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An Agent-based Model for Crisis Liquidity Dynamics

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Abstract

Financial crises are often characterized by sharp reductions in liquidity followed by cascades of falling prices. Researchers are making progress in work to understand the levels of liquidity on a daily basis, but understanding the vulnerability of liquidity to market shocks remains a challenge. We develop an agent-based model with the objective of evaluating the market dynamics that lead the market supply of liquidity to recede during periods of crisis. The model uses a limit-order-book framework to examine the interaction of three types of traditional market agents: liquidity demanders, liquidity suppliers, and market makers. The paper highlights the implications of changes in market makers' ability to provide intermediation services and the heterogeneous decision cycles of liquidity demanders versus liquidity suppliers for crisis-induced illiquidity.

Keywords: Liquidity, agent-based modeling, price impact, limit orderbook, market making
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1 Introduction

Financial crises are often characterized by sharp reductions in liquidity followed by cascades of falling prices. Researchers are making progress in measurement of liquidity day-to-day, but little is understood on the vulnerability of liquidity to crisis-like shocks. Unfortunately, it's difficult to evaluate the vulnerability of markets to the liquidity shocks that accompany such events. Most research on asset liquidity focuses on day-to-day market functioning during noncrisis periods, employing measures based on market statistics such as bid-ask spreads and daily volumes drawn from these typical market periods. But these data provide only limited insight into large liquidations during periods of sharp price declines and related fire-sale dynamics.

Modeling liquidity during market crises is difficult because of the complex, nonlinear dynamics of market participants interacting. As a result, measuring the normal relatively small transactions does not give insight into the impact of larger ones during crisis. In addition, the infrequent and sporadic natures of liquidity declines make for limited knowledge of what defines a large order and where the limits of liquidity supplier lie.

Rather than extrapolating from noncrisis to crisis periods, we propose a model in this paper meant to incorporate market participant dynamics into the liquidity assessment process. Using a similar technique as Kyle's (1985) seminal paper, we build a continuous market model using an agent-based modeling frame work to test the concerns of liquidity shocks. We specifically examine two important characteristics of liquidity specific to crises periods that cannot typically be assessed through statistical measures of spread, depth, or resiliency.

The first is the effect of the crisis on the balance sheet of market makers, reducing their ability to take on inventory. Notably, market making issues can be seen on both liquid exchanges and less liquid over-the-counter markets. In electronic exchanges, like those of equity, options, and futures, automated algorithmic trading systems have come to dominate the liquidity supply, acting as market makers in many ways but having much tighter balance sheets and no requirements on making markets in illiquid periods. In many of the less liquid over-the-counter markets, regulatory changes have required bank/dealers to maintain a larger balance sheet, which reduces the incentive for market making by limiting their ability to trade on their own account against the market making flows. The second is the heterogeneous decision cycles among those in the market, specifically the difference in timeframes between the liquidity demanders that require immediacy and the liquidity suppliers that continue to have a longer-term decision cycle. The 1987 crash and the failure of Long Term Capital Management (Bookstaber, 2007) showed the effect of the difference in the operating speed of liquidity demanders and suppliers.

The remainder of the paper is organized as follows. Section 2 discusses issues underlying liquidity measurement during crises. Section 3 presents the model. Section 4 presents model validation. Section 5 presents the liquidity features of interest that the model produces and the implications of the model for liquidity measurement during crisis periods. Section 6 concludes and discusses the use of the model in the OFR's larger agent-based model project.

2 Background

The literature on market liquidity has generally focused on daily liquidity during normal periods of equity markets and shown that a relationship can be drawn between price impact and transactions volume. Much of the literature suggests that price impact follows stable proportionality rules, market impact being proportional to the square root (Gabi et al., 2006, Bouchaud et al., 2008, Hasbrouck and Seppi, 2001, and Toth et al., 2011) or cube-root (Kyle and Obizhaeva, 2011a, 2011b) of transaction size.

Stable, proportional relationships also broadly underlie the literature on liquidity measures ranging from standard measures such as turnover and bid-ask spreads to more sophisticated measures that seek to address relationships and common factors of liquidity across markets (Chordia et al., 2000, Karolyi et al., 2012, Huberman and Halka 2001). Gabrielsen (2011) classifies many of these liquidity measures: volume-based such as the turnover ratio; price variability-related, such as variance ratio; or transaction costs-related, such as bid-ask spread.

Much of this work and discussion derives from Kyle's (1985) seminal paper, which builds a dynamic model of trade with a sequential auction model that resembles a continuous market. He uses three agent types: a random noise trader, a risk neutral insider, and a competitive risk natural market maker. By doing so he is able to create a market model allowing questions about liquidity and information to be tested.

The reason it is essential to model these events rather than just measuring them is not a lack of crisis samples, but as Kyle shows, market participant decisions are necessary to capture the dynamics of pricing. Incorporating market participant behaviors is at the core to addressing the nonlinearities, endogeneity, and feedback that are manifest in market pricing during market events.

The importance of this is stressed by Duffie (2010), who presents a model highlighting the impact of inattentive investors with particular interest on its implications for the 2007-09 financial crisis. Duffie suggests that for some markets it can take a week for enough supply to arrive sufficient to support the market. The difference in timing represents a heterogeneous decision cycle of investors, which influences price because only a subset of investors actively participate in providing a pricing signal to the market. As he shows, this will have implications for price dynamics even during typical day-to-day market activity. But it will have particularly adverse effects during times of stress.

For the market participants, sudden price dislocation can lead to forced selling because of margin calls, redemptions, and other pressures.[1] Such liquidity demanders exhibit greater immediacy. That is, the time frame of the liquidity demanders drops, and there is more of a focus on the speed of liquidation than on price. On the other hand, those in a position to supply liquidity do not face such pressures and continue to be price sensitive, and more critically, do not share the same short-term focus on trading. Many liquidity suppliers have a longer decision cycle and are not even monitoring the markets with the intent to do trades with that frequency.

The impacts that inventory constraints can have compounds the difference between the speed of the liquidity demanders and liquidity suppliers. For example, during the 1987 stock market crash, asset managers using portfolio insurance programs set off a flurry of selling in equity futures. As a result, specialists in the cash equity market, who were trying to sell through program trades but could not find buyers, had a rapid growth in inventory, overwhelming their capital. Investors with ready cash did not quickly enter the market to take advantage of the rapidly falling prices. This inaction led to further price drops and triggered even more portfolio insurance selling (Bookstaber, 2007). [2]

[1]Margin calls and forced sales are the most broadly treated motivation for a demand for immediacy in the face of a market drop, but several papers show other pathways for a drop in liquidity with a decline in asset prices. Morris and Shin (2003) show gaps in liquidity can emerge when traders hit price limits, with the effect of one investor's actions then feeding back to affect the decision of other investors. Vayanos (2004) argues for a similar dynamic through the path of anticipated mutual fund redemptions. Investors redeem from mutual funds when asset prices and fund performance drops to a low enough level, so when the mutual fund is close to a point where redemptions will start to occur, it will take actions to increase fund liquidity. Kyle and Xiong (2001) show how a decline in prices can lead to liquidations because of decreasing absolute risk aversion.

[2]Market breakdowns after the collapse of Long Term Capital Management in 1998 (Lowenstein, 2001) and during

The fragility of the market to liquidity events, and the capacity of liquidity to pass as an emergent phenomenon from the orderly day-to-day, is also manifest in two recent market breaks that occurred in a far shorter time frame, and without any crisis pushing them over the cliff. During the Flash Crash of May 2010, the time disintermediation spanned fractions of a second as market limit orders overwhelmed the depth of the order book more quickly than the liquidity suppliers in this case, mostly high-frequency traders with microsecond reaction speed could act to infuse further liquidity into the market (Kirilenko et al., 2011, CFTC & SEC, 2010). More recently, in October 2014, the U.S. Treasury market, one of the most liquid markets in the world, dropped more than 30 basis points in one hour. The key points we must understand if we are to successfully model the dynamics of liquidity during market crises include the nature of the demanders and suppliers: What are their decision cycles? How much are these affected by market dislocations? How critical is the market under stress to their portfolio adjustments? Of the market makers, what is their capacity for taking on inventory? How long are they willing to hold these positions? What is the cycle of feedback for how are these affected by the market dislocations and how they in turn further affect funding, leverage, and balance sheet?

3 The Model

Agent-Based Models (ABM) offer the ability to incorporate key features of the real-world financial system that are often abstracted away in standard economic analysis, namely that agents are heterogeneous, operate on heuristics, act in ways that alter the environment, and interact and affect one another's actions (LeBaron, 2001). Additionally, they are of particular relevance in modeling financial crisis event are presented in Farmer and Geanokoplos (2009), and Bookstaber (2012) with specific focus on assessing vulnerabilities.

In a typical ABM of a financial market, the market participants are agents, the market mechanism is the topology and the exogenous flow of information that is relevant to the market is the environment. As complex a behavior as pricing is, the ability to directly correlate its macro-level

the 2008 financial crisis have similar story lines. Large declines in asset prices were exacerbated by sales from levered investors (and borrowers), overwhelming the balance sheets and capacity of traditional market makers. The result was accelerating price declines, more deleveraging, and in the worst cases, a breakdown of market functioning. These breakdowns can occur based on common factors as well as the assets themselves, as demonstrated by the rapid drying up of liquidity and price declines due to rapid forced selling by a group of leveraged equity funds that were following similar factor-based strategies in 2007 (Khandani and Lo, 2011).

behavior with the changes in the underlying micro level behavior and parameters diminishes. This requires that the micro-level behavior and parameters be more fully examined and defined to understand their relationship to the macro outcomes. An example of this was the Nasdaq Market Simulation done by the Bios group in 2001 to test the regulatory changes that came with decimalization and how it would impact market function (Darley and Outkin, 2007).

Traditional economic approaches in building agents rely on the role of intelligent (rational) agents that seek to maximize a utility function, as Kyle's model does with the insider trader observing private information about liquidity and previous prices. Though these agents may have private information, the ability of a single agent to estimate this and continue to exist as a monopolist in this fashion is unlikely. Furthermore, rational economic theory relies on the abstraction of equilibrium methods to solve these problems, which are rarely easily observed in financial markets (Farmer and Foley, 2009).

Zero-intelligence agent descriptions, which do not rely on utility functions, have been used to represent aggregate market participant behavior by stochastically sampling empirically generated distributions to characterize the agent decisions. First introduced by Gode and Sunder (1993) in a double auction market, they found that the allocation efficiency of the market derived larger from its structure and was indicative of trader's motivation, intelligence, or learning. This methodology has come to dominate the ABM limit order book literature with several expansions of the models and found to explain the majority of general dynamics like spread variance and price diffusion rates.[3]

3.1 Agents

Our model takes as a starting point Paddrik, et al. (2015), which models trading of various classes of agents over a heterogeneous decision cycles and combines it with Kyle's model (1985). We do so by creating three classes of agents with zero intelligence characteristics that are risk neutral, consistent with Kyle's model setup. A Liquidity Demanding agent type is similar to Kyle's noise trader with its placement of market orders on either side of the market and going often to market with the purpose to transact. A market maker agent type is competitive, similar to Kyle's market maker, by taking the position of straddling both sides of the market, taking long and short

[3] Maslov, 2000; Challet and Stinchombe, 2001; Farmer et al., 2005; Preis et al., 2006.

positions on an asset and placing their best bid and ask one price tick apart. The market maker does this by creating a uniformly distributed book of contracts it is willing to buy or sell at each price point away form the best bid-ask. A Liquidity Supplying agent type has a private view on price, similar to the insider trader, but is a passive order supplier and such that they participates when price moves away with best bid-ask.

Each of these three types of agents has decisions determined by four variables:

1. Order Arrival Rate (ρ): A new order is placed by an agent by an arrival process that follows a Poisson distribution with a mean observed from the empirical order submission rate of a class.[4]

2. Order Size (δ): An order's size is randomly selected based on the empirical distribution of order sizes of a class. In the case of the theoretical agents, liquidity demanders and liquidity supplier have an inverse log normal distribution. The market makers have set target order sizes that they continually try to keep at the target size.

3. Order Placement: An order's price is randomly selected based on the empirical distribution of orders placed based of the distance from the current best bid and best ask of a class. In the case of the theoretical agents, liquidity demanders have an inverse log normal probability distribution, such that the probability of placement is closer to the best bid-ask. The liquidity suppliers have a log normal probability distribution, such that the probability of placement is further from the best bid-ask. The market makers have a uniform distribution of order placement.

[Figure 1]

4. Order Duration: The length of time an order stays in the book that follows a Poisson distribution with a mean observed from the empirical order submission rate of a class.

Additionally we presume market makers and liquidity demanders have one additional characteristic each to fully characterize the agents in the definitions stated above, which requires using

[4] Agents only manage a single order at a time. If an agent has an old order still in the order book at the time it is scheduled to place a new order, it cancels the old order before adding the new order to the market.

features from the near-zero intelligence literature (Paddrik et al., 2012, 2014; Wah and Wellman, 2013). These characteristics include which side of the book to place an order (i.e. place a buy or sell order) and whether the agents have limits on their inventory. These characteristics are partially connected because the probability of a buy or sell order depends on how close an agent is to a position limit.

[Figure 2]

Market Maker Position Limit: The decision to give an agent a limit is based on the averages derived from Kirilenko et al. (2011). From these observations market makers all appeared to have some form of risk control built into their behavior that limited the positions (I_{max}) that they took throughout the day. A governing algorithm is placed on top of the uniformly distribution order book of each market maker so as its positions (I) get closer to its limit, it decreases the amount of contracts its willing to buy or sell. Figure 2 shows the market maker position and related order book histogram demonstrates the behavior of the market maker market demand/supply curve with respect to their position (or market risk).

$$\delta = \left\{ \begin{array}{ll} \text{If I} = 0 & \begin{array}{c} \delta_B \\ \delta_A \end{array} \\ \text{If I} > 0 & \begin{array}{c} \frac{I_{max} - I}{I_{max}} \delta_B \\ \delta_A \end{array} \\ \text{If I} < 0 & \begin{array}{c} \delta_B \\ \frac{I_{max} + I}{I_{max}} \delta_A \end{array} \end{array} \right\} \tag{1}$$

Liquidity Suppliers Supply: liquidity suppliers, in contrast to market makers, look to provide liquidity based on some assessment of the underlying fundamental value of the asset they are buying. The farther the price gets from this value, the larger the supply of liquidity they are willing to provide with respect to bid side if they believe the price of the asset is undervalued, or sell side if overvalued. The increment that they increase supply by is a ratio of the distance the current price is from the fundamental value, where we set the fundamental value is the price at the beginning of the simulations run.

$$\delta = \left\{ \begin{array}{ll} \text{If I} = 0 & \begin{array}{c}\delta_B \\ \delta_A\end{array} \\ \text{If } \dfrac{\Delta P}{\omega} < 0 & \begin{array}{c} -\dfrac{\Delta P}{\omega}\,\delta_B \\ \delta_A \end{array} \\ \text{If } \dfrac{\Delta P}{\omega} > 0 & \begin{array}{c}\delta_B \\ \dfrac{\Delta P}{\omega}\,\delta_A\end{array} \end{array} \right\} \tag{2}$$

We assume that liquidity suppliers participate in the market when they observe asset prices move away from a fundamental value, P_0. The liquidity supplier uses a threshold percentage, ω, to govern the supply they are willing to offer based on how far the current price P_t is from the fundamental price, P_0, Equation 2 uses this relationship to affect the order size, $delta_B$ and δ_S. Figure 3 shows this relationship relative to the order book histogram and demonstrates the behavior of the liquidity supplier market demand/supply curve with respect to current price P_t.

[Figure 3]

Liquidity Demander Order Distribution: The liquidity demanders place market orders and will execute close to the current bid-ask. Because the bid-ask is only for a limited quantity, the actual price of execution may vary from the bid-ask. We draw the distance from the bid-ask for the liquidity demander as an inverse log normal.

3.2 Market Topology

The design of the simulated market follows a traditional price-then-time order book market using a set of agents to insert transactional messages to the order book. This mechanism works by allowing agents every period, to make a decision as to place a buy or sell order, cancel an existing order, or do nothing. The decision as to where to place a new order with in the order book and the size is drawn at random from the distributions of that agent class. Between each agents decision in a period the order book is checked for if a trade should occur due to a buy and sell order crossing. By selecting this topography, asset price creation on the part of the market participants can take place organically through their individual actions (order, cancel), and the market matching engine connecting them (execute).

This format is consistent with a continuous double auction market, a bilateral search process where traders seek partners for mutually beneficial bilateral transactions, traditionally used in financial and commodities markets (Friedman and Rust, 1993). We have implemented a traditional limit order rule set which allows for agents to place market and limit order into the book.

As a starting point, let us consider a graphical representation of three important characteristics of market liquidity: market depth, measured by the range of orders both above and below the trading price of an asset; breadth, measured by the volume of buy and sell order; and resiliency, measured by the change in the flow of orders in response to price changes (Baker, 1996).

[Figure 4]

The order book depth and breadth can be seen as a representation of the demand and supply for contracts in the model. Figure 4 presents the progression of the depth of open interest through time at different price points by the width of the buy and sell order bands, and the breadth by the shading of the bands, where darker blocks of blue and orange represent large amounts of liquidity. The lighter blocks generally represent the interest provided by the market makers at various price points which help prevent small demand shocks from moving price easily. The darker blocks representing the interest of liquidity supplier agents that help prevent medium demand shocks from drastically moving price in any one instance. The resiliency of the market can be seen by the extent the bands maintain similar depth and breadth in the face of price changes.[5]

[Figure 5]

Figure 5 shows the path of the order book when there is alignment of the arrival frequency of liquidity demanders and suppliers and is consistent with the Figure 4, but the liquidity suppliers have a lower frequency of arrival. Comparing the two figures, the drop in the frequency of liquidity supply reduces market depth (the prices above and below the bid-ask), market breadth (the order size at each level), and resilience (the changes in market price needed to generate order flow to fulfill liquidity demand).

[5]The figures follow the methodology of Paddrik et al. (2014) in how to visualize limit order book markets.

4 Model Calibration and Validation

The empirical targets for calibrating and validating a statistical and agent-based model are known as stylized facts (Kaldor, 1961), which reflect empirical statistical moments of a market's pricing series. While many researchers have conducted empirical analyses of financial markets, Mandelbrot (1963) was one of the first to identify that price returns do not necessarily follow a Gaussian random walk. Many have built upon Mandelbrot's foundational work as seen in Cont (2001) which provides a comprehensive review of these empirical analyses that identify several stylized facts of financial markets. These stylized facts have been shown to be robust across the price time series of various markets over varying time intervals. The features traditionally included the distribution of price returns, volatility clustering, absence of autocorrelation of price returns, and aggregation of price returns (Maslov, 2000; Challet and Stinchombe, 2001).

An initial step in formulating the model requires calibrating the parameter set to reflect the market of interest by initially calibrating the model to contain the correct numbers of participants and their stochastic distributional behaviors with rest of the market. This can be done using general descriptive statistics about the traders of a market and the data of aggregate group's number of trades, order, and cancelations. This allows for a check of individual agent behaviors before looking at the aggregate results of the simulated markets behavior.

To fit the model to a set of parameters that are not observable, we use Indirect Inference, a simulation-based method of parameter estimation, to calibrate the arrival rate of the agents so we can fit the stylized facts. It is particularly useful for estimating complex models whose likelihood and moments cannot be found in closed form but can be easily simulated.

[Table 1]

Table 1 shows the parameter set we have selected for the testing of the model. In this example, we have calibrated the model so there is an equal distribution of market depth offered by liquidity suppliers, liquidity demanders, and market making agents. Then we applied the indirect inference method to extract the order arrival rates that displayed the most reasonable fit to the stylized facts. The following subsections describe the stylized facts we use for fitting the model.

4.1 Distribution of Price Returns

It has been widely observed that the empirical distributions of financial returns and log returns are fat-tailed. Mandelbrot (1963) observed that the tails of a distribution of price changes are extraordinarily long and the sample second moment of price typically varies in an erratic fashion. This has caused various suggestions regarding the form of the distribution, ranging from the Student-t, hyperbolic, normal inverse Gaussian, and others, but no general consensus exists for the form of the tails for all markets. In the observed data from the agent-based model, the normality of the distribution of price returns, as seen in Figures 6, illustrates that the data diverge from normality at the tails.

[Figure 6]

4.2 Volatility Clustering

The characteristic of volatility clustering is seen in the absolute price returns for securities that have slowly decaying autocorrelation in variance (i.e. price changes tend to follow other price changes of the same size). This was first noted by Mandelbrot (1963) and was finally translated into agent-based models by Kirman and Teyssiere (2002), when they discovered a model would exhibit autocorrelation patterns in the absolute returns if a variable was herded by the positive or negative opinion of an asset.

[Figure 7]

The model captures this same herding variable through the influence of the market maker and liquidity supplier agent decision to supply more or less liquidity (see Figure 7). This creates a similar autocorrelation pattern in the absolute returns seen in the real minute price returns.

4.3 Absence of Autocorrelation

In validating that markets are efficient, it has been common practice to show there is no predictability in the price returns of assets. To demonstrate this, the autocorrelation of price returns should show that there is no correlation in the time series of returns. Figure 8 illustrate that this property is observable in the simulated market data when the price is near its fundamental value.

[Figure 8]

4.4 Aggregation of Returns

The final stylized fact, aggregation of returns, shows that as one increases the time scale over that one measures price returns, the distribution approaches the Gaussian form. This cross-over phenomenon was noted first by Kullmann et al. (1999), where the evolution of the Pareto exponent of the distribution with the time scale was studied. Kyle and Obizhaeva (2013) show that this distributional change is a result of the aggregation of informational units (orders) such that they will only look the same if the velocity (the rate of orders are submitted to a market) is equivalent.

[Figure 9]

Figure 9 illustrates the standardized distributions of returns the agent-based model, for one minute, five minutes, one hour, and one day increments. As we had hoped, the longer the time scale, the more Gaussian both sets of distributions become, and the observed distributions have the same moments reflecting that the velocity of orders flow in the two markets match informational value.

5 Liquidity Model Dynamics

In examining our model's reaction to liquidity events, it is important that we consider which parameters of the model to shock exogenously. These shocks can come from various places (e.g. changes in agent behavior, changes in topology rules, etc.) and can have very different implications for market participants implications. In the following subsection we will consider different situations where this model framework could be helpful to regulator interested in understanding how market conditions, ecology and policy can influence price impact.

To keep the shock simple for the experiments, we change the liquidity demander's preference in buying or selling from 50/50 to 40/60 and implement this until the number of shares sold minus bought equals equal n, seen in equation 3. As we change this preference, we can look at how this simple dynamic leads complex behavior in the market makers decision to offer short-term liquidity and what the rate of liquidity supplier has to be to soften or worsen the impact of demand shocks.

$$
\begin{aligned}
\Theta &= \sum_{t=0}^{T} \#\text{ of Shares Sold}_t^{LD} - \#\text{ of Shares Bought}_t^{LD} \\
&\text{If } \Theta < n \quad \text{then} \quad \mathrm{P}_t^{LD}(\text{Sell}) = 60\% \\
&\text{If } \Theta \geq n \quad \text{then} \quad \mathrm{P}_t^{LD}(\text{Sell}) = 50\%
\end{aligned} \tag{3}
$$

5.1 Liquidity and Market Impact during a Market Sell Off

In this section, we look at one run of a simulation to illustrate the dynamics of liquidity during a market event. We illustrate this using two visualizations. We first show the progress of liquidity using the limit order book heat map, in Table 2. In this figure, time period A is an example of the order book before a market shock that precipitates the selloff of the asset.

Period B shows the order book after the start of the event, when there is a sudden demand in selling such that n is 1,000. In period C, the liquidity demander sees a lack of response to his price concessions as being an indication that the price level is still too high, rather than understanding that the lack of response is due to a lack of attentiveness or a lag in the ability of the other side of the trade to make a decision. It will lead to a further drop in prices that in both unnecessary and ineffective because those who might be willing to take on the other side of the trade might literally be out to lunch.

[Table 2]

Table 3 provides another visualization of this same simulation run, focusing on the interaction of the liquidity demanders, market makers, and liquidity suppliers over the course of the event. The large box to the left has a cell for each of the 250 liquidity demand agents in this simulation; the circles in the center represent the five market makers, with the size of the internal black circles representing the amount of free inventory capacity; and the box to the right has cells for each of the twenty-five liquidity suppliers. For both the liquidity demanders and liquidity suppliers, the color of each cell shows the amount of order flow in the time period and whether the order is to buy (green) or to sell (red).

[Table 3]

Table 3 accompanies the previous figure on the progression of the order book. This gives a view of the three types of agents for each of the four periods over the course of the liquidity event. The liquidity demanders are the cells in the left-most box, and the color of the cells, like those for the liquidity suppliers in the right-most box indicate the extent of their activity. The capacity of the five market makers is indicated by the extent of fill in the circles.

In Period A, representative of a typical day's order book, there is low-level buying and selling across many of the liquidity demanders. The market makers, which provide liquidity based on the extent its balance sheet allows it to take on inventory, have high capacity. During Period B as the selling by the liquidity demanders heats up, there is a large increase in the flows to the market makers, requiring them to hold inventory for longer periods. The market makers come close to their inventory limits by Period C. In Period C the liquidity suppliers begin to take the other side of the trades, and by Period D the flows have equilibrated and the inventory of the market makers is reduced.

5.2 Models Implication on Price Impact Analysis

The key output of the model is the price impact as a function of the liquidity demand presented in Figure 10. The analysis presented in this is based on many runs of the model where the red line is the mean of the price impact, and the dotted lines in these figures show the 5 percent and 95 percent bands on the price effect across the many runs. Figure 10 shows the market impact is modest and increases linearly as the size of liquidity demand rises, but then hits an inflection point and increases at an increasing rate similar to the cubic form of the measure for liquidity suggested by Kyle and Obizhaeva (2013).

This inflection point, sometimes known as a phase transition, is suggestive of the point where the market is unable to maintain a stable price discovery process, which results in the abrupt loss of either buyers or sellers willing to participate. This is particularly strikingly seen in the lower tail of the distribution of market impact, where the nonlinear effect leads to a drop double that of the mean by 28 percent. This illustrates the difficulties of determining the extent of liquidity effect during a market event. The effect is both nonlinear and subject to large error on the downside.

[Figure 10]

There are two components to determining liquidity demand: the ability of the market makers to take on inventory, and the time frame for the liquidity suppliers to arrive to take on the other side of the trade. The effect of restricting each of these classes of agents is shown in Figure 11. As would be expected, the absence of either a market maker or a liquidity supplier causes an increase in market impact, with the effect becoming increasingly severe as the liquidity demand increases. Less obvious is the effect of the market maker and liquidity supplier on the spread of prices.

[Figure 11]

Table 4 further illustrate the effects of the market makers and liquidity suppliers on market impact, in particular on the standard deviation of prices. We in the top portion of the table a normal period in which the market impact as a function of the number of liquidity suppliers, liquidity demanders, and market makers during periods of low liquidity demand (e.g., approximately 2 percent of average daily volume), and in the lower portion a sharp sell off period of high liquidity demand. For the low volume periods, the number of market makers is the most significant determinant of market impact. However, as we move to periods of high liquidity demand, the number of liquidity suppliers takes on the dominant effect. The importance of the roles of the three types of agents in dictating market impact varies based on the nature of the market dynamic and the market environment at the time of the market event, which agents are under pressure, how concentrated the various agents are to the assets, how the actions of the liquidity demanders bleeds into the ability of the market makers to hold inventory.

[Table 4]

These findings are consistent with a recent study done by Getmansky et al. (2014) who examined the role of short- and long-term traders in liquidity provision during normal times and during crashes in the spot market for stocks of the National Stock Exchange using a unique dataset that has trader identities. They observe that short term traders, similar to our market makers, carried little inventories and put in buy orders when prices declined and sold when prices rose, providing liquidity to the market during normal periods. However, during two flash crash days, the inventories of short-term traders were high beforehand, indicating limited capital capacity and market fragility.

The market recovered because of long-term traders, liquidity suppliers who acted to stabilize the markets during these abrupt liquidity shocks.

However the smooth monotonic relationships of Figure 10 belie the complexity of the effect of the liquidity demand on market impact. To appreciate this, we need to follow the liquidity dynamic through the course of the crisis. Figure 12 looks at market impact over 1,440 periods, which cover the beginning of the demand shock to its end. The market impact has a downward trend over the course of the crisis and also has less variability as time passes. And, as expected, the market impact increases with an increase in the quantity sold, though due to the sampling size, it is not monotonic in this figure. But the figure manifests considerable variability and nonlinearity, with sudden dips and peaks. This is the result of the lumpiness in the arrival of liquidity supply and in the occasional need of the market makers to become liquidity demanders themselves when their inventory is large in the face of a shrinking balance sheet. As would be expected, this is most pronounced for the higher levels of liquidity demand.

[Figure 12]

5.3 Models Implications for Policy Analysis

There is consensus that, at a minimum, trading platforms should be prepared to handle the surges in activity that characterize periods of market distress and ensure speedy execution of trades. The 1987 stock market crash spurred a series of measures aimed at improving trading capacity and order execution (Lindsey and Pecora, 1998). The model we propose here offers channels for the introduction and analysis of several types of policy actions to deal with periods of illiquidity, ranging from incentives and market participant behaviors to market structural design, such as order types and execution methods.

In particular, policy action can target any of the three agent types. Policies to combat illiquidity can seek to reduce the speed and size of liquidity demand, increase the capacity and holding period of the market makers, and increase the speed and size of the liquidity suppliers. The first of these, addressing the liquidity demanders, has taken the form of circuit breakers or a slowing of the cycle of margin calls. The second has taken the form of infusions of funding to allow the broker-dealers to apply a larger balance sheet to their market making activities. Increased funding also reduces the pressure on leveraged investors, possibly stemming mushrooming liquidity demand, and adds more funding for liquidity suppliers to enter and take larger positions. The third, addressing the liquidity suppliers, has taken the form of government policy to step in as a liquidity supplier of last resort, buying up assets when ready liquidity supply from the marketplace is flagging. Additionally. policy measures and incentives maybe useful to increase the willingness of liquidity suppliers to more rapidly enter the market in the face of disruptions well.

The policy response to emerging illiquidity cannot have a one-size-fits-all solution, because liquidity is dynamic and varies in terms of the importance of the three agent types. As an extension to this paper, we plan to examine how different policy responses affect the costs and benefits to market participants.

6 Conclusion

The agent-based model we present here is designed to simulate the dynamics of asset pricing and assess the liquidity of markets during times of crisis. The nature of this agent-based approach allows the model to employ agents designed to represent the varied, heterogeneous institutions in the financial system. In that sense, it is an engineering model, with its starting objective being application.

To get a fuller picture of liquidity during market events, the liquidity model presented in this paper can be incorporated as a component of a broader model, a model that combines the effects of liquidity with those of leverage and forced selling.[6] The forced selling due to leverage constraints has a price impact that reduces liquidity, and that reduction in liquidity then feeds back to induce greater effects on prices from subsequent forced selling. The interactions between the financial impact of leverage constraints and the price impact of reduced liquidity are illustrated in Figure 13.

[6]The interaction of funding and asset liquidity are discussed by Amihud and Mendelson (1988, 1991); Brunnermeier and Pedersen (2009) provide an integrated framework for the two. Bookstaber, et al. (2014) presents an agent-based model for fire sales that projects the dynamics of market shocks on the path of leverage, funding, and capital, tracing the cascades and propagation as the initial shock works its way through the system. That fire sale model represents the market impact of the forced selling by a simple linear relationship. The model presented in this paper is intended to replace that function as a component in the fire-sale model. The result is an interaction between the financial effects of leverage and dwindling capital on the leveraged investors and on the balance sheet of the bank/dealers, and the liquidity demand and market making capacity central to the market impact of the liquidity demand.

[Figure 13]

As Figure 13 shows, market liquidity is intricately linked to the funding and capital structure of the market, and when these become stressed they alter the ability of the markets to provide liquidity, which in turn further fuels the funding components of the process. Indeed, the interaction of asset liquidity and funding leverage are necessary in the fire sale dynamics. With perfect liquidity, the forced selling from leverage will not lead to a price cascade; with no leverage, even an illiquid market will not lead to a fire sale because there will be no forced selling.

This model is centered on the actions of three types of market participants: liquidity demanders, market makers, and liquidity suppliers. The decision cycles of market participants vary between the liquidity providers and suppliers, and the market makers' capacity and willingness to hold inventory is impaired during these periods. Forced liquidation will dictate the immediacy of liquidity demanders, the ability of the market maker to take on inventory will change depending on how the crisis affects it; and the suppliers will be differentially affected from one market to another, depending on the patience they have, and the diversity of their holdings.

Liquidity models that focus on the day-to-day characteristics of bid-ask spreads, trading volume, and price behavior will have the same limitation in assessing liquidity during times of crisis as the risk models, such as VaR (value-at-risk), that depend on historical behavior do in assessing risk during these periods. A model that attempts to project the course of liquidity during a crisis without explicit knowledge of the behavior of these agents and without taking into account the real-time specifics of leverage, balance sheet, portfolio construction, and decision process is likely to fail in practice.

References

Amihud, Y. & H. Mendelson. 1988. "Liquidity and asset prices: Financial management implications." *Financial Management*, 17(1):5-15.

Amihud, Y. & H. Mendelson. 1991. "Liquidity, maturity, and the yields on US Treasury securities." *The Journal of Finance*, 46(4): 1411-1425.

Baker, H. K. 1996. "Trading location and liquidity: An analysis of U.S. dealer and agency markets for common stocks." Cambridge, MA: Blackwell Pubs.

Bookstaber, R. 2007. "A demon of our own design: Markets, hedge funds, and the perils of financial innovation." J. Wiley.

Bookstaber, R. 2012. "Using agent-based models for analyzing threats to financial stability." Office of Financial Research Working Paper, 12-03.

Bookstaber, R., M. Paddrik, & B. Tivnan. 2014. "An agent-based model for financial vulnerability." Office of Financial Research Working Paper, 14-05.

Borio, C. 2004. "Market distress and vanishing liquidity: anatomy and policy options." BIS Working Paper.

Brunnermeier, M. 2009. "Deciphering the liquidity and credit crunch 2007-2008." *Journal of Economic Perspectives*, 23(1): 77-100.

Brunnermeier, M. & L. Pedersen. 2009. "Market liquidity and funding liquidity." *Review of Financial Studies*, 22(6): 2201-2238.

CFTC & SEC. 2010. "Findings regarding the market events of May 6, 2010." September 30, 2010

Challet, D. & R. Stinchcombe. 2001. "Analyzing and modeling 1 + 1d markets." *Physica A: Statistical Mechanics and its Applications*, 300(1): 285-599.

Cont, R. 2001. "Empirical properties of asset returns: stylized facts and statistical issues." *Quantitative Finance*, 1(2): 223-236.

Darley V. & A. Outkin. 2007. “NASDAQ market simulation: insights on a major market from the science of complex adaptive systems.” World Scientific Publishing Co. Inc, River Edge.

Duffie, D. 2010. “Presidential address: Asset price dynamics with slow-moving capital.” *Journal of Finance*, 65(4): 12371267.

Farmer, J., P. Patelli, & I. Zovko. 2005. “The predictive power of zero intelligence in financial markets.” Proceedings of the *National Academy of Sciences of the United States of America*, 102(6): 2254-2259.

Farmer, J. & D. Foley. 2009. “The economy needs agent-based modelling.” *Nature*, 460(7256): 685-686.

Farmer, J. & J. Geanakoplos. 2009. “The virtues and vices of equilibrium and the future of financial economics.” *Complexity*, 14(3): 11-38.

Friedman, D. & J. Rust. 1993. “The Double Auction Market: Institutions, Theories, and Evidence.” Redwood City, WA: Addison-Wesley Co.

Gabrielsen, A., M. Marzo, & P. Zagaglia. 2011. “Measuring market liquidity: An introductory survey.” Working Paper, Universita di Bologna.

Gallant, A. & G. Tauchen. 1996. “Which moments to match?.” *Econometric Theory*, 12(4): 657-681.

Geithner, T. 2014. “Stress test: reflections on financial crises.” Random House.

Getmansky, M., R. Jagannathan, L. Pelizzon, & Schaumburg E. 2014. “Liquidity Provision and Market Fragility.” NSE-NYU Stern Working Paper.

Gigerenzer, G. 2010. “Rationality for Mortals.” Oxford University Press.

Gode, D. & S. Sunder. 1993. “Allocative effciency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality.” *Journal of Political Economy*, 101(1): 119-137.

Gourieroux, C., A. Monfort, & E. Renault. 1993. “Indirect inference.” *Journal of Applied Econometrics*, 8(S): 85-118.

Huberman, G. & D. Halka. 2001. “Systematic liquidity.” *Journal of Financial Research*, 24(2): 161-178.

Kaldor, N. 1961. “Economic growth and capital accumulation.” The Theory of Capital, Macmillan, London.

Kirilenko, A., A. Kyle, M. Samadi, & T. Tuzun. 2011. “The flash crash: The impact of high frequency trading on an electronic market.” Available at SSRN: http://ssrn.com/abstract=1686004.

Kirman, A., & G. Teyssiere. 2002. “Microeconomic models for long memory in the volatility of financial time series.” *Studies in Nonlinear Dynamics Econometrics*, 5(4): 1-23.

Khandani, A. & A. Lo. 2011. “What happened to the quants in August 2007? Evidence from factors and transactions data.” *Journal of Financial Markets*, 14(1): 1-46.

Kullmann, L., J. Tyli, J. Kertesz, A. Kanto, & K. Kaski. 1999. “Characteristic times in stock market indices.” *Physica A: Statistical Mechanics and its Applications*, 269(1): 98-110.

Kyle, A. 1985. “Continuous auctions and insider trading.” *Econometrica*0, 53(6): 1315-1335.

Kyle, A. & A. Obizhaeva. 2011a. “Market microstructure invariants: Empirical evidence from portfolio transitions.” Working Paper, University of Maryland.

Kyle, A. & A. Obizhaeva. 2011b. “Market microstructure invariants: Theory and implications of calibration.” Working Paper, University of Maryland.

Kyle, A. & A. Obizhaeva. 2012. “Large bets and stock market crashes.” Working Paper, University of Maryland.

Kyle, A. & A. Obizhaeva. 2013. “Market microstructure invariance: Theory and empirical tests.” Working Paper, University of Maryland.

Kyle, A. & W. Xiong. 2001. “Contagion as a wealth effect.” *The Journal of Finance*, 56(4): 1401-1440.

LeBaron, B. 2001. “A Builders Guide To Agent-Based Financial Markets.” *Quantitative Finance*, 1(1): 254-261.

Lindsey, R. & A. Pecora. (1998). "Ten years after: regulatory developments in the securities markets since the 1987 market break. *Journal of Financial Services Research*, 13(3): 283-314.

Lowenstein, R. 2001. "When Genius Failed: The Rise and Fall of Long-Term Capital Management." Random House.

Mandelbrot, B. 1963. "The variation of certain speculative prices." *Journal of Business*, 36(4): 394-419.

Maslov, S. 2000. "Simple model of a limit order-driven market." *Physica A: Statistical Mechanics and its Applications*, 278(3): 571-578.

Morris, S. & H. Shin. 2004. "Liquidity black holes." *Review of Finance*, 8(1): 1-18.

Paddrik, M., R. Hayes, A. Todd, S. Yang, W. Scherer, & P. Beling. (2012). "An Agent-based Model of the E-Mini SP 500 and the Flash Crash." In Proceedings of the *IEEE Computational Intelligence for Financial Engineering and Economics*.

Paddrik, M., R. Haynes, A. Todd, P. Beling, & W. Scherer. 2014. "The Role of Visual Analysis in the Regulation of Electronic Order Book Markets. Office of Financial Research Staff Discussion Paper, 2014-02.

Paddrik, M., R. Hayes, P. Beling, & W. Scherer. 2015. "Effects of Limit Order Book Information Level on Market Stability Metrics. *Journal of Economic Interaction and Coordination*, Forthcoming

Preis, T., S. Golke, W. Paul, & J. Schneider. 2006. "Multi-agent-based order book model of financial markets." *Europhysics Letters*, 75(3): 510-516.

Smith, A. 1990. "Three essays on the solution and estimation of dynamic macroeconomic models."

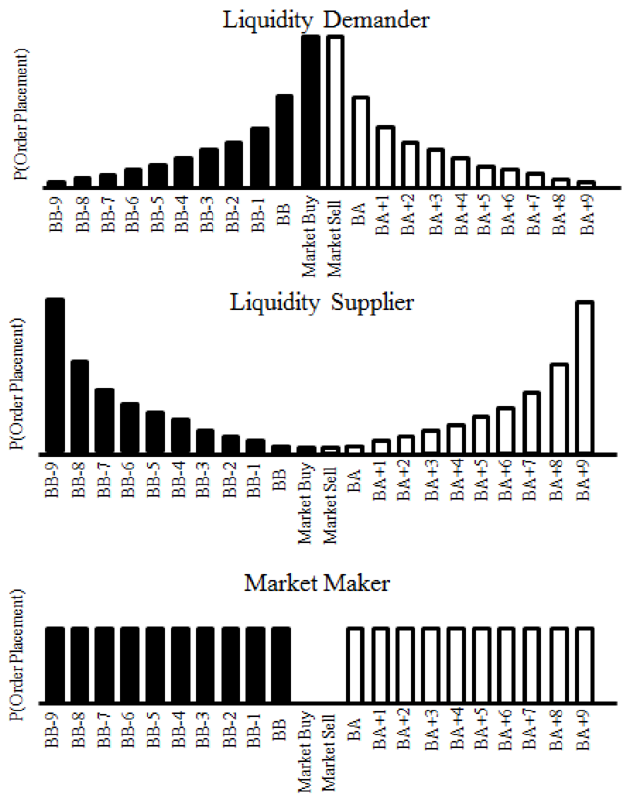
Smith, A. 1993. "Estimating nonlinear timeseries models using simulated vector autoregressions." *Journal of Applied Econometrics*, 8(S): S63-S84.

Tirole, J. 2011. "Illiquidity and all its friends." *Journal of Economic Literature*, 49(2): 287-325.

Vayanos, D. 2004. "Flight to quality, flight to liquidity, and the pricing of risk." Working Paper, National Bureau of Economic Research.

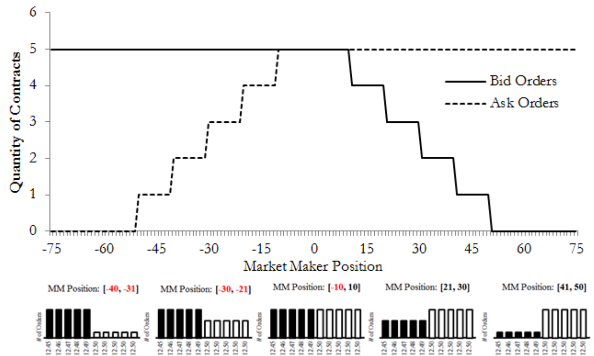
Wah, E., & M. Wellman. (2013). Latency arbitrage, market fragmentation, and efficiency: a two-market model.” In Proceedings of the *14th ACM conference on Electronic Commerce*: 855-872, ACM.

Figure 1: Distribution Likelihood of Order Placement by Agents



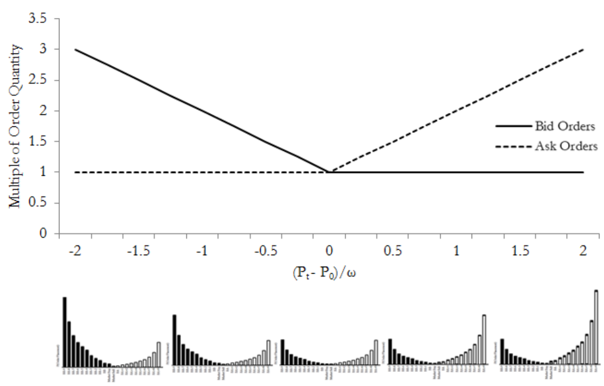
Source: Authors' calculations.

Figure 2: Position Limit and Order Placement for Market Makers



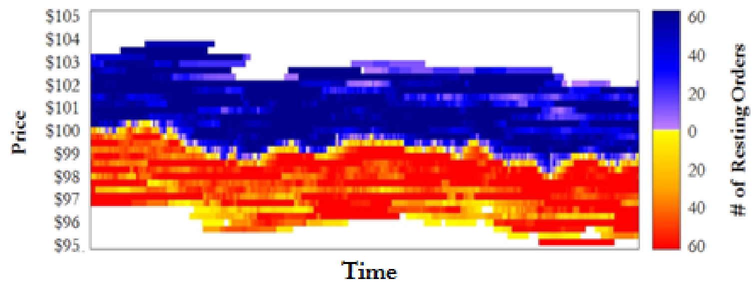
Source: Authors' calculations.

Figure 3: Position Limit and Order Placement for Liquidity Suppliers



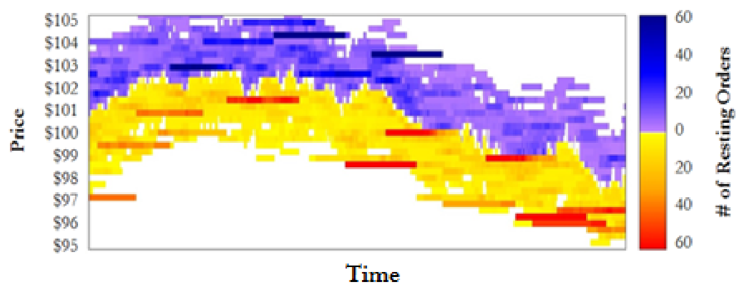
Source: Authors' calculations.

Figure 4: Order Book Depth of Liquid Market



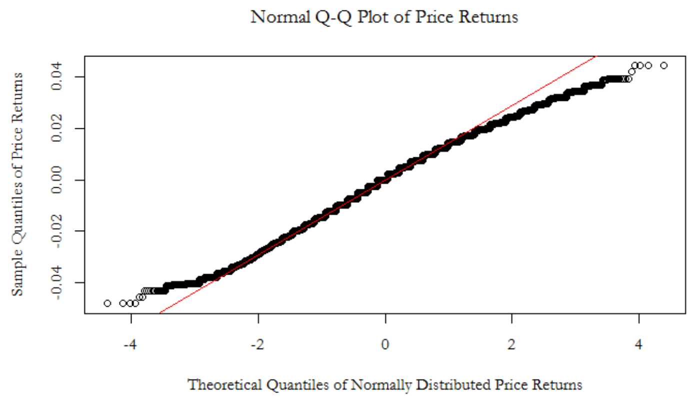
Source: Authors' model.

Figure 5: Order Book Depth of Illiquid Market



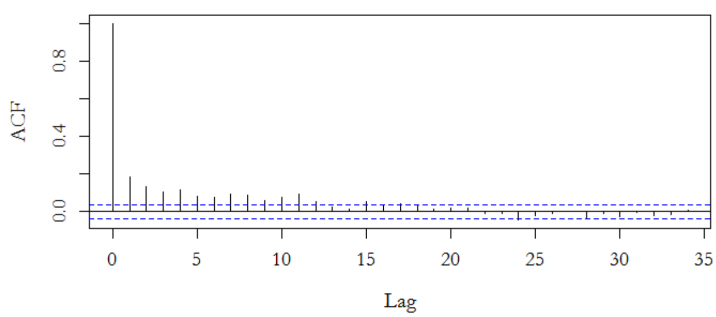
Source: Authors' model.

Figure 6: Normality Test of ABM Price Returns



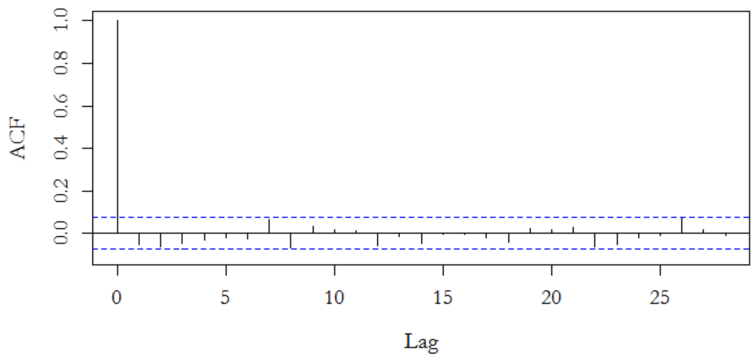
Source: Authors' model.

Figure 7: Volatility Clustering of Absolute Price Returns for the Agent-based Model



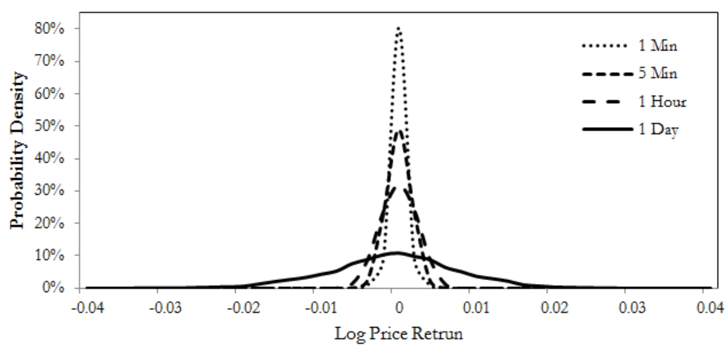
Source: Authors' model.

Figure 8: Absence of Autocorrelation for Price Returns for Agent-based Model



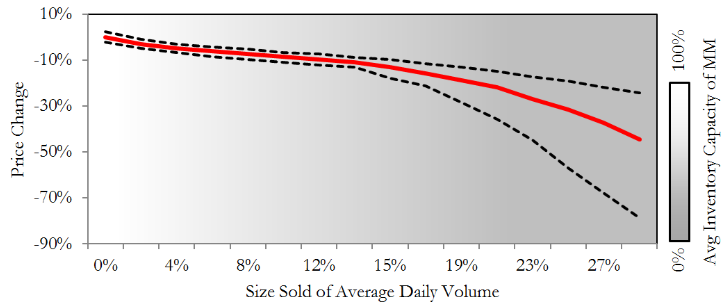
Source: Authors' model.

Figure 9: Aggregation of Price Returns for Agent-based Model



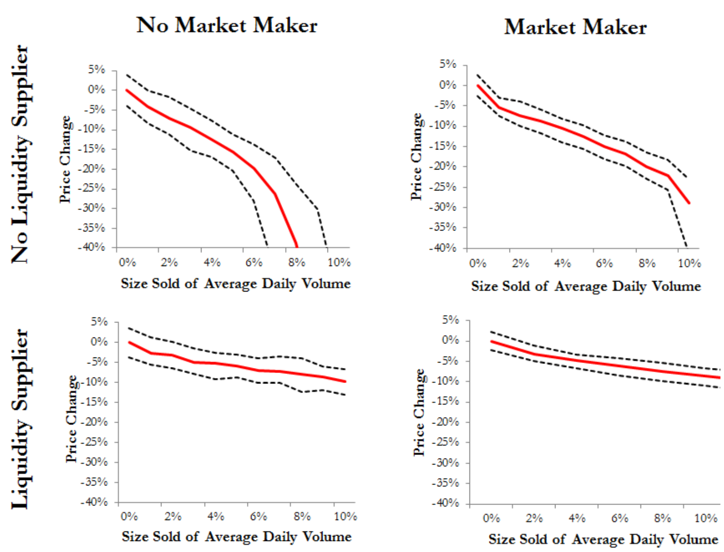
Source: Authors' model.

Figure 10: Price Impact



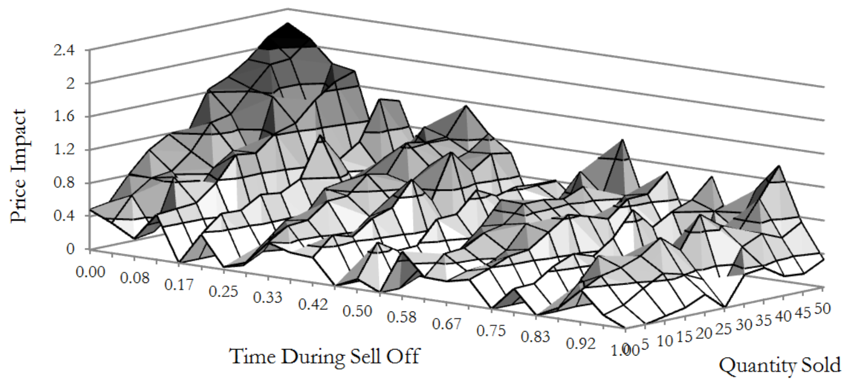
Source: Authors' model.

Figure 11: Market Ecology



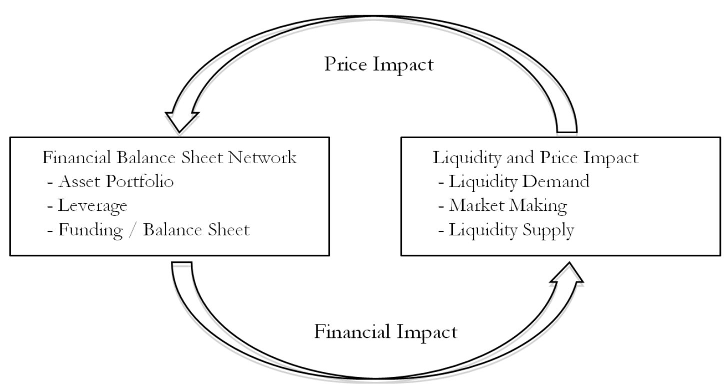
Source: Authors' model.

Figure 12: Market Impact Dynamics



Source: Authors' model.

Figure 13: Feedback cycle of Liquidity



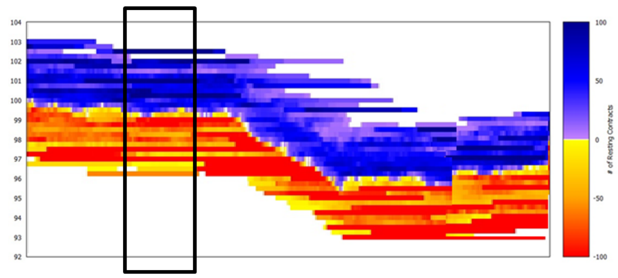
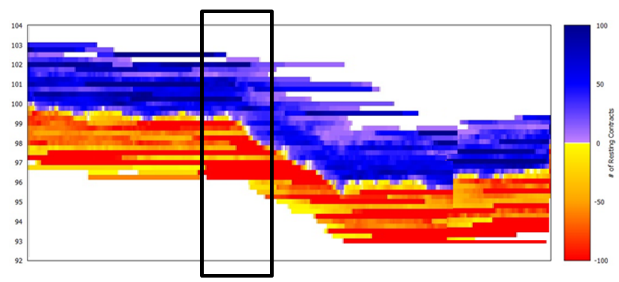
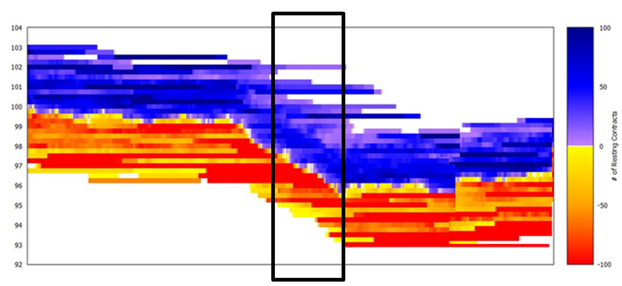
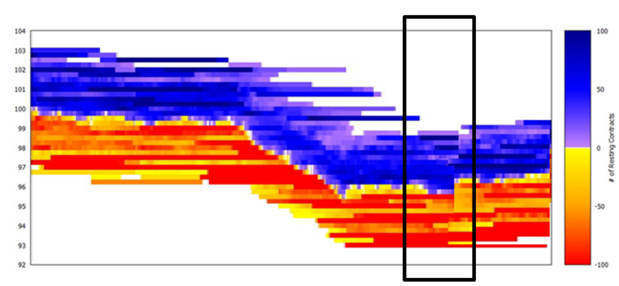
Source: Authors' model.

Table 1: Model Order Flow Statistics

Trader Type	#	Order Arrival	Order Size	Order Placement	Order Duration	Agent Specific
Liquidity Demanders	250	120	1	Poisson ($= 1$)	60	-
Liquidity Suppliers	25	480	10	1 - Poisson ($\lambda = 1$)	240	$\omega = 5$
Market Makers	5	10	5	Uniform	5	$I_{max} = 50$

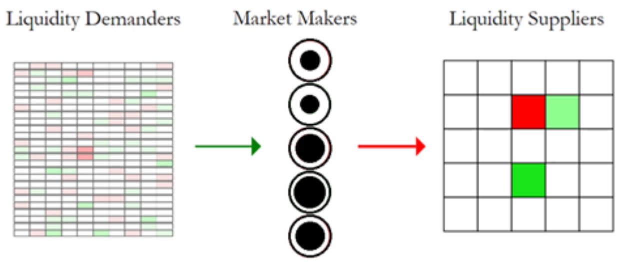
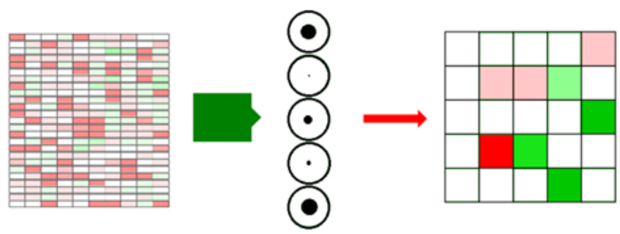
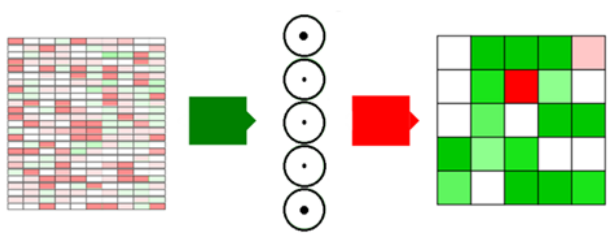
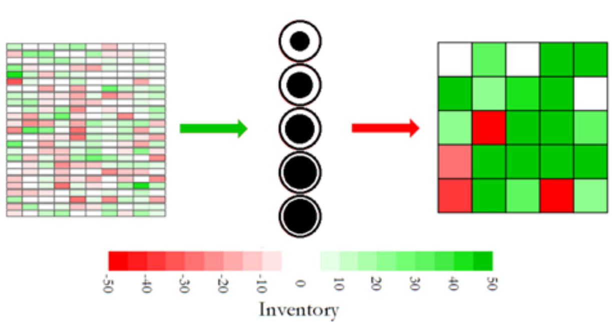
Source: Authors' model input.

Table 2: Liquidity Shock to the Order Book

Order Book Heat Map of Demand Shock	Description
Period A 	The market is in an equilibrium state where the buyer and seller demand offset one another; the depth of supply is also offset by equal amounts in the order book. Price Δ : 0% Volume: 652 Bid-to-Ask Depth Ratio: 1-to-1
Period B 	At the beginning of this period a large quantity of selling order demand begins to sweep through the order book of suppliers. Prices begin to shift and volumes in trade increase sharply. Price Δ : -1.7% Volume: 1931 Bid-to-Ask Depth Ratio: 1-to-1.5
Period C 	During the second half of the price drop, the amount of selling volume decreases, but the amount of price drop doubles showing the difficulty of supply catching up with demand in the market. The depth of market indicates a liquidity shortage in the market. Price Δ : -3.6% Volume: 1028 Bid-to-Ask Depth Ratio: 1-to-3.2
Period D 	The market hits its low as liquidity suppliers begin to support price by creating a wall of supply, preventing price from dropping further. The liquidity supplier's interest also helps to push prices back up marginally. Price Δ : 1% Volume: 869 Bid-to-Ask Depth Ratio: 1.8-to-1

Source: Authors' model.
Note: This figure shows the progression of the order book over the course of a liquidity event, starting before the event, then during two periods in its development, and finally after the event has resolved. This is based on a single run of the agent-based model simulation. This figure is illustrative; to get a better picture of the evolution of the order book requires many runs.

Table 3: Liquidity Shock to the Inventory of Participants

Order Book Heat Map of Demand Shock	Description
Period A 	The market is in an equilibrium state where the buyer and seller demand offset one another. The supply held by market makers is low and the averages of liquidity supplier and demander inventories are close to zero. Price Δ : 0% MM Bid Order Depth: 250 LS Order Depth: 172
Period B 	At the beginning of this period a large quantity of selling order demand begins to sweep through the order book of market makers who marginally start offsetting their positions with willing liquidity suppliers. Price Δ : -1.7% MM Bid Order Depth: 50 LS Order Depth: 156
Period C 	During the second half of the price drop, the amount of selling volume is nearly entirely passed directly to liquidity suppliers. This is due to the market makers having diminished supply capacity. Price Δ : -3.6% MM Bid Order Depth: 5 LS Order Depth: 65
Period D 	The market hits its low as liquidity suppliers begin to support price by creating a wall of supply and absorbing all the demand, preventing price from dropping further. This allows market makers to offset their positions and return to providing liquidity. Price Δ : 1% MM Bid Order Depth: 220 LS Order Depth: 432

Source: Authors' model.

Table 4: Market Ecology Influence

	Normal Market Periods			
	Coefficients	Standard Error	t Stat	P-value
Intercept	0.8993	0.0655	13.7336	4E-31
# Liquidity Suppliers	-0.0027	0.0028	-0.9738	0.3313
# Liquidity Demanders	0.0006**	0.0003	2.2966	0.0226
# Market Makers	-0.0919***	0.0141	-6.5345	4.67E-10
	Sharp Sell Off Periods			
	Coefficients	Standard Error	t Stat	P-value
Intercept	14.9129	2.1299	7.0018	3.28E-11
# Liquidity Suppliers	-0.4298***	0.0915	-4.6962	4.76E-06
# Liquidity Demanders	-0.0441***	0.0092	-4.8207	2.73E-06
# Market Makers	0.2224	0.4576	0.4861	0.6274

* = 90%, ** = 95%, *** = 99% Significance

Source: Authors' model.