R^{\frac{1-\gamma}{\gamma}} \right) + \delta^{\frac{1}{\gamma}} \right) \rho^N x_1^*$$

This inequality will hold for some $n > 0$ if and only if

$$\tau > \left(\frac{\delta}{R}\right)^{\frac{1}{\gamma}} \rho^N x_1^* \quad \equiv \underline{\tau}. \tag{24}$$

To show that this $\underline{\tau}$ lies below the upper bound $\bar{\tau}$, note that using the first-order conditions in equation (4) and the utility function in equation (1), we can write the incentive compatibility condition in equation (5) as

$$\rho^N < \bar{\rho}^N \equiv \frac{x_2^*}{x_1^*} = \left(\frac{R}{E\left[\rho_1^{1-\gamma}\right]} \right)^{\frac{1}{\gamma}}.$$

Combining this inequality with the definition of $\underline{\tau}$ in equation (24) yields

$$\begin{aligned} \underline{\tau} &< \left(\frac{\delta}{R}\right)^{\frac{1}{\gamma}} \left(\frac{R}{\mathbb{E}\left[\rho_i^{1-\gamma}\right]} \right)^{\frac{1}{\gamma}} x_1^* \\ &= \left(\frac{\delta}{\mathbb{E}\left[\rho_i^{1-\gamma}\right]} \right)^{\frac{1}{\gamma}} x_1^* = \bar{\tau}, \end{aligned} \tag{25}$$

where the last equality uses the definition of the the upper bound on fiscal capacity in equation (6) together with the utility function in equation (1). □

Proposition 2. *With no CBDC, there exists $\underline{\rho}^N < \bar{\rho}^N$ such that $\rho^N > \underline{\rho}^N$ implies $\alpha^N = 1$ is the unique equilibrium of the withdrawal game when σ and n are sufficiently large.*

Proof. The proof of Proposition 1 establishes $\hat{x}_2^N$ is strictly decreasing in α. To show $\alpha^N = 1$ is the unique equilibrium of the withdrawal game, therefore, it suffices to show that a non-mover will prefer to withdraw early even if no other non-movers run, that is,

$$\hat{x}_2^N(\alpha = 0) < \rho^N x_1^*.$$

Using equations (22) and (23), together with $\hat{\pi}(0) = 0$ from equation (9), we can write this inequality as

$$\frac{R^{\frac{1}{\gamma}}}{(1-\pi)R^{\frac{1-\gamma}{\gamma}}+\frac{1}{n}\delta^{\frac{1}{\gamma}}}\left(1-\sigma-\pi x_1^*+\frac{\tau}{n}\right)<\rho^N x_1^*.$$

If (n,σ) are sufficiently close to their maximal values of $n=1$ and $\sigma=1-\pi x_1^*$, this inequality is satisfied whenever

$$R^{\frac{1}{\gamma}}\tau<\left((1-\pi)R^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}\right)\rho^N x_1^*.$$

Using the upper bound on fiscal capacity $\bar{\tau}$ in equation (25), the previous inequality is satisfied if

$$R^{\frac{1}{\gamma}}\left(\frac{\delta}{\mathbb{E}\left[\rho_i^{1-\gamma}\right]}\right)^{\frac{1}{\gamma}}x_1^*<\left((1-\pi)R^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}\right)\rho^N x_1^*$$

or

$$\rho^N>\underbrace{\left(\frac{R}{\mathbb{E}\left[\rho_i^{1-\gamma}\right]}\right)^{\frac{1}{\gamma}}}_{=\bar{\rho}^N}\underbrace{\frac{\delta^{\frac{1}{\gamma}}}{(1-\pi)R^{\frac{1-\gamma}{\gamma}}+\delta^{\frac{1}{\gamma}}}}_{<1}\equiv\underline{\rho}^N. \tag{26}$$

Equation (5) shows the first term in the middle expression is equal to $\bar{\rho}^N$, while the second term is clearly smaller than 1. It follows immediately that $\underline{\rho}^N<\bar{\rho}^N$, which establishes the result. □

Proposition 3. *When CBDC is introduced, $x_1^*\left(\rho^C\right)$ is strictly decreasing and $x_2^*\left(\rho^C\right)$ is strictly increasing for $\rho^C\in\left(\underline{\rho},\bar{\rho}\right)$.*

Proof. Using the utility function in equation (1), the first-order condition in equation (14) can be written as

$$\hat{\mathbb{E}}\left[\rho_i^{1-\gamma}\mid\rho^C\right]x_1^{-\gamma}=Rx_2^{-\gamma}$$

where

$$\hat{\mathbb{E}}\left[\rho_i^{1-\gamma}\mid\rho^C\right]\equiv\left(\rho^C\right)^{1-\gamma}F^C\left(\rho^C;\rho^C\right)+\int_{\rho^C}^{\bar{\rho}}\rho_i^{1-\gamma}dF(\rho_i;\rho^C)\text{ for }\underline{\rho}\leq\rho\leq\bar{\rho}. \tag{27}$$

It is straightforward to show that this expression is strictly increasing in ρ^C on $\left(\underline{\rho},\bar{\rho}\right)$. Combining this condition with the resource constraint in equation (3) and solving yields

$$x_1^*=\left(\frac{\hat{\mathbb{E}}\left[\rho_i^{1-\gamma}\mid\rho^C\right]}{B}\right)^{\frac{1}{\gamma}}\quad\text{and}\quad x_2^*=\left(\frac{R}{B}\right)^{\frac{1}{\gamma}}, \tag{28}$$

where

$$B\equiv\left(\pi\hat{\mathbb{E}}\left[\rho_i^{1-\gamma}\mid\rho^C\right]^{\frac{1}{\gamma}}+(1-\pi)R^{\frac{1-\gamma}{\gamma}}\right)^{\gamma}. \tag{29}$$

The fact that $\gamma > 1$ implies $\hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]$ is strictly decreasing in ρ^C. It is then straightforward to show from equations (28) and (29) that x_1^* is strictly decreasing and x_2^* is strictly increasing in ρ^C over this range, as desired. $\square$

Proposition 4. *$\alpha^I = 0$ is an equilibrium if and only if $\alpha^C = 0$ is an equilibrium.*

Proof. Using equation (17) and focusing on $\rho^C \geq \rho^N$, we have that $\alpha^C = 0$ is an equilibrium if and only if

$$\hat{x}_2^C\left(0, \rho^C\right) \geq \rho^C x_1^*\left(\rho^C\right).$$

From equation (21), $\alpha^I = 0$ is an equilibrium if and only if

$$\hat{x}_2^I\left(0, \rho^C\right) \geq \rho^C x_1^*\left(\rho^C\right).$$

Because the constrained-efficient allocation $x_1^*\left(\rho^C\right)$ is the same under both policy regimes, establishing the result is equivalent to establishing

$$\hat{x}_2^C\left(0, \rho^C\right) = \hat{x}_2^I\left(0, \rho^C\right).$$

The first-order conditions in equation (16), which apply under both regimes C and I, can be written as

$$\hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right](\hat{x}_1)^{-\gamma} = R\,(\hat{x}_2)^{-\gamma} = \delta\left(\tau - n\hat{b}\right)^{-\gamma},$$

where $\hat{\mathbb{E}}[\cdot]$ is as defined in equation (27). Combining these conditions with the resource constraint in equation (8) yields an expression for $\hat{x}_2$ that applies in both regimes,

$$\hat{x}_2 = \left(\frac{R}{A(\theta, \hat{\pi})}\right)^{\frac{1}{\gamma}}\left(1 - \sigma - \theta x_1^*\left(\rho^c\right) + \frac{\tau}{n}\right), \tag{30}$$

where

$$A(\theta, \hat{\pi}) \equiv \left((1-\theta)\left(\hat{\pi}\hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]^{\frac{1}{\gamma}} + (1-\hat{\pi})\,R^{\frac{1-\gamma}{\gamma}}\right) + \frac{1}{n}\delta^{\frac{1}{\gamma}}\right)^{\gamma} \tag{31}$$

and where θ is equal to π in regime C and determined by equation (18) in regime I. In addition, $\hat{\pi}$ is given by equation (9) in regime C and by equation (19) in regime I.

When $\alpha = 0$, equation (18) shows that $\theta = \pi$ holds under both regimes. Equations (9) and (19) then show that $\hat{\pi} = 0$ holds in both regimes, which implies that the constant A defined in equation (31) takes the same value in both regimes. Given that both A and θ take the same value under both regimes, equation (30) shows that $\hat{x}_2$ takes the same value under both regimes as well, which establishes the result. $\square$

Proposition 5. *The set of (σ, n) for which $\alpha^I = 1$ is an equilibrium is strictly smaller than the set for which $\alpha^C = 1$ is an equilibrium.*

Proof. Using equation (17) with $\rho^C \geq \rho^N$, we have that $\alpha^C = 1$ is an equilibrium if and only if

$$\hat{x}_2^C\left(1, \rho^C\right) \leq \rho^C x_1^*\left(\rho^C\right), \tag{32}$$

while, from equation (21), $\alpha^I = 1$ is an equilibrium if and only if

$$\hat{x}_2^I\left(1, \rho^C\right) \leq \rho^C x_1^*\left(\rho^C\right). \tag{33}$$

To establish the result, it suffices to show that

$$\hat{x}_2^C\left(1, \rho^C\right) < \hat{x}_2^I\left(1, \rho^C\right). \tag{34}$$

This inequality is sufficient to establish the proposition because it implies (i) if (σ, n) is such that equation (33) is satisfied, equation (32) is also satisfied, and (ii) there exist (σ, n) such that equation (32) is satisfied but equation (33) is not.

When $\alpha = 1$, equations (9) and (19) show that $\hat{\pi} = \pi$ holds under both policy regimes. Using equations (30) - (31), we can then write $\hat{x}_2$ in both regimes as

$$\hat{x}_2 = R^{\frac{1}{\gamma}} \frac{1 - \sigma - \theta x_1^*\left(\rho^c\right) + \tau/n}{(1-\theta)B^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}/n}, \tag{35}$$

where B is given by equation (29). The difference between the two regimes comes from the value of θ, which is equal to π under regime C but is strictly less than π under regime I when $\alpha = 1$, as shown by equation (18). We can, therefore, establish the result by showing that the expression for $\hat{x}_2$ in equation (35) is strictly decreasing in θ on $(0, \pi)$.

Differentiating equation (35) with respect to θ yields

$$\frac{d\hat{x}_2}{d\theta} = R^{\frac{1}{\gamma}} \frac{B^{\frac{1}{\gamma}}\left(1 - \sigma - x_1^*\left(\rho^C\right)\right) + \left(\frac{1}{n}\right)\left(B^{\frac{1}{\gamma}}\tau - \delta^{\frac{1}{\gamma}} x_1^*\left(\rho^C\right)\right)}{\left((1-\theta)B^{\frac{1}{\gamma}} + \delta^{\frac{1}{\gamma}}/n\right)^2}.$$

We will show that this derivative is strictly negative. Because $x_1^*\left(\rho^C\right) > 1$ holds for all $\rho^C \in \left[\underline{\rho}, \bar{\rho}\right]$, the first term in the numerator is clearly negative. Therefore, it suffices to show that the second terms in the numerator is also negative, or

$$B^{\frac{1}{\gamma}}\tau < \delta^{\frac{1}{\gamma}} x_1^*\left(\rho^C\right). \tag{36}$$

Using equation (6) and the utility function in equation (1), the upper bound on the government's fiscal capacity, $\tau \leq \bar{\tau}$, can be written as

$$\delta \tau^{-\gamma} \geq \hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right] \left(x_1^* \left(\rho^c\right)\right)^{-\gamma},$$

where $\hat{\mathbb{E}}\left[\cdot\right]$ given by equation (27), or as

$$\tau \leq \left(\frac{\delta}{\hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]}\right)^{\frac{1}{\gamma}} x_1^* \left(\rho^c\right).$$

A sufficient condition for inequality (36) to hold, therefore, is

$$B^{\frac{1}{\gamma}} < \hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]^{\frac{1}{\gamma}}$$

or, using the definition of B in equation (29),

$$\pi \hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]^{\frac{1}{\gamma}} + (1-\pi) R^{\frac{1-\gamma}{\gamma}} < \hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right]^{\frac{1}{\gamma}}$$

or

$$R^{1-\gamma} < \hat{\mathbb{E}}\left[\rho_i^{1-\gamma} \mid \rho^C\right] \tag{37}$$

The fact that $\rho^C \leq R$ and that $\rho_i \leq \bar{\rho} \leq R$, with strict inequality for some i, together with $\gamma > 1$, implies that inequality (37) holds, as desired. $\square$

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