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Central Counterparty Default Waterfalls and Systemic Loss

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Abstract

Central counterparty default waterfalls act as last lines of defense in over-the-counter markets by managing and allocating resources to cover defaults of clearing members and clients. However, central counterparties face competing objectives in setting up their default waterfalls. In this paper we evaluate the trade-offs between default waterfall resiliency and central clearing, using a unique and comprehensive dataset containing all U.S. cleared and bilateral credit default swap positions. We evaluate the resiliency of different default waterfall designs, accounting for the interconnectedness of payments in the system, the presence of client clearing obligations for members, and the distribution of losses among market participants.

Keywords: central counterparty, systemic risk, default waterfall, financial networks, credit default swaps

JEL Classification Numbers: G10, G23, G28, L14

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Central counterparty (CCP) clearing in over-the-counter (OTC) financial markets has grown substantially since the 2007-09 financial crisis, from nearly nonexistent in 2007 to more than 70 percent of new interest rate derivatives and index credit default swaps volume in the United States in 2019 (Financial Stability Oversight Council (2019)). Financial regulators have encouraged this growth in cleared products as a way to reduce the financial stability risks posed by large counterparty failures. CCPs help to ensure the continuity of payments within these markets and reduce the potential losses that taxpayers suffer (Financial Stability Board (2017)). They do so by performing risk management and maintaining *default waterfalls*, financial resources that cover losses generated by counterparty default.

Although default waterfalls are critical to a CCP's risk management, there is little consensus on the optimal structure of default waterfalls globally. After the introduction of central clearing to numerous previously non-cleared markets, many new CCPs were created with a tremendous degree of variation in how they source default waterfall resources. Such variations reflect the conflicting objectives that the default waterfall must serve. As a CCP's default waterfall is its last line of defense in times of stress, it is important that the waterfall be resilient against market shocks. But it is also necessary for the waterfall to account for the incentives of participants, as requiring large contributions can be costly for clearing members and discourages clearing through the CCP (Ghamami and Glasserman (2017)). Lower rates of central clearing could in turn decrease financial system resilience.

Assessing the systemic risk implications of CCP default waterfall designs is difficult for financial regulators, CCPs, and market participants alike due to the historical rarity of CCP member defaults and the complex interrelationship of payments within both cleared and non-cleared portions of derivatives markets (Duffie (2015); Cont (2015)).[1] Spillover effects can arise through both the cleared and non-cleared network of exposures and from fire sales of illiquid collateral, causing greater losses in a crisis. As firms see a limited view of the overall market, they face difficulties in accurately evaluating the extent of these spillover effects and determining the cost-benefit of a particular default waterfall structure (Cox and Steigerwald (2017)). Although groups such as the International Swaps and Derivatives Association (ISDA) and standard-setting bodies such as

[1]Kroszner (1999), Cox (2015), and Bignon and Vuillemey (2020) examine in depth a few historical examples of large clearing member default at derivatives CCPs.

the Committee on Payments and Market Infrastructures (CPMI) and International Organization of Securities Commissions (IOSCO) have written reports that qualitatively discuss the merits of different default waterfall designs (Elliott et al. (2014); ISDA (2013); CPMI-IOSCO (2014)), there has been limited theoretical modeling of these mechanisms or empirical testing using market data.

In this paper we evaluate the merits of existing CCP default waterfall designs through a structural modelling approach that can account for these complexities. Unlike the theoretical works of Biais et al. (2012), Amini et al. (2015) and Wang et al. (2020) that have investigated optimal counterparty risk exposures through trade-offs in CCP risk-sharing, we take the network of exposures as given and focus on how the CCP's default waterfall influences financial system loss. We calibrate this model using a unique and comprehensive dataset on U.S. credit default swap (CDS) CCP transactions to assess the resilience of the system against large market shocks. We also assess counterfactual default waterfall structures and determine their impacts on CCP stability and overall financial system resilience.

We examine the impact of the CCP's default waterfall and recovery mechanisms on both total and individual losses suffered from variation margin owed by (and to) the CCP members of a major CDS central counterparty. We also incorporate the hundreds of clients that clear through CCP members, which the members are responsible for in the event of their default, and examine the impact of client defaults on systemic resiliency. client clearing losses have been highly significant historically. For instance, clients were heavily responsible for the default of the CCP Caisse de Liquidation des Affaires en Marchandise in 1974 (Bignon and Vuillemey (2020)). As recently as March 2020, a large client of the CME clearing member ABN Amro defaulted and caused the member an estimated \$200 million in losses (Mourselas and Smith (2020)).

This work makes several contributions to the literature on central clearing and risk sharing. First, we quantify how losses are allocated across a financial payment system. We do so by providing a comprehensive measure of systemic loss and examining how losses are influenced by the scale of market shocks and the liquidity of collateral. We find substantive spillover effects due to network contagion that dramatically elevate losses from large shocks. Though asset fire sales can also intensify the level of systemic losses, in line with Duarte and Eisenbach (2018), we find that current collateral standards are high enough to prevent major liquidation losses.

Second, we analyze how different waterfall structures that vary in the proportion of losses allocated to individual CCP members vs. the shared collective of all CCP members affect the quantity of capital needed to sustain the CCP against market shocks. We compare our results with a unique data collection on the default waterfall designs of more than 60 global derivatives CCPs. This allows us to analyze how global variations in funded resources, including initial margins, CCP capital, and guarantee funds, influence expected losses for market participants and the CCP's resilience under stress. These data provide us with a measure of the preferences of CCPs and clearing members in selecting a default waterfall's resource allocation.

Finally, we consider the CCP members' responses to changes in waterfall structure. As more default waterfall resources are required, clearing members may become more hesitant to participate in central clearing due to the heightened costs. We estimate the resilience of the default waterfall if participation in central clearing is also reduced. We find that the changes in central clearing participation can have a large impact on the resilience provided by requiring more waterfall resources. In the case of more minor market shocks, requiring more waterfall resources will not counterbalance the consequences of decreased central clearing participation. This result highlights the downward pressure on waterfall resources that a CCP faces in periods of market calm, which is in line with the downward trends in CCP waterfall resources in the decade following the 2008 financial crisis.

Previous papers that examined CCP default waterfall designs include Capponi et al. (2017), which examines the CCP's role in attracting less risky membership and the consequences for risk sharing from allocating risk to themselves in the default waterfall. In contrast, we consider the aggregate loss to firms and counterparty externalities, similar to Acharya and Bisin (2014), Ghamami (2015) and Ghamami and Glasserman (2017), by incorporating the impacts of several layers of the waterfall on client clearing and non-cleared positions into our analysis. Importantly, previous papers in this literature have not considered the full network implications of the CCP's default waterfall, nor have they had access to the detailed transaction-level market data that we use.

Other papers such as Huang (2019) have studied the conflicting objectives CCPs may have in determining their waterfall structures. As commercial enterprises with profit-making incentives, CCPs compete for the clearing business of members and their client positions (Glasserman et al. (2016)). This is likely to drive how much and where waterfall resources are allocated, as collecting collateral has direct short-term costs for participants and could disincentivize participation in central clearing. Our paper is complementary to these other works and helps determine the magnitude of the stability benefits provided by default waterfall resources, which must be balanced against the costs imposed on CCP members.

The rest of this paper is divided into the following sections. Section 1 provides a background on CCP default waterfalls and how they have been implemented. Section 2 describes our CCP payments model and how the waterfall is incorporated into it. Section 3 describes how to compute systemic losses and individual firm losses using the model. Sections 4 presents an empirical test of CCP waterfall resiliency using U.S. CDS market data. Section 5 presents counterfactual analysis that considers the impact of various segments of the default waterfall. Section 6 concludes.

1 CCP Default Waterfall Structure

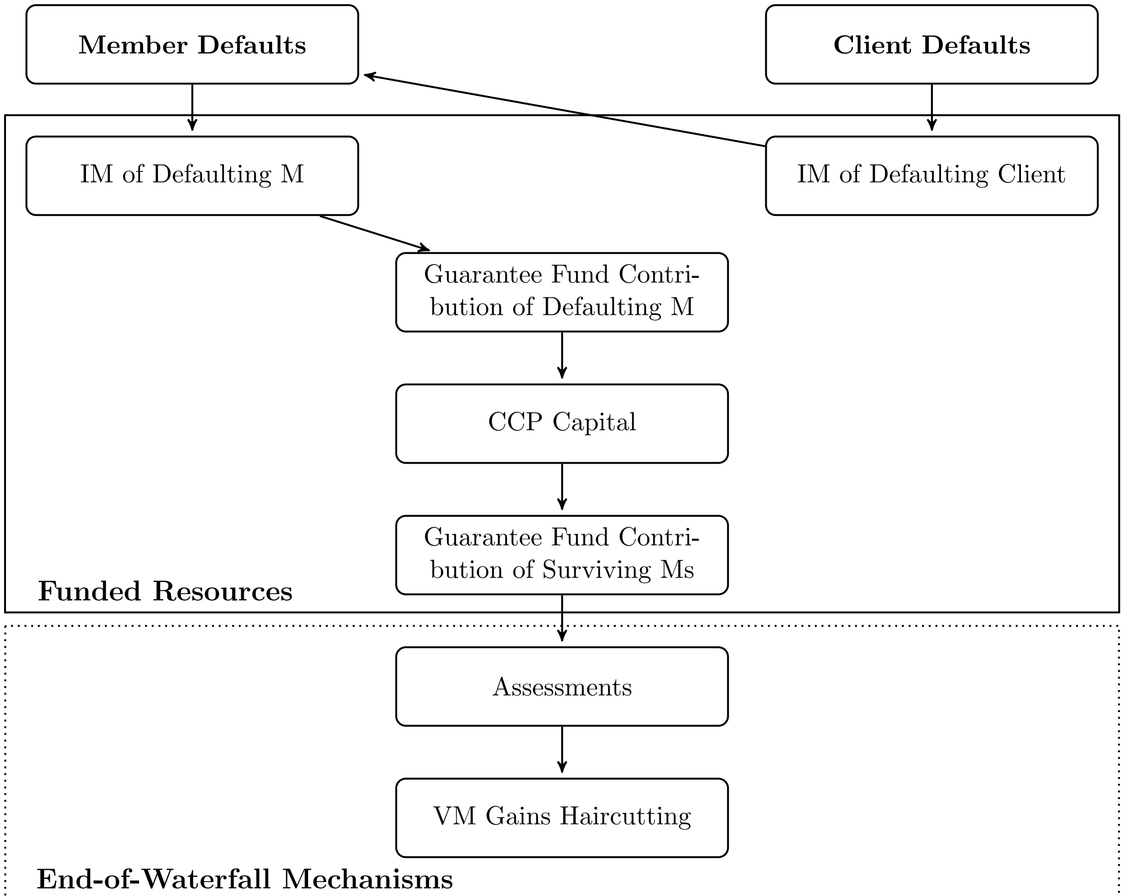
A CCP's recovery plan to deal with clearing member or client defaults is known as its *default waterfall*. The default waterfall provides a detailed list of resources that the CCP will use in attempting to recoup losses from clearing member defaults. While the exact rules of default waterfalls vary across CCPs, their overall structures are similar and follow from standard industry guidelines (ISDA (2013), ISDA (2015)). The stages of a typical default waterfall are depicted in Figure 1.

1.1 Default Waterfall Resources and Mechanisms

The first several stages of the default waterfall are present in nearly all CCPs, and they involve widely used mechanisms. These stages are known as the *funded* waterfall stages because their resources are contributed before the shock occurs. Thus the amount available to use is independent of the shock. Since these are the first stages to be used, there is more historical precedent for them than for the final stages, and they are thus better understood and tested.

The first stage of the default waterfall is the initial margin (IM) of the defaulting clearing member. IM is held at the CCP in case a clearing member defaults. IM can be used when the clearing member does not fulfill its payment obligations. The IM amount is usually set at a certain Value-at-Risk (VaR) level, such as 99 percent, but may also have additional components, like concentration and liquidity (Capponi et al. (2020)). IM is also collected for non-centrally cleared transactions. However, the margin period of risk (MPOR) used in IM calculations typically differ, with derivative CCPs typically using a 5-day MPOR while bilateral trades typically use a 10-day

Figure 1. Stages of CCP Default Waterfall



Note: The chart depicts the series of resources and mechanisms in the waterfall which will be accessed if previous ones are insufficient to cover total default losses in the event of a clearing member (M) or client default. The solid arrows depict the most common set of waterfall resource contingencies. A defaulting clearing member's, or client's, obligation is first covered by their initial margin (IM). Positions of defaulting clients are the responsibility of the associated clearing member, who has to cover any shortfalls in variation margin (VM) payments owed for those positions. If the clearing member cannot fulfill this obligation, the clearing member may be put into default. If the clearing member's IM is insufficient to cover its obligations, the resources of the following stages will be used.
Source: Authors' creation.

MPOR.

The second stage of the default waterfall is the guarantee fund contribution of the defaulting clearing member. Guarantee fund contributions are collected from all clearing members and held at the CCP. A clearing member's contribution is usually proportional to its VaR, and is thus also proportional to its IM. The CCP's total guarantee fund amount is typically sized according to the "Cover 2" rule, which states that the guarantee fund should cover the default of the two largest clearing members of the CCP. However, alternative risk-based rules can also be used. We will empirically test the resilience offered by variations in the ratio of guarantee funds to IM. The guarantee fund is more versatile than IM because it can be used to cover the losses of any clearing member, but this versatility also opens up non-defaulting clearing members to losses.

The next stage is the CCP's own capital contribution.[2] This is commonly referred to as "skin in the game" and is intended to reduce moral hazard on the part of the CCP. CCP capital contributions are typically small relative to the total IM or the guarantee fund, and are generally one to three times smaller in relative magnitude, as we will discuss in the following section. The final stage of funded resources is the loss mutualization of surviving clearing members' guarantee fund contributions. The guarantee fund covers the defaulted payments *pro rata* across each of the clearing members.

In the event that the funded resources are entirely deployed, a few different end-of-waterfall mechanisms can be implemented either to raise fresh funds, via *assessments*, or reduce obligations, via *variation margin gains haircutting* (VMGH). Assessments allow the CCP to request additional funds from non-defaulting clearing members, whereas VMGH allows the CCP to temporarily reduce the VM payments on its obligations. These mechanisms have rarely been used in practice and may have alterations made to them to further support the CCP. For the purpose of focus, we will not analyze these mechanisms in the main body of the text, but we do discuss and analyze them in Appendix D.[3]

1.2 Empirical Comparison of CCP Default Waterfall Resources

Though CCPs use the same types of default waterfall resources and mechanisms globally, the amount of resources collected at each stage varies significantly in practice. To highlight the differences empirically, we collected a unique data sample of default waterfall resources from the Principles for Financial Market Infrastructures (PFMI) filings of 60 global CCPs from the fourth quarter of 2017. Our data show a large degree of heterogeneity in how resources are allocated along the waterfall. Particularly, there are major differences in the amount of resources CCPs have available through their IM, guarantee funds, and CCP capital contributions.

Such differences are significant because the relative levels of IM and guarantee fund can have a

[2] This stage may come in one or two parts depending on the CCP. Some CCPs allow for a second part that comes after the guarantee fund stage. We use one part in our analysis for simplicity, but having a second part would not materially change our model.

[3] We compare their assessment rules, specifically the cap on assessments as a function of the guarantee fund contribution sizes, and whether their rules permit usage of variation margin gains haircuts (VMGH) and initial margin haircuts (IMH). Each choice of rules can have a significant effect on the size of the shock the CCP can withstand.

tremendous impact on the resilience of the CCP, its clearing members, and the overall market. We empirically test the magnitude of these effects below in Section 5 using positions-level data from a major U.S. CCP. Additionally, though for the sake of focus we do not analyze CCP end-of-waterfall mechanisms in the main body of the text, we do discuss and empirically analyze them in Appendix D.

The CCP data collection is summarized in the following sequence of tables. Table I shows the average percentage of resources for different stages of the default waterfall across CCPs grouped by asset class. Commodity CCPs make the highest percentage of capital contributions, whereas Interest Rate CCPs make the lowest percentage. Credit CCPs have relatively high levels of IM and guarantee funds but low levels of capital contribution relative to other CCP asset classes. This waterfall structure shifts the losses from the CCP onto the clearing members. The table also shows the maximum assessment limit as a percentage of total guarantee funds.[4] These values are due to caps on assessments set by each CCP as a function of guarantee fund size.

Table I. Waterfall Resources by CCP Asset Class

	Interest Rate	Currency	Commodity	Credit	Equity
Number of CCPs	13	12	16	6	13
Funded Resources					
Initial Margin	79.2	73.6	77.2	77.9	81.1
Guarantee Fund	19.2	21.8	13.7	20.1	13.4
CCP Capital	1.6	4.6	9.1	2.0	5.5
End-of-Waterfall Resources					
Assessments	86.5	96.9	75.8	60.2	124.9

Note: The table presents the mean percentage of funded resources collected at each stage, and the maximum assessment a CCP can make on its clearing members relative to the guarantee fund size, grouped by the asset class a CCP clears. Looking across CCP types, initial margin makes up the majority of resources collected, ranging from 70 to 81 percent, followed by the guarantee fund with 13 to 22 percent. The CCP's contribution is minimal, ranging from 1 to 9 percent. More generally we find that no particular asset class appears to have any unique preference in assigning resources.
Source: CCPView Clarus Financial Technology; authors' analysis.

Table II shows a similar summary grouped by the location of the CCP. There does not seem to be a global consensus on the optimal waterfall design for minimizing systemic risk or ensuring incentive compatibility for CCP members. The ratios vary dramatically across regions. Asian and European CCPs have significantly lower percentages of IM than North American CCPs. European CCPs have larger levels of guarantee funds, while Asian CCPs have larger CCP capital. These

[4] Some CCPs allow for a greater assessment amount if there are multiple clearing member defaults versus a single clearing member default.

differences across regions can have an important impact on the CCP's resilience under periods of market stress. Our empirical analysis in Section 5 shows that CCPs with higher IM relative to guarantee funds and capital are less resilient to market shocks.

Table II. Waterfall Resources by Jurisdictional Region

	Asia	Europe	North America	Oceania	South America
Number of CCPs	27	20	12	2	2
Funded Resources					
Initial Margin	69.2	74.0	85.2	90.1	97.7
Guarantee Fund	18.7	25.3	13.5	2.2	2.2
CCP Capital	12.2	0.7	1.3	7.7	0.1
End-of-Waterfall Resources					
Assessments	75.5	122.3	77.5	300.0	73.6

Note: The table presents the mean percentage of funded resources collected at each stage, and the maximum assessment a CCP can make on its clearing members relative the guarantee fund size, grouped by the continental jurisdiction a CCP resides. Looking across CCP jurisdictions, we see wide variation in funded resources and assessments, suggestive of jurisdictional regulatory preferences influencing CCP's default waterfall allocations.
Source: CCPView Clarus Financial Technology; authors' analysis.

An additional dimension to the waterfall structure is the liquidity of the collateral resources held by the CCP and used in the event of default. Although intra-firm payments are made in cash, holding IM and guarantee fund collateral in cash alone creates significant costs for clearing members. As a result, other forms of collateral that pay higher interest rates are typically held, or the CCP may rely on credit lines in case of short-term delays in payments. Table III highlights the percent of collateral and credit lines held by 30 CCPs as of the fourth quarter of 2017.

Table III. Funded Resource Collateral and Credit Lines

	Mean	Median	Std Dev	Min	Max
Collateral					
Secured Cash Deposits	44.3	47.5	35.0	-	100.0
Unsecured Cash Deposits	14.6	1.8	31.0	-	100.0
Repo Lent Cash/Securities	10.3	-	21.1	-	81.4
Government Securities	28.2	21.0	30.2	-	99.0
Other	2.6	-	13.5	-	74.2
Unsecured Credit Lines	8.1	-	23.4	-	121.8

Note: The table presents the percentage of collateral and liquidity resources held by 30 OTC derivative CCPs as of the fourth quarter of 2017. The majority of CCP collateral holdings are in cash, repo, or government securities. A small percentage of holdings are in other less liquid assets. Additionally some CCPs have unsecured credit lines, which we present as a percentage of their total collateral holdings, that they may draw on in times of short-term liquidity impairment.
Source: CCPView Clarus Financial Technology; authors' analysis.

In the event of a default, non-cash collateral will need to be liquidated before payments can be made, likely at a reduced price in times of market stress. These liquidation costs and the ability to raise cash in a timely fashion are additional elements of risk that need to be considered. As a result CCPs in general hold very high quality collateral, in cash or cash equivalents, as Table III highlights, and require haircuts. Though CCPs may take wider forms of collateral if needed, they generally encourage collateral delivered to be of high quality through applying steep haircuts relative to the general market.

2 Model of CCP Default Waterfall

CCP stress can arise via many forms. For instance, a substantial shock such as the Comprehensive Capital Analysis and Review (CCAR) shock can lead to a situation where some firms owe much more VM than they expect to receive. These VM obligations are supposed to be satisfied within a very short time – typically within a few hours – and can put severe stress on the firms' trading desks. This stress may prevent a firm from fulfilling all its obligations to its counterparties. This, in turn, increases the stress on the firm's downstream counterparties, amplifying the impact of the shock through the network of CDS exposures. The model assumes that payments are all made simultaneously as margin payments are made intra-day, though it can take several days or weeks for some portfolio liquidations if necessary.

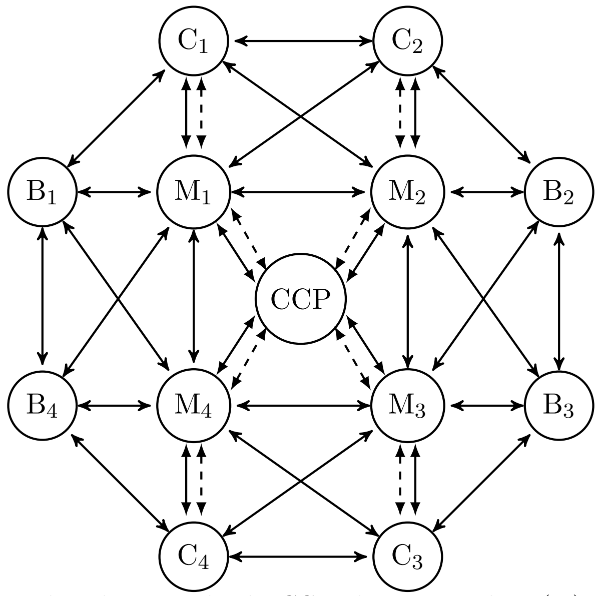
In the next two sections, we introduce a network model to account for this type of contagion, and we show how to incorporate the different components of the CCP waterfall into the model.

2.1 Basic Setup and Payments

The setup is based on the framework of Glasserman and Young (2015), which in turn builds on the model of Eisenberg and Noe (2001). There are $N+1$ agents in the market, with 0 indexing the CCP and $i \in 1, ..., N$ indexing the non-CCP firms. Each pair of counterparties i, j can have a set of contracts between them, which we assume are in a single asset class, for instance CDS. Contracts can represent both bilateral and centrally cleared transactions.

There are three types of non-CCP firms in our model: clearing members M, clients C, and bilateral firms B. These firms differ in their use of central clearing. Clearing members may clear through the CCP directly. Clients cannot clear through the CCP directly but must instead submit

Figure 2. Example Network of Cleared and Non-cleared Obligations



Note: Four firm types are depicted in the network: the CCP, clearing members (M), clients (C), and bilateral firms (B). The links represent obligations, both direct (solid) and client clearing (dashed). These firms differ in their use of central clearing. Clearing members may clear through the CCP directly. Clients cannot clear through the CCP directly but must instead submit trades through a clearing member. Clearing members pass through the payments received or owed from the client to the CCP, and clearing members must cover any shortfalls in these payments out of their own funds.
Source: Authors' creation.

trades through a clearing member. Clearing members pass through the payments received or owed from the client to the CCP, and clearing members must cover any shortfalls in these payments out of their own funds. We give the details of how this process works below. We denote C_k as the set of client firms of a clearing member k. Also, we denote M_i as the set of clearing members of a client firm i.[5] Finally, some non-members are purely bilateral firms that do not participate in any centrally cleared transactions and instead engage only in bilateral transactions.

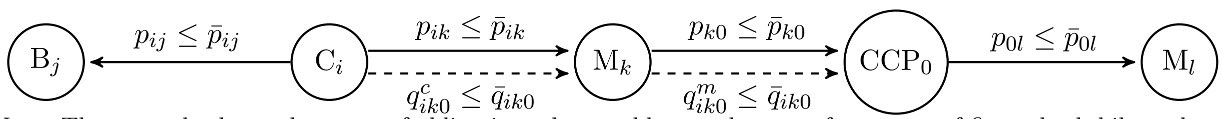
Given a market shock, we can calculate the variation margin (VM) payment obligations for each individual contract. We then net all the VM payments owed on contracts between every pair of counterparties. This produces a net VM payment that needs to be made between this pair. We exclude client clearing trades in this summation, and will instead consider them separately next. We write the net VM payment obligations as an *obligation matrix* $\bar{P} = (\bar{p}_{ij})_{i,j \in N_0}$, where $\bar{p}_{ij}$ is the net amount of VM owed by node i to node j in the aftermath of the shock. We denote the indices of the first row and column of this matrix as 0 to account for the CCP. Note that if $\bar{p}_{ij}$ is positive

[5]In the data, we do not observe the identity of the clearing member that a client uses, but we assume that the client clears through the clearing member with which it has the most bilateral transactions.

then $\bar{p}_{ji}$ is zero. Further, $\bar{p}_{ii} = 0$ for all i.

Given the market shock, firms may only be able to satisfy part of their payment obligations due to shortfalls in funds. We denote the actual VM payments made between firms using the *realized payment matrix* $P = (p_{ij})$.

Figure 3. Example of Normal and Realized Payments



Note: The example shows the types of obligations that could occur between four types of firms, both bilateral and client cleared. The figure helps to illustrate the dependencies that firms can have on one another, as clearing member k's ability to pay CCP 0 may depend on client i's ability to fulfill its obligations. In the figure, if the realized payments from client i, $p_{ij}, p_{ik}, q^c_{ik0}, q^m_{ik0}$, are less than its obligations, clearing member k and pure bilateral firm j will suffer direct losses. Additionally, the impact of client i's realized payments may lead to clearing member k reducing its payments, causing potential losses to the CCP as well.
Source: Authors' creation.

There are also client clearing transactions in the network that induce a set of corresponding client clearing obligations. We denote the net VM client clearing obligations with a set of two matrices $\bar{Q} = (\bar{q}_{ik0}, \bar{q}_{0ki})_{i \in C, k \in M}$. In this notation, $\bar{q}_{ik0}$ denotes the net amount owed by client i to the CCP through clearing with member k, and $\bar{q}_{0ki}$ denotes the net amount owed by the CCP to client i through clearing with member k.[6] Note again that if one of these obligations is positive then the other is zero. Because client clearing must involve a clearing member as an intermediary, $\bar{q}_{ik0}$ is fulfilled by having client i submit a payment to clearing member k, q^c_{ik0}, and that clearing member k then submitting a payment to the CCP, q^m_{ik0}. The reverse obligation $\bar{q}_{0ki}$ is fulfilled by the CCP submitting a payment to clearing member k, q_{0ki}, and clearing member k submitting a payment to client i, q^m_{0ki}. A clearing member must pass through any payments that it receives in the course of client clearing, even if the clearing member is in default. Therefore $q^m_{ik0} \geq q^c_{ik0}$ and $q^m_{0ki} \geq q_{0ki}$. We describe how the payment values are determined in equilibrium over the next few sections.

2.1.a Initial Margin and Capital Buffer

In most transactions, IM will be held to help cover deficiencies in VM payments. We denote the IM that i holds from counterparty k as z_{ki}. This IM value will always be positive unless the

[6] The CCP does not net the transactions of a client that clears through multiple different clearing members. The client is in effect treated as a different entity across each clearing member with which it clears.

counterparty k is the CCP itself as the CCP does not post IM, $z_{0i} = 0 \; \forall i$. If counterparty k fails to pay VM to i in a timely manner, the position will be closed out and the IM will be applied to the shortfall in VM payments. The value of IM held is calculated on a portfolio level using a VaR estimate based on historical look back.[7]

For client clearing transactions, IM will be held by the CCP for a transaction with client i clearing through clearing member k. We denote the value of this IM by z_{ik0}. The IM is used to cover shortfalls in payments if the client is unable to cover its obligations. The IM is then passed through by the clearing member to the CCP to assist with the clearing member's passthrough obligation if necessary. The passthrough payment of clearing member k must be weakly greater than the amount it receives, $q^m_{ik0} \geq (q^c_{ik0} + z_{ik0})$. The CCP does not post IM for the client clearing obligations that it owes. For a given firm i, we denote the total amount of IM that it contributes across bilateral and client clearing obligations as z_i.

Beyond IM, firms are assumed to hold some quantity of assets on hand, which we term as *capital buffer*, $b_i > 0$. This b_i is a function of the firm's risk management policies, non-CDS positions, or available cash. Firms will use their capital buffers to help meet their VM obligations.

Generally collateral in the financial system can be treated as cash, particularly in the case of cleared IM, as CCPs require IM to be in sovereign bonds or currency. As a result, if collateral z_{ij} is seized by node j from node i, node i's payment obligation is reduced by exactly z_{ij}. Similarly we assume there is no liquidation cost for the capital buffer b_i that is used to node i's payment obligations. However, in the non-cleared derivatives market, a wide range of less liquid securities, including corporate bonds, foreign-denominated bonds, and equities are accepted as collateral, and these types of securities are also used as collateral in repurchase agreements.

We note that our model can be adjusted to incorporate the potential consequences of collateral illiquidity. To do so, we assume that z_{ij} and b_i are held in an asset with a normalized price π. The value of π can range from zero to one, with zero indicating a full reduction in value and one indicating no reduction in value (i.e. cash). In turn, πz_{ij} is the reduced value of the IM and πb_i the reduced value of the capital buffer. Following the price-impact formulation used in Cifuentes et al. (2005) and Amini et al. (2016), we posit that π is a strictly decreasing function $G(1, \Delta)$ of

[7] We estimate VM and IM following the formulation of Luo (2005) and applying the framework adopted in Appendix I of Paddrik et al. (2020).

the total proportion of collateral asset liquidated Δ, with the first argument of G indicating the initial price of 1. To be concrete, we set

$$\pi = G(1, \Delta) = e^{-\lambda\Delta} \tag{1}$$

for some $\lambda > 0$. A larger λ corresponds to a less liquid asset. Though this setting makes the simplifying assumption that all collateral is held in a single illiquid asset, any overstatement can be offset through a smaller value of λ. We analyze the impact of collateral illiquidity in Appendix C, and we show that the current collateral standards are sufficient to prevent major losses. To reduce our notation in the subsequent sections we will present the case where IM and capital buffers are held in cash, $\pi = 1$.

2.1.b Payments for Bilateral Firms

We first derive payment equations for nodes other than the CCP. We start with the simplest case of a bilateral firm i, and we write out the payments p_{ij} that i makes to each counterparty j conditional on the payments that it receives from each counterparty k, p_{ki}.

The resources available to firm i are its capital buffer b_i, the payments from each counterparty k, p_{ki}, and the IM that i holds from each counterparty k, z_{ki}. The total obligations of firm i to its counterparties are $\sum_{k\neq i} \bar{p}_{ik}$. If this exceeds the resources of firm i, it will not be able to fully pay its counterparties. It will thus need to reduce its payments to its counterparties proportionally. We describe next how this reduction is implemented.

For a bilateral firm $i \in B$, we define the *stress* at i, s_i, to be the amount by which i's payment obligations exceed the incoming payments from i's counterparties and IM held:

$$s_i = \left[\underbrace{\sum_{k\neq i,0} \bar{p}_{ik}}_{\text{Obligations}} - \underbrace{\left(\sum_{k\neq i,0} ((p_{ki} + z_{ki}) \wedge \bar{p}_{ki}) + b_i \right)}_{\text{Resources}} \right]^+ . \tag{2}$$

Let $\bar{p}_i = \sum_{j\neq i} \bar{p}_{ij}$ be the total payment obligations of i to all other nodes. In the following definitions, we restrict attention to the nodes i in the system such that $\bar{p}_i > 0$. The others do not

[7] In general, $x \wedge y$ denotes the minimum of two real numbers x and y.

have payment obligations and thus do not transmit payment shortfalls. In the CDS market that we consider, such firms would be buyers of CDS protection (not sellers) and under our CCAR shock they would not have VM obligations. Shortfall and payment equations are thus not necessary for such nodes.

We can use these VM payment obligations to define the *relative liability* of node i to node j as

$$a_{ij} = \bar{p}_{ij}/\bar{p}_i. \tag{3}$$

We assume that the stress of firm i is transmitted to i's counterparties pro rata the size of its payment obligations. We can now define the payment functions for bilateral firms. Given any input vector $p \in \mathbb{R}^{2n+2}$ such that $0 \leq p_{ij} \leq \bar{p}_{ij}$ for all $0 \leq i, j \leq n$, the payment functions $p_{ij}(p)$ for a bilateral firm i are given by:

$$p_{ij} = \bar{p}_{ij} - a_{ij} s_i. \tag{4}$$

2.1.c Payments for Client Firms

Now we define the payments for client firms. Client firms have similar payment functions as bilateral firms, but they also have client clearing transactions with the CCP. Recall that for these transactions, clients need to form a contract with a clearing member, which then intermediates transactions with the CCP. If the client owes money to the CCP, the client pays the clearing member , which then passes the payment onto the CCP. If the client is owed money by the CCP, the CCP will pay the clearing member, which then passes the payment onto the client. Clients also post IM to cover shortfalls in their payments. However, the CCP does not post IM for the client.

The total resources of a client firm i are given by the sum of its payments received from bilateral and client clearing transactions along with its capital buffer. The total obligations of client i are given by the sum of its bilateral and client clearing obligations. If this amount exceeds the resources of firm i, it will not be able to pay its counterparties in full and will instead pass on partial payments.

For a client firm $i \in C$, we define the *stress* at i, s_i, to be the amount by which i's payment obligations exceed the incoming payments from i's counterparties and IM held:

$$s_i = \left[\underbrace{\sum_{k \neq i,0} \bar{p}_{ik} + \sum_{k \in M_i} \bar{q}_{ik0}}_{\text{Obligations}} - \underbrace{\left(\sum_{k \neq i,0} ((p_{ki} + z_{ki}) \wedge \bar{p}_{ki}) + \sum_{k \in M_i} q^m_{0ki} + b_i \right)}_{\text{Resources}} \right]^+ . \tag{5}$$

Unlike bilateral firms, clients will also have client clearing payment obligations. When the client faces stress, we assume that these payments are reduced pro rata the total payment obligations owed by the client across all types of transactions.

Denote $\bar{p}^g_i = \sum_{k \neq i} \bar{p}_{ik} + \sum_{k \in M_i} \bar{q}_{ik0}$ as the sum over all payment obligations for client i. We use these combined payment obligations to derive client i's relative payment liability to different firms.

$$a_{ik} = \bar{p}_{ik} / \bar{p}^g_i \quad \forall k \neq i, \tag{6}$$

$$a^c_{ik0} = \bar{q}_{ik0} / \bar{p}^g_i \quad \forall k \in M_i. \tag{7}$$

The stress of client i is transmitted to i's counterparties pro rata the size of its combined payment obligations. Given any vector $p \in \mathbb{R}^{2n+2}$ such that $0 \leq p_{ik} \leq \bar{p}_{ik}$ for all $0 \leq i, k \leq n$, the payment functions $p_{ik}(p), q^c_{ik0}(p)$ for a client firm i are given by:

$$p_{ik} = \bar{p}_{ik} - a_{ik} s_i, \tag{8}$$

$$q^c_{ik0} = \bar{q}_{ik0} - a^c_{ik0} s_i. \tag{9}$$

2.1.d Payments for Clearing Members

Now we define the payments for clearing members. Clearing members may engage in bilateral trades, centrally cleared trades, and client clearing trades. Each of these three types of trades will entail different obligations and resources for the clearing member. The total obligations of clearing member k to its counterparties are given by $\sum_{i \neq k} \bar{p}_{ki} + \sum_{i \in C_k} (\bar{q}_{0ki} + \bar{q}_{ik0})$. If this amount exceeds the resources of clearing member k, it will not be able to pay its counterparties in full.

Member k's stress is given by:

$$s_k = \Bigg[\underbrace{\sum_{i \neq k} \bar{p}_{ki} + \sum_{i \in C_k} (\bar{q}_{0ki} + \bar{q}_{ik0})}_{\text{Obligations}} - \underbrace{\left(\sum_{i \neq k} ((p_{ik} + z_{ik}) \wedge \bar{p}_{ik}) + \sum_{i \in C_k} ((q^c_{ik0} + z_{ik0}) \wedge \bar{q}_{ik0}) + q_{0ki}) + b_k \right)}_{\text{Resources}} \Bigg]^+. \quad (10)$$

The way that a clearing member k passes on stress is special because for centrally cleared transactions it must pass through at least the amount that was given to it by a client i for the CCP, $((q^c_{ik0} + z_{ik0}) \wedge \bar{q}_{ik0})$, or the CCP for a client i, q_{0ki}. If these amounts are sufficient to cover the original obligations, $\bar{q}_{ik0}$ and $\bar{q}_{0ki}$ respectively, then the clearing member will make the payment in full. However, if the amount received by the clearing member is less than the obligation, the clearing member must cover the remainder out of its own funds or be in default. If the clearing member is under stress, then any remaining obligations will be cut pro rata based on the clearing member's stress.

Define $\bar{p}^r_k = \sum_{i \neq k} \bar{p}_{ki} + \sum_{i \in M_k} (\bar{q}_{0ki} - q_{0ki} + [\bar{q}_{ik0} - q^c_{ik0} - z_{ik0}]^+)$ as the sum of the remaining payments over all entities. Because the passthrough payments must always be made, we deduct them in determining the clearing member's relative liability.

$$a_{ki} = \bar{p}_{ik} / \bar{p}^r_i \quad \forall k \neq i, \quad (11)$$

$$a^m_{0ki} = (\bar{q}_{0ki} - q_{0ki}) / \bar{p}^r_i \quad \forall i \in C_k, \quad (12)$$

$$a^m_{ik0} = [\bar{q}_{ik0} - q^c_{ik0} - z_{ik0}]^+ / \bar{p}^r_i \quad \forall i \in C_k. \quad (13)$$

The stress of clearing member k is transmitted to k's counterparties pro rata the size of its remaining payment obligations. We can now define the payment functions for clearing member k.

$$p_{ki} = \bar{p}_{ki} - a_{ki} s_k \quad \forall i \neq k, \quad (14)$$

$$q^m_{0ki} = \bar{q}_{0ki} - a^m_{0ki} s_k \quad \forall i \in C_k, \quad (15)$$

$$q^m_{ik0} = \bar{q}_{ik0} - a^m_{ik0} s_k \quad \forall i \in C_k \quad (16)$$

We have defined the payment functions for all entities other than the CCP. We will define the CCP's equilibrium payment functions and received payments in the next section. In equilibrium, the payments received by a firm and the payments made by a firm must be balanced. Firms that are under stress will cut their payments pro rata, while unstressed firms will make their payments in full. The equilibrium payment vector accounts for contagion effects from one firm failing to pay its counterparties and propagating stress further down the network. Such contagion effects have the potential to be very large, as shown in Paddrik et al. (2020).

2.2 CCP Default Waterfall

In this section, we derive the CCP's payment function from the funded stages. Recall that the CCP is indexed as node 0. The CCP's payment obligations are given by $\sum_{k\in M}\left(\bar{p}_{0k}+\sum_{i\in C_k}\bar{q}_{0ki}\right)$. The resources that the CCP has available come from the CCP's default waterfall and the VM payments it receives. As described in the introduction, the funded layers of the CCP default waterfall consist of the following stages: IM of defaulting clearing members, guarantee fund contributions of defaulting clearing members, CCP capital, and guarantee fund contributions of surviving clearing members.

IM is the first stage used in the default waterfall. It is used to cover shortfalls in the payments of clearing member obligations, including a clearing member's client clearing obligations. z_{k0} denotes the IM collected by the CCP from clearing member k. Including IM, the CCP receives total resources from clearing member k of:

$$\left(p_{k0}+\sum_{i\in C_k}q^m_{ik0}+z_{k0}\right)\wedge\left(\bar{p}_{k0}+\sum_{i\in C_k}\bar{q}_{ik0}\right). \tag{17}$$

The next several stages of the default waterfall utilize the guarantee fund of the CCP. Funds are first taken from contributions of the defaulting clearing members. If that is insufficient to cover the CCP stress, funds are next taken from the CCP's capital contribution b_0, and then from the remaining guarantee fund contributions of all the clearing members.

We define some notation to describe the guarantee fund. Let γ be the total size of the CCP's guarantee fund. We assume that each clearing member k contributes an amount γ_k to the guarantee fund. Thus the total guarantee fund is given by the following equation:

$$\gamma=\sum_{k\in M}\gamma_k. \tag{18}$$

The guarantee fund contributions γ_k are computed by examining the riskiness of the portfolio of each clearing member. We approximate the contribution γ_k by using the proportion of IM of clearing member k held by the CCP , as z_{k0} is a reflection of the relative amount of risk clearing member k contributes to the total portfolio of the CCP.[8]

$$\gamma_k = \gamma \frac{z_{k0}}{\sum_{j \in M} z_{j0}} \tag{19}$$

In section 1.2 we will present a survey of the default waterfall sizes of real-world CCPs. There exists a large degree of variation in guarantee fund sizing, as CCPs follow different rules such as Cover 1 or Cover 2 (CPMI-IOSCO (2012)). We examine the resiliency and trade-offs of these different rules in our empirical tests.

The CCP also provides its own capital contribution to cover losses, which we denote by b_0. This capital contribution is used after the guarantee fund of defaulting clearing members and before the guarantee fund of non-defaulting clearing members.

The CCP prefunds the first four stages of the default waterfall. The amount that is available is thus fixed and independent of the market shock. In addition, implementing these stages does not create additional stress for the network.[9]

The remaining CCP stress after the funded waterfall layers, denoted s_0, is given by the following equation:

$$s_0 \equiv \left[\sum_{k \in M} \left(\bar{p}_{0k} + \sum_{i \in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i \in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0}) \right) - \gamma - b_0 \right]^+ . \tag{20}$$

The CCP's stress is zero if it can cover its payment obligations without using up its own capital b_0 and the guarantee fund contributions of all clearing members γ. Only after the guarantee fund is fully used up (in the fourth stage) will the CCP stress be greater than zero. After the guarantee fund is used up, the CCP passes on this level of stress to its counterparties by cutting its payments pro rata. In Appendix D, we analyze the impact of including the end-of-waterfall mechanisms to

[8] γ reflects the expected loss to the CCP if the two largest clearing members were to default, after accounting for the clearing members beyond their z_{k0}. For the purposes of this model, we use public disclosures of γ, discussed in Section 4.1.

[9] Clearing members must eventually replenish the IM and guarantee fund contributions that are used, but this will usually come later. The PFMI regulations require that the IM and guarantee fund payments be replenished by the start of the next business day to ensure compliance with the cover two rule.

cover the remaining stress.

Given the CCP's stress s_0, we define the combined payment obligations for the CCP to derive its payments. We let $\bar{p}_0^g = \sum_{i\neq 0} \bar{p}_{ki} + \sum_{k\in M} \bar{q}_{0ki}$.

We use these combined payment obligations to derive the CCP's relative payment liability to different firms.

$$a_{0k} = \bar{p}_{0k}/\bar{p}_0^g, \tag{21}$$

$$a_{0ki} = \bar{q}_{0ki}/\bar{p}_0^g. \tag{22}$$

$$p_{0k} = \bar{p}_{0k} - a_{0k}s_0 \quad \forall k \in M, \tag{23}$$

$$q_{0ki} = \bar{q}_{0ki} - a_{0ki}s_0 \quad \forall k \in M. \tag{24}$$

Similar to other firms, the CCP prorates its outgoing payments proportionally based on its level of stress.

2.3 Existence of Payment Equilibrium

In this section, we prove that a payment equilibrium will exist for any default waterfall design in the financial system we have proposed. Compared with the classical network model of Eisenberg and Noe (2001), our model features the additional complexities of IM, client clearing obligations, and CCP default waterfall obligations. However, we show in the following theorem that a maximal payment equilibrium still exists for our model. A maximal payment equilibrium refers to an equilibrium with the largest value of payments made by all the firms. We will focus on the analysis of the maximal payment equilibrium for our model in the empirical section of the paper. We state our existence result in the following theorem.

THEOREM 1. *A maximal payment equilibrium exists for the financial clearing system.*

Proof. To prove the existence theorem, we build on the results obtained in Paddrik et al. (2020). Proposition 1 of that paper guarantees existence of a greatest fixed point for a function that is upper-semicontinuous, monotone, and order-preserving on a bounded lattice. To apply this result to our model, we start by showing that our payment functions satisfy these properties. The payment functions in our model are given by Equation (4) for purely bilateral firm payments, Equations (8)-(9) for client payments, Equations (14)-(16) for clearing members, and Equations (23)-(24) for the CCP. We define the mapping function $\Phi(p, q)$ as the set of all the p and q payment functions for all firms in our model.

First, we note that all of the payment functions are continuous, and thus upper-semicontinuous. To show that the payment vectors are also monotone and order preserving, note that all the payment functions are decreasing in the amount of stress at each entity. We therefore need to show that lower payments received will increase the stress in each of these equations. For the non-CCP equations, this is straightforward to see directly from the definitions of stresses: Equation (2) for purely bilateral firms, Equation (5) for the clients, and Equation (10) for the clearing members. Stress at the CCP is given in Equation (20), which also shows that it is also a decreasing function of payments. Since each individual payment function is upper-semicontinuous, monotone, and order preserving, the function $\Phi(p, q)$ satisfies these properties as well.

Since the function $\Phi(p, q)$ is upper-semicontinuous, monotone and order preserving, we can apply Proposition 1 in Paddrik et al. (2020) to show that starting from $\bar{p}$ and $\bar{q}$, the sequence of payment vectors will converge to the greatest fixed point payment vector. Therefore a maximal payment equilibrium will exist. ■

In the empirical applications, we will compute the maximal equilibrium by choosing values of the capital buffers $\{b_0, b_1 ..., b_n\}$ and then recursively computing a fixed point of this system by taking the limit of the sequence $(p^1, q^1) = \Phi(\bar{p}, \bar{q}), (p^2, q^2) = \Phi(p^1, q^1), ..., (p^{n+1}, q^{n+1}) = \Phi(p^n, q^n)$,... The resulting fixed point is a maximal equilibrium of the network model. In Appendix A, we describe this fictitious algorithm in more detail and how it is used to calculate the equilibrium payment vector in the empirical settings. Finally, we note that both our existence result and our fictitious algorithm can be extended in a straightforward way to cover collateral illiquidity.[10]

[10] In the case where πz_{ij} and πb_i are held in an illiquid asset, rather than cash, and the asset follows the price-impact function given in Equation (1), finding the greatest fixed point for the clearing payments and the illiquid asset value may require further iterations of the fictitious algorithm. However, a greatest fixed point is guaranteed to exist given the monotonicity of π by Tarski's fixed point theorem (1955), as applied in Cifuentes et al. (2005), Elliott et al. (2014), Gofman (2017), and Amini et al. (2016).

3 Measuring Losses

A social planner concerned with financial system risk must select a desirable default waterfall design by assessing the consequences of different waterfall structures on overall system losses and how those losses are distributed. We use *systemic loss*, defined as the total of bankruptcy losses that creditors suffer, which follows the interpretation applied in Eisenberg and Noe (2001) and Glasserman and Young (2015). However, unlike the measures used in these works, which consider only the shortfall of payments, we also consider bankruptcy costs resulting from client clearing and the CCP's default waterfall. Clearing members are responsible for defaulted client obligations and clearing members' default resources are used by the CCP to cover defaulting member obligations. Thus clearing members can become creditors to the defaulted firms as if they were directly receiving payments.[11] Systemic loss captures the overall level of payment disruption to the system felt across all firm types.

We next discuss how losses are calculated among the individual participant types. Then we describe how total systemic losses can be derived in our model. This measure will allow us to later empirically measure the social welfare implications of different default waterfall structures.

3.1 Losses for Different Types of Firms

We now write out the total losses for the different types of firms. For each firm x, we denote the loss as l_x. These loss equations will be used in the empirical section to compute the amount of losses suffered by individual market participants based on different shock sizes and default waterfall structures.[12]

For a purely bilateral firm i, losses given a payment equilibrium are composed only of bilateral losses. These bilateral losses are equal to the difference in expected payments versus received payments plus IM in bilateral transactions.

$$l_i = \underbrace{\sum_{k \neq i} [\bar{p}_{ki} - (p_{ki} + z_{ki})]^+}_{\text{bilateral}}, \quad \forall i \in B. \tag{25}$$

[11] An example of this bankruptcy cost on clearing members can be seen in the default of a clearing member of Nasdaq OMX CCP in late 2018, which caused clearing members to become creditors to the order of €107 million.

[12] In Appendix B we provide a detailed description of the different types of losses in our model.

For a client firm j, losses given a payment equilibrium are composed of bilateral plus client clearing losses, which occur when clients do not receive their full obligations in client clearing transactions.

$$l_j = \underbrace{\sum_{k \neq j} [\bar{p}_{kj} - (p_{kj} + z_{kj})]^+}_{\text{bilateral}} + \underbrace{\sum_{k \in M_j} \left(\bar{q}_{0kj} - q^m_{0kj}\right)}_{\text{client clearing}}, \quad \forall j \in C. \tag{26}$$

For a clearing member k, losses given a payment equilibrium are composed of bilateral and direct clearing, client clearing, and waterfall losses, which occur when waterfall contributions are used to cover obligations of a separate defaulting clearing member. We denote the waterfall losses by $\hat{\gamma}_k$, and we show how they can be computed in Appendix B.

$$l_k = \underbrace{\sum_{i \neq k} [\bar{p}_{ik} - (p_{ik} + z_{ik})]^+}_{\text{bilateral and cleared}} + \underbrace{\sum_{i \in C_k} \left([\bar{q}_{ik0} - (q^c_{ik0} + z_{ik0})]^+ + \bar{q}_{0ki} - q_{0ki}\right)}_{\text{client clearing}} + \underbrace{\hat{\gamma}_k}_{\text{waterfall}}, \quad \forall k \in M. \tag{27}$$

3.2 Losses for the CCP

We now describe the losses of the CCP, which may be different from the losses of other types of firms. The CCP's losses can come from two channels. First, the CCP has a capital contribution of b_0 that it could lose if used in the default waterfall. We denote the amount used in equilibrium by $\hat{b}_0$, which is equal to the amount the CCP needs to cover in excess of received payments, IM, and own default fund contributions of defaulting clearing members:

$$\hat{b}_0 = \min\left(\sum_{k \in M}\left(\bar{p}_{0k} + \sum_{i \in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i \in C_k} q^m_{ik0} + z_{k0} + \gamma_k) \wedge (\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0})\right), b_0\right) \tag{28}$$

The CCP can also suffer losses from payment shortfalls that exceed its total waterfall resources (IM, guarantee fund, and capital contribution). Since the CCP has a balanced book, the remaining shortfall in the payments it receives after using its default waterfall is equal to the equilibrium stress that it suffers. Recall that s_0, defined in Equation (20), denotes the stress of the CCP. The CCP's losses given a payment equilibrium are thus:

$$l_0 = \hat{b}_0 + s_0. \tag{29}$$

3.3 Systemic Losses

With losses defined for all firms in the system, we can sum up all losses to get an aggregate *systemic loss*. This term, which we denote as L, also captures the total shortfall in payments received across all types of firms.

$$\begin{aligned} L = &\sum_{i \neq 0} \left[\sum_{k \neq i} \bar{p}_{ki} - \sum_{k \neq i} \left((p_{ki} + z_{ki}) \wedge \bar{p}_{ki} \right) \right] + \sum_{k \in M} \left[\sum_{i \in C_k} \left(\bar{q}_{ik0} - (q^c_{ik0} + z^0_{ik}) \wedge \bar{q}_{ik0} + \bar{q}_{0ki} - q_{0ki} \right) \right] \\ &+ \sum_{i \in C} \left[\sum_{k \in M_i} (\bar{q}_{0ki} - q^m_{0ki}) \right] + \sum_{k \in M} \left[\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0} - (p_{k0} + \sum_{i \in C_k} q^m_{ik0} + \gamma_k) \right]^+ . \end{aligned} \tag{30}$$

This equation can be derived by summing Equations (25), (26), (27) and (29) across all clearing members, clients, bilateral firms, and the CCP. Note that this equation is decreasing in all of the payments made in the system. Therefore a waterfall structure that increases payments made in equilibrium will also lower systemic losses. We will use this systemic loss metric extensively in the empirical section for social welfare analysis, along with the individual losses defined above for different types of firms.

4 Empirical Implementation: U.S. Credit Default Swaps Market

To assess the implications of the default waterfall structure we perform an empirical exercise using data from the U.S. credit default swap (CDS) market. The U.S. CDS market provides an ideal setting as it has one active CCP, ICE Clear Credit, and an active non-cleared portion of the market.[13] As a result, we can assess the potential losses this market faces and the consequences of

[13]This is a privately held, for-profit company that cleared more than 97 percent of the notional value of CDS contracts on the date of our analysis. The only other CCP in this market is CME Clearing, which in 2014 cleared less than 3 percent of the contracts and has since announced its exit from the market.

changes in the default waterfall or central clearing rates without concerns about CCP competition (Glasserman et al. (2016)).

The analysis employs CDS transaction and position level data from the Depository Trust & Clearing Corporation (DTCC), which includes all CDS contracts that involve at least one U.S. counterparty or a U.S. reference entity. Table IV provides a summary snapshot of market participant positions and counterparty relationships as of October 3, 2014, the date that corresponds to the stress scenario we discuss in the subsequent section. In accordance with previous empirical literature, we find a core-periphery network structure with the CCP intermediating a few highly interconnected clearing members (30) and many sparsely connected clients (364) and bilateral firms (534).

Table IV. Average Values for CDS Participants and Counterparty Network

	#	*Reference Entities*		*Gross Notional*		*Net Notional*		*Counterparties*
		Cleared	Bilateral	Cleared	Bilateral	Cleared	Bilateral	
CCP	1	435	-	6,688,393	-	0	-	389
Member	30	141	1,051	98,843	473,078	38	1,775	162
		(145)	(839)	(132,625)	(628,684)	(10,706)	(20,687)	(177)
Client	364	4	35	2,127	3,555	-3	-280	6
		(5)	(78)	(6,817)	(15,510)	(3,614)	(9,227)	(4)
Bilateral	534	-	12	-	1,305	-	116	3
		(-)	(32)	(-)	(13,795)	(-)	(1,269)	(3)

Note: The table presents summary statistics on the four types of firms, CCP, Members, Clients, and Bilateral firms, in the credit default swaps market as of October 3, 2014. The statistics present the mean (standard deviations) number of CDS reference entities traded, and gross notional and net notional positions divided by whether the CDS contract was cleared or remained bilateral (non-cleared). Gross notional and net notional figures are presented in \$ millions, with positive (negative) net notional values referring to CDS protection purchases (sales). Standard deviations for all values are in the parentheses.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation, and CCPView Clarus Financial Technology.

Table IV highlights the distribution of CDS positions and contract types intermediated through the network. We find that clearing members and bilateral firms are generally purchasers of protection, whereas clients are sellers. This is consistent across both cleared and bilateral positions. These positions suggest that clients may pose a risk under a stress scenario, given that they would likely to be making more payments than they would be receiving under a market shock scenario. An additional point of note is the large standard deviations for the net notionals, which indicate that there is significant heterogeneity in positions such that making broad generalizations is difficult.

One limitation of the DTCC data is information on which clearing members clear on behalf of client firms. The data indicate the direct notional position that each client firm has with the CCP, but do not indicate the clearing member that the position cleared through. We assume that each client firm clears through only a single clearing member.[14] If a client has traded with only one clearing member, we assume the client clears through that clearing member as well. In cases where clients have historically traded with multiple clearing members, we select the clearing member with which the client has historically held the largest portion of their positions. This assignment rule divides the client firms across the clearing members fairly evenly, with a minimum to maximum range of 7 to 37 clients across the 30 clearing members.[15]

4.1 Calibrating the Model and the Shock

We calibrate the parameters of the model based on the historical CDS positions, prices, and market conventions of the data period. To calibrate the IM, we compute a 99.5% VaR estimate with a 10-day margin period of risk for bilateral margin (BCBS and IOSCO (2015)). We use public 10-K filing amounts for ICE Clear Credit, as reported in Table V, to calibrate the CCP's default waterfall. In the case of cleared IM we divide the margin across clearing members and clients using the estimation method in Equation (19) using an estimated 99.5% VaR with a 5-day margin period of risk. This margin calculation method is used for the CCP to capture the fact that CCP margin calculations can be based on factors beyond VaR.[16] Finally, we assume that client clearing IM follows the same method as the CCP, though clearing members in practice can charge higher IMs than the CCP.

To calibrate the capital buffer, we use weekly inflows and outflows of VM at the firm level from 2010 through 2016 following from Section 5.3 in Paddrik et al. (2020). The method involves first computing for each firm the net VM payments divided by the gross notional value of all its contracts over the period of observation. We then find the largest negative change for each firm, and assign the capital buffer this value with respect to the portfolio values on the date of the shock. Our method assumes that larger capital buffers are applied to firms with historically higher VM payment variance.

[14] This assumption is in line with empirical evidence by the European Central Bank on client clearing (Kahros et al. (2020)), which shows that the vast majority of clients clear through only one clearing member.

[15] From public quarterly reporting, ICE reports that only 13 of its clearing members actually clear on behalf of clients.

[16] Several additional factors beyond portfolio volatility have been considered such as liquidity, default, concentration, and correlation. Some CCP risk models have introduced correlation uncertainty risk charges. See Li and Cheruvelil (2019) for details on how CDS portfolio risk measures can change across correlation regime shifts.

Table V. Principal Elements of the Waterfall Structure of ICE Clear Credit

Tranche	Total Amount
Initial Margins	\$14.1 billion
Guarantee Fund	\$2.4 billion
CCP Capital	\$50 million
Member Assessments	Up to 3 times nondefaulting members' guarantee fund contributions

Note: The table presents the waterfall resources and assessment power of ICE Clear Credit as of December 2014. The initial margins and the guarantee fund are made up of U.S. Treasuries and cash (USD, CAD, EUR, GBP, JPY). *Source*: SEC EDGAR 10-K Filing.

We next apply a stress scenario on the portfolios of CDS positions to estimate the size of VM payments. Specifically, we apply the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) severely adverse global trading book shock, which prescribes a sudden widening of credit spreads for corporate, state, municipal, and sovereign debt according to their rating class (Federal Reserve Board (2016)). This systemic shock is applied to the positions data and widens CDS spreads in line with the largest historically observed single-day spread increase.[17] The widening of credit spreads results in a series of netted VM payments that need to be settled between firms in the system.

Table VI summarizes the estimated aggregate VM payments owed between firm types based on the calibrated CCAR shock. Each firm type owes roughly as much as it is collectively owed, in contrast to what the notional position summaries suggested regarding clients. This difference in client positions and VM flows highlights why notional position size on its own does not inform the riskiness of a firm. Overall, the VM flows suggest no single firm type is a particular risk to the financial system. However, it is worth noting that in the event that payments from clients to the CCP are not fulfilled, these payment obligation become the clearing members' responsibility. Given the scale of client cleared payment obligation relative to those of members, there is a size amount of payments members could become responsible for if several clients were to default.

[17]The date of October 3, 2014 is selected because the 2015 CCAR scenario is designed to be implemented on positions from this week.

Table VI. Variation Margin Payments under 2015 CCAR

		To				
		CCP	Members	Clients	Bilateral	**Total**
From	CCP	-	1.556	7.191	-	8.748
	Members	1.712	5.249	6.029	3.198	16.188
	Clients	7.036	6.157	0.008	0.003	13.204
	Bilateral	-	2.051	0.003	6.036	8.090
	Total	8.748	15.013	13.231	9.237	

Note: The table presents the aggregate variation margin payment obligations owed (in $ billions) by each firm type (rows) to each firm type (columns) under the 2015 CCAR.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

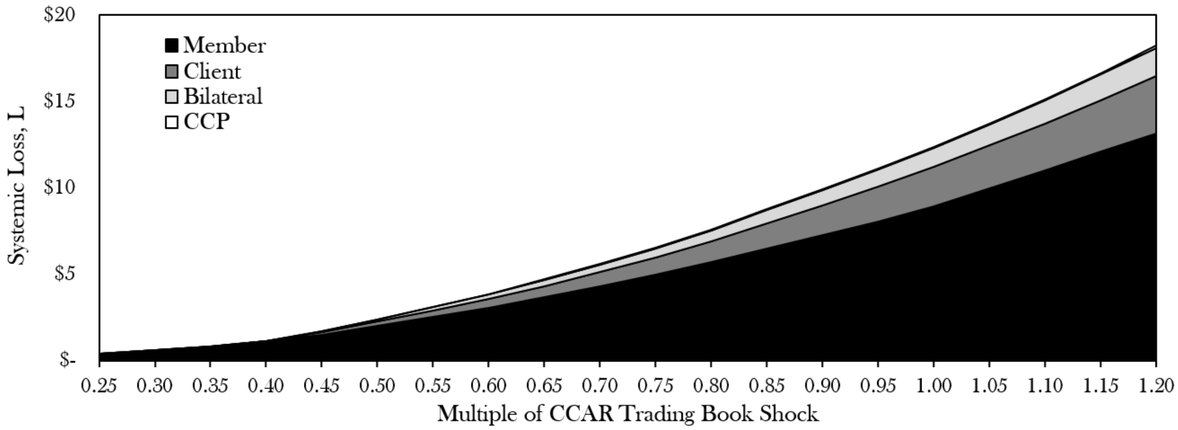
4.2 Estimating Systemic Loss for Different Shock Sizes

By computing the equilibrium for the calibrated model, we are able to estimate the systemic losses, L, under a range of shocks scaled using the 2015 CCAR scenario. A scaling factor $\alpha > 0$ is applied to all VM payments under the CCAR scenario, which results in different values of L in equilibrium.

Figure 4 depicts losses by firm type as a function of shock size up until the CCP's funded resources are completely consumed at an α around 1.2. At lower levels of α, most if not all losses are concentrated on clearing members, with only 9 and 18 percent of the total losses of $12.4 billion suffered by bilateral and client firms at the original CCAR shock ($\alpha = 1$). Member losses are much higher than client losses, and we break down the sources of these member losses below. Of note is the nonlinearly increasing scale of losses as α increases, indicating that contagion is playing a significant factor at high shock levels. Due to contagion and spillover effects, losses grow rapidly as the default of one firm causes its counterparties to default. The influence of these spillovers is affected by the level of interconnectedness in the network.

As the CCAR shock may cause losses to swell from different avenues, we further break down losses by type. Figure 5 separates the losses suffered by clearing members into those suffered from non-cleared positions, client clearing, and the CCP's waterfall. The most notable finding is the large degree of client clearing losses generated by client defaults at all levels of α. This demonstrates the risk that these indirect obligations pose for clearing members. A second takeaway is the large contribution of non-cleared positions to systemic losses when α is large. This highlights the outsized impact of network contagion on non-cleared (bilateral) losses under large shocks. Finally,

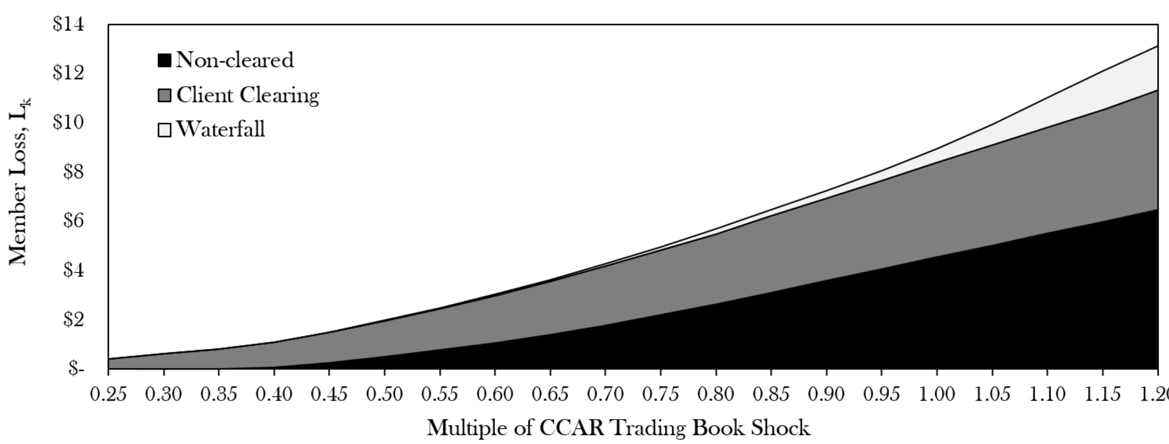
Figure 4. Aggregate Losses to the CDS Market under the 2015 CCAR



Note: The figure plots the aggregate amount of systemic losses (in \$ billions), split on firm type, under variations of the 2015 CCAR severely adverse global shock scenario. At lower multiples of the shock scenario most if not all losses are concentrated on clearing members and only 9 and 18 percent of the total losses of \$12.4 billion are suffered by bilateral and client firms at the original CCAR stress. The aggregate losses are nonlinearly increasing with the shock multiplier, indicating that contagion is playing a factor.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

clearing members suffer few losses from the default waterfall initially, but the level of losses grows substantially as α increases.

Figure 5. Member Losses under the 2015 CCAR



Note: The figure plots the aggregate amount of clearing member losses (in \$ billions), split by cause, under variations of the 2015 CCAR severely adverse global shock scenario. At lower multiples of the shock scenario most if not all losses are caused from covering client failures from client clearing positions. At higher multiples aggregate losses are caused mainly from non-cleared positions, and there are growing losses caused from the use of shared waterfall resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

While additional features such a collateral liquidity could create additional effects, we will assume in our model that the CCP requires cash for simplicity and ease of exposition. However, if CCPs were to lower their collateral requirements, then the losses in our model would likely increase. We provide some further insights into how this would affect systemic losses in Appendix C.

5 Counterfactual Analysis of Default Waterfall Structures

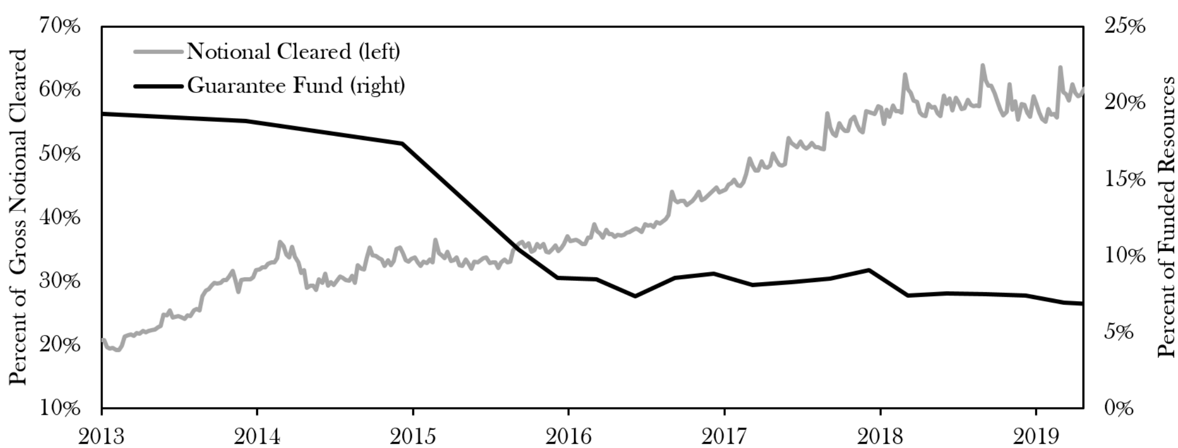
A guiding principle in the mandated introduction of central clearing to OTC derivatives markets is the belief that CCPs improve risk management for large exposures and reduce the consequences on the financial system of a large counterparty's default. It is thus important to determine to what degree the financial system is improved from the mandated incorporation of central clearing. Variations in the waterfall structure affect the benefits of clearing and systemic resilience, and no single standard has emerged globally. In this section, we assess default waterfall structures from the perspective of the social planner (i.e. financial regulator) whose objective is to promote financial system resilience by reducing systemic losses. We analyze this objective by examining how IM, guarantee funds, and central clearing rates each influence CCP resilience and how financial system loss is divided.

Of first order importance to the social planner is how the waterfall affects the payment continuity of the CCP, as this influences the scale of shocks the CCP can withstand. We test the effects of marginal changes to the CCP's allocation of guarantee funds vs. IM on payment continuity. Though logic may suggest that a default waterfall structure that holds a larger proportion of guarantee fund resources to IM can maintain payment continuity under greater levels of stress than one with less (holding all else equal), how economically meaningful this differential in structure is on losses is not obvious *ex ante*. We use our data on global CCP waterfall structures to highlight the large differences in IM to guarantee fund ratios in practice, and we assess the resilience provided by different regions' waterfall structures.

In addition, the social planner must also consider the tradeoff between greater default waterfall resources and lower central clearing rates. Requiring more waterfall resources from members places additional costs on them, which can lead them to reduce their central clearing participation. As Figure 6 highlights, the U.S. CDS market has witnessed substantial changes in waterfall resources and central clearing rates since the central clearing mandate took effect in early 2013. As the percentage of cleared positions increased, the percentage of the CCP's cross-subsidizing resources (i.e. guarantee funds) simultaneously declined. The figure provides evidence that incentivizing greater central clearing may have required lowering the proportion of guarantee fund resources.

Thus, it is important to analyze whether a financial system with greater waterfall resources but less clearing is more resilient than a financial system with less clearing but more waterfall resources. Our analysis shows that the answer depends on the scale of the shock. Though increased central clearing is generally more valuable in decreasing systemic loss, under the most extreme conditions greater waterfall resources are more beneficial relative to greater clearing rates. In addition, we find that as central clearing increases, clearing members' relative loss rates also increase, which may further intensify the pressure to reduce waterfall resources.

Figure 6. Central Clearing Rates and Guarantee Fund over Time



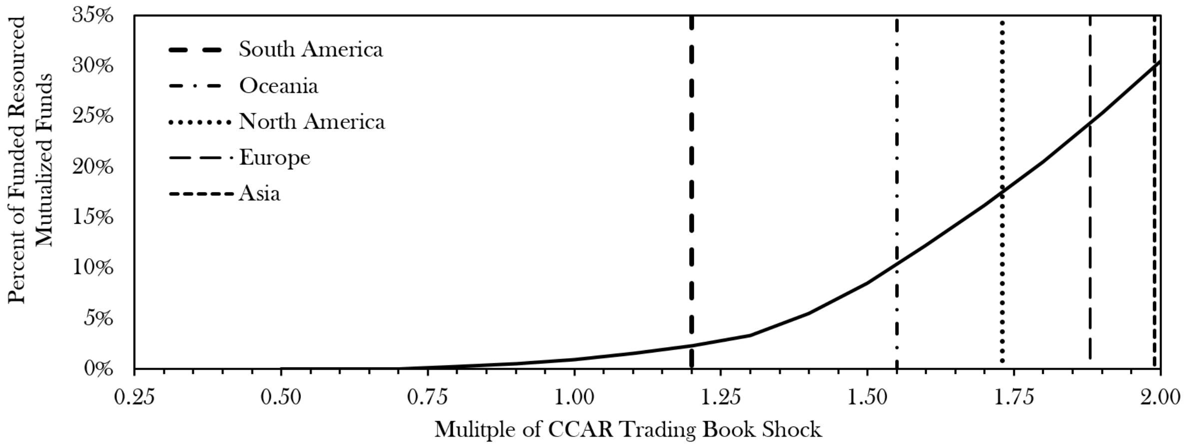
Note: The figure plots the percent of gross notional positions cleared and the amount of waterfall resources that can be mutualized.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation; CCPView Clarus Financial Technology; SEC EDGAR 10-K Filing.

5.1 Impact of Waterfall Resource Allocation

The social planner's first consideration in making the financial system resilient is to contemplate how the allocation of waterfall resources influences systemic loss. We begin by testing the impact of variations in the ratio of IM to mutualized funds (guarantee fund and CCP capital) on CCP resilience. Mutualized funds are more versatile than IM, as they can be used to cover the losses of any clearing member whereas IM can be used to cover only the losses of the clearing member that defaulted.[18] However, guarantee fund contributions may also be more costly to clearing members for this reason, and so CCPs may be reluctant to impose high levels of contributions on clearing members.

In practice, CCPs across different regions have settled on significantly different ratios of mutualized funds to IM. Our CCP survey data shows that guarantee funds and CCP capital make up around 31% of the total default waterfall for Asian CCPs, 26% for European CCPs, and 15% for North American CCPs (see Table II). We compare the resilience provided by these different choices by computing a *default frontier* for the CCP under different ratios of mutualized funds to total funded resources. The resulting curve, derived by finding the boundary at which $\dot{s}_0$ goes from zero to positive in Equation (20), indicates the minimum percentage of mutualized funds to total funded resources needed to maintain payment continuity for a fixed level of funded resources as a function of the shock size.

Figure 7. CCP Default Frontier: Payment Continuity vs. Cross-Subsidization



Note: The figure plots the CCP's default frontier, the point at which the CCP would either need to implement assessments or reduce its payments, under multiples of the 2015 CCAR severely adverse global shock scenario. As the shock increases in size, a larger percentage of mutualized funds are needed to maintain payments. The vertical lines represent the level of shock resilience the CCP would have at each of the regional average levels.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

Figure 7 shows that the default frontier for different regions varies significantly. At the low end, South American CCPs are resilient against shocks up to $\alpha = 1.2$, whereas on the high end Asian and European CCPs can survive shocks of greater than $\alpha = 1.75$. Emerging market CCPs thus

[18] In the appendix we evaluate Initial Margin Haircutting (IMH), which allows the CCP to use the IM of non-defaulting clearing members at the end of the waterfall.

tend to have less resilient default waterfall structures than those in advanced economies.

Overall, the CCP is more resilient when the ratio of mutualized funds to IM is large. Our results thus suggest that higher levels of mutualized funds relative to IM can provide benefits to clearing members by increasing the resilience of the CCP and ensuring payment continuity. However, clearing members also pay the costs of the guarantee fund contributions, which must be weighed against the benefits they receive. Therefore, requiring high levels of contributions could also disincentivize participation in central clearing. We analyze the impact of changes in central clearing rates in the next section.

5.2 Impact of Central Clearing Participation

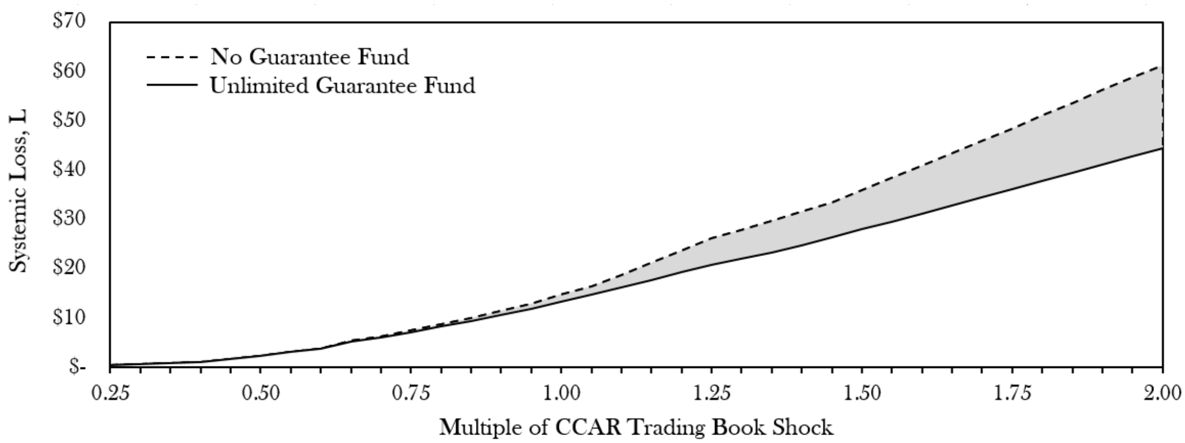
The social planner must also consider the tradeoff between central clearing participation and default waterfall resources. Although requiring greater waterfall resources increases the CCP's resilience, it also imposes substantial capital costs on clearing members. Such costs have been well established in the literature, for instance in Ghamami and Glasserman (2017). If central clearing becomes too costly compared to non-cleared positions, then clearing members may switch away from central clearing. This decrease in central clearing rates in turn makes the system less resilient. In such a case, the net impact of requiring higher waterfall contributions could actually be harmful to the financial system.

Before comparing the impact of variations in central clearing rates, let us first compute the maximum impact the default waterfall structure can have on systemic losses. We compute the systemic loss, L, under two extreme waterfall structures in Figure 8. The first scenario sets the guarantee fund resources to zero. The second scenario allows for unlimited guarantee fund resources such that the CCP can always maintain payment continuity. Comparing these two scenarios gives a measure of the maximum decrease in systemic losses that default waterfall resources could provide.

As Figure 8 highlights, once IM held by the CCP are depleted due to covering shortfalls in payments, the difference in losses (in gray) grows quite rapidly if the shared guarantee fund resources are limited. The difference between the two scenarios under the original CCAR shock level is 11%, but the difference grows to over 50% at twice the CCAR shock level.

Of economic importance is the rate of central clearing at the time of our study: 31 percent of notional CDS positions as of October 6, 2014. As Figure 6 highlighted, the rate of clearing has

Figure 8. CCP Resilience and Systemic Losses



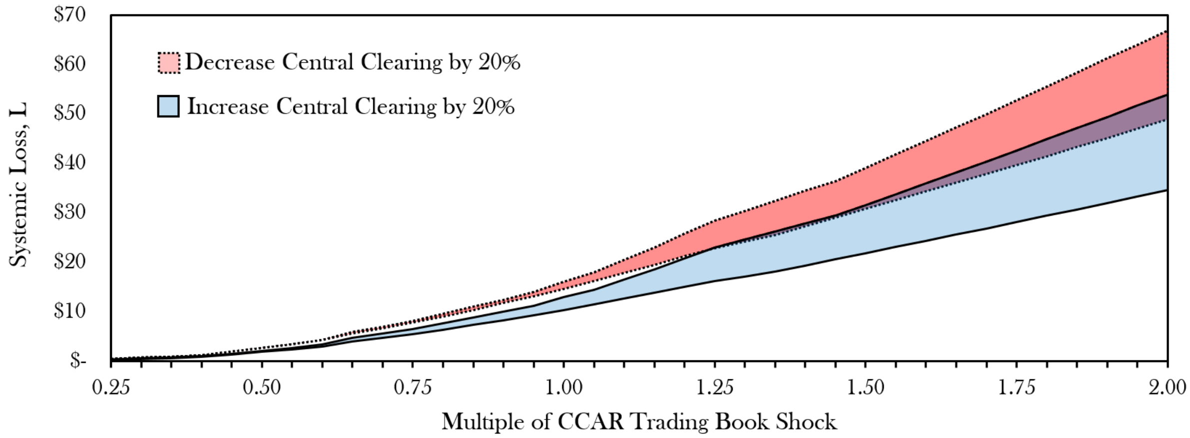
Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. The solid line represents the systemic loss under the scenario where guarantee funds are unlimited. The dotted line represents the systemic loss under the scenario where guarantee fund is set to zero. The gray area represents the difference in systemic losses suffered under different degrees of risk sharing (through the sizing of the guarantee fund). The additional losses show how consequential the CCP's full payment continuity is to the entire financial system of payments.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

grown substantially in more recent years to 60 percent; however, the proportion of default waterfall resources has declined. The large magnitudes of these shifts can make it difficult for a financial regulator to determine whether potential systemic losses have increased or decreased, holding all else equal.

The social planner must weigh this tradeoff in default waterfall resources versus central clearing rates while considering the impact it may have on firms' choices. As we do not directly model the endogenous responses of clearing members to the default waterfall structure, as done in Wang et al. (2020), we can only compute the bounds of outcomes by exogenously varying the rates of central clearing and looking at the extremes of default waterfall structure on systemic loss. Figure 9 plots two perturbations on the rate of central clearing, one increasing (blue) and one decreasing (red) clearing by 20 percent. We do this by increasing (decreasing) the relative size of payments and initial margins proportionally depending on whether the position was centrally cleared or not.

At low α levels, the increased central clearing wedge is below the reduced central clearing wedge irrespective of the default waterfall structure. When $\alpha < 1.25$, the benefits of the higher clearing rate thus dominate the impact of the lower waterfall resources. However, under more extreme

Figure 9. Central Clearing Rate and Systemic Losses



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario, shaded by two regions representing the difference in systemic losses suffered under differing degrees of risk sharing (through the sizing of the guarantee fund). The two regions represent what happens if there was a hypothetical increase (blue) or decrease (red) in the rate of central clearing positions. The figure highlights that the rate of central clearing is an important determinant in the size of the systemic loss the financial system suffers irrespective of the default waterfall. However, the strength of the waterfall plays a more significant role the higher the rate of central clearing is, as depicted by the difference in the width of the blue region versus the red region.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

conditions where $\alpha \geq 1.25$, default waterfall resources begin to impact losses. A 20 percent lower central clearing rate with a strong default waterfall results in less losses than a 20 percent greater central clearing rate with a weak default waterfall. To protect against higher shock levels, it is thus important that the social planner enforce a sufficient guarantee fund, even if it lowers the rates of central clearing. This figure suggests that recent trends towards higher central clearing rates but lower guarantee fund levels may have decreased systemic losses against small shocks while increasing systemic losses against larger shocks.

6 Conclusion

The design of a default waterfall is critical to a CCP's ability to fulfill its payment obligations under stress and reduce the potential of systemic losses from derivatives trading. However, there are inherent difficulties in measuring the impact and conflicts of interest in how waterfall structures allocate losses. The unsettled nature of this issue is reflected in the wide variety of default waterfalls adopted by CCPs around the world.

In this paper, we present a methodology to measure the systemic losses that can arise in derivatives markets and how these losses are influenced by the resilience of the CCP's default waterfall in covering its payment obligations. Measuring systemic loss is difficult due to the highly interconnected nature of derivatives trading firms and the spillover effects linked to contagion and collateral fire sales. We overcome this challenge by incorporating various waterfall elements into an equilibrium model that distributes losses among individual firms for centrally cleared and bilateral OTC derivatives positions and allows for the consideration of fire sales.

We perform a series of counterfactual analyses that evaluate how different default waterfall elements such as the ratio of initial margin to guarantee funds and the rate of central clearing affect systemic losses. We find that higher relative levels of guarantee fund contributions increase CCP resilience, reduce systemic loss, and even reduce clearing member losses. However requiring high levels of waterfall resources may also disincentivize clearing members from clearing trades due to the additional capital costs.

CCPs face a choice between greater waterfall resources and potentially lower rates of central clearing, or having less waterfall resources but with higher rates of central clearing. We find that due to the non-linear nature of contagion, the benefits of greater central clearing rates generally dominate the benefits of increased CCP waterfall resources, especially at lower levels of shocks. However, this means that regulators should be wary of the incentives of clearing members and continue to enforce minimum CCP resiliency standards, as periods of market calm are likely to exert downward pressure on guarantee funds.

Our results highlight several areas for further investigation. First, a better understanding is needed of potential interactions among multiple CCPs operating in the same marketplace. CCPs are likely to affect each other through interconnections with shared clearing members. A disruption in clearing member payments across multiple CCPs and funding networks could cause additional sources of contagion and fire sales. Netting down these payments across CCPs may reduce short term payment demands and collateral sales. Second, further analysis can be performed on how systemic losses are influenced by the structure of different derivatives relationships and the types of central clearing models available. Finally, clearing incentives are not simply based on risk sharing allocation; other clearing costs such as additional margin add-ons can play a role that should be studied in future research.

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Appendix A Fictitious Stress Algorithm

In this section we describe the fictitious stress algorithm that can be used to find the clearing vectors for p and q. The essence of the algorithm is simple. First, determine each market participant's payout, assuming that all other market participants satisfy their obligations. The iterative algorithm starts with the assumption that no market participants are under stress. If this is a feasible outcome, then it is the outcome of the clearing equilibrium. If, however, some market participants are stressed, then we update the payment vector given the stress and check for additional stress. The algorithm terminates when the level of additional stress added to the financial system is below a given tolerance threshold.

I At step m of the algorithm, let Λ_m be the set of stressed market participants. Denote p_h and q_h as the payment vector for firm h. Initialize $\Lambda_0 = \{\}$ and $p_h = \bar{p}_h, q_h = \bar{q}_h$.

II Compute the stress s_h at each firm h given the payment vectors. Recalculate Λ_m as the set of all market participants such that entry h of the stress vector is positive, i.e. $s_h > 0$.

III Terminate if there are no stressed firms, or if the increase in total stress $S_m \equiv \sum_h s_h$ is lower than a minimum threshold, $S_m - S_{m-1} < \epsilon$.

IV Otherwise determine the clearing payments by recalculating the maximal values of p_h, q_h $\forall h \in \Lambda_m$. Iterate $m \to m + 1$ and repeat starting at II.

Following Theorem 3.1 from Rogers and Veraart (2013), the stress algorithm above produces a well-defined sequence of payment vectors p_h, q_h which reaches the clearing vector of Φ. Similar algorithms have been used to find the clearing vector(s) in Blume et al. (2011) and Elliott et al. (2014).

We note that this algorithm can be modified to incorporate collateral illiquidity as well. This would involve extra steps in the algorithm to recalculate the updated price of the collateral depending on how much was liquidated in the previous round.

The updated algorithm with illiquidity is the following:

I At step m of the algorithm, let Λ_m be the set of stressed market participants. Denote p_h and q_h as the payment vector for firm h. Let Δ_m be the total value of collateral liquidated. Initialize $\Lambda_0 = \{\}$, $p_0 = \bar{p}, q_0 = \bar{q}$, and $\Delta_m = 0$.

II Compute the stress s_h at each firm h given the payment vectors and the collateral liquidated. Recalculate Λ_m as the set of all h market participants such that entry h of the stress vector is positive, i.e. $s_h > 0$.

III Terminate if there are no stressed firms, or if the increase in total stress $S_m \equiv \sum_h s_h$ is lower than a minimum threshold, $S_m - S_{m-1} < \epsilon$.

IV Otherwise determine the remaining clearing payments and the collateral liquidated by recalculating the maximal values of $p_h, q_h \ \forall h \in \Lambda_m$ given a value of collateral liquidated Δ_{m-1}. Then compute the corresponding total liquidations Δ_m from these new payments. Iterate $m \to m+1$ and repeat starting at II.

Appendix B Types of Losses

In this Appendix we provide a detailed description of the different types of losses in our model. Losses in our model represent shortfalls suffered by an institution as a result of the shock. For firms other than the CCP, there are three types of losses that are possible in our model: bilateral losses, client clearing losses, and default waterfall losses. A bilateral loss arises when a firm does not receive all the payments it is owed by another firm in a bilateral transaction. A client clearing loss arises when a client or clearing member does not receive the resources it is owed in a client clearing transaction. A default waterfall loss arises when a clearing member has its guarantee fund contribution used to cover another clearing member's shortfall to the CCP. We define each type of loss precisely below.

The first type of loss comes from bilateral transactions. For a firm i, these losses are given by

$$\text{Bilateral Loss} = \sum_{k \neq i} [\bar{p}_{ki} - (p_{ki} + z_{ki})]^+. \tag{B.1}$$

Bilateral losses can accrue to clearing members, clients, and bilateral firms. For CCP members, we also include direct clearing losses from the CCP in this category.

The second type of loss comes from client clearing transactions. Such losses are experienced by clients that have shortfalls in funds owed to them by the CCP. Clearing members also experience client clearing losses when they do not receive the entirety of the funds that they are liable to pass through for the obligation. client clearing losses are given by the following set of equations

$$\text{client clearing Losses} = \begin{cases} \sum_{k \in M_i} (\bar{q}_{0ki} - q^m_{0ki}) & \forall i \in C, \\ \sum_{i \in C_k} ([\bar{q}_{ik0} - (q^c_{ik0} + z_{ik0})]^+ + \bar{q}_{0ki} - q_{0ki}) & \forall k \in M, \end{cases} \tag{B.2}$$

where the first equation is the losses for clients and the second equation is the losses for clearing members.

Finally, the third type of loss comes from clearing members that have their guarantee fund contributions used by the CCP to cover another clearing member's default. We denote the default waterfall loss for firm k as $\hat{\gamma}_k$.[19]

We now describe how $\hat{\gamma}_k$ can be calculated. Recall that in calculating losses, we consider only funds used by a clearing member to cover obligations of *other* clearing members. Guarantee funds that are used to cover a clearing member's own default are thus not counted in losses, but guarantee funds used to cover another clearing member's default are counted in losses.[20]

Recall that the total guarantee fund contributions of a clearing member k are denoted γ_k. We denote the amount of guarantee fund used to cover a member's *own obligations* as $\dot{\gamma}_k$. This guarantee fund is taken from clearing member k when clearing member k is short payments to the CCP and owes more than its IM z_{k0} can cover. The value of $\dot{\gamma}_k$ is given by

$$\dot{\gamma}_k \equiv \min\left(\left[\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0} - \left(p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}\right)\right]^+, \gamma_k\right), \quad \forall k \in M. \tag{B.3}$$

Suppose clearing member k does not use all of its own contributions γ_k, and losses to the CCP from other firms go beyond the resources of the CCP's capital contribution b_0. Clearing member k would have part, if not all, of its guarantee fund contribution used to cover the obligations of other clearing members. Guarantee fund contributions will be taken pro rata the remaining contributions of each clearing member to cover the CCP's shortfall.

Let the additional amount that the CCP needs to cover if it uses the preceding waterfall layers be denoted as g_0:

$$g_0 \equiv \left[\sum_{k\in M}\left(\bar{p}_{0k} + \sum_{i\in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0}) - \dot{\gamma}_k\right) - b_0\right]^+. \tag{B.4}$$

As a function of g_0, the guarantee fund losses for clearing member k, $\hat{\gamma}_k$, are given by the following:

$$\text{Default Waterfall Loss } \hat{\gamma}_k = \min\left(\frac{\gamma_k - \dot{\gamma}_k}{\sum_{j\in M} \gamma_j - \dot{\gamma}_j} g_0, \gamma_k - \dot{\gamma}_k\right), \quad \forall k \in M. \tag{B.5}$$

[19] In Appendix D.4 we also consider default waterfall losses due to the use of assessments and IMH at the end of the waterfall.

[20] Note that our notion of losses is not directly equivalent to our previous notion of stresses. While the CCP does not transmit stress to its clearing members until the default waterfall is fully depleted, clearing members will suffer default waterfall losses as soon as their guarantee fund is used to cover other clearing member obligations.

Appendix C Collateral Liquidity

In this appendix, we assess the impact of collateral quality on systemic losses. Forced fire sales of illiquid collateral can create deadweight losses. Equation 1 states that, for a given proportion of collateral liquidations Δ, the final normalized collateral price is equal to $\pi = G(1, \Delta) = e^{-\lambda\Delta}$ for some $\lambda > 0$. A larger λ corresponds to a less liquid asset. The value of Δ is derived from the payment vector and the stress equations. Note that as the value of π affects the final payment equilibrium itself, this entails an additional fixed point calculation for the final payment vector along with π. Details of the calculation are provided in Appendix A.

Given π, Δ and a total initial collateral value of IM and capital buffers, the total deadweight loss in our system, D, can be defined as

$$D = \Delta(1 - \pi)\left[\sum_i \sum_k z_{ik} + \sum_{i \in C} \sum_{k \in M_i} z_{ik0} + \sum_i b_i\right] \tag{C.1}$$

This notion of deadweight loss corresponds entirely to the amount of liquidations of collateral in the payment equilibrium, and can be thought of as the net welfare loss of the system. The other forms of losses are transfers between agents, and are thus not net welfare losses, whereas the deadweight loss represents money that is taken out of the system. Note that we only consider the reduction in value of assets that are actually liquidated, and not reductions for collateral that is unliquidated. This is because we assume in the long run the collateral will recover its value. Such an assumption is standard in the literature, see for instance Allen and Gale (2000) and Acemoglu et al. (2015).

Next, we re-estimate losses by incorporating the influence of collateral illiquidity and fire sales. We employ the Federal Reserve's CCAR scenario once again, which contains estimates of the price impact consequences of the scenario on collateral assets by credit quality.[21] Table C.1 presents the price impacts on corporate lending in advanced economies, which we use as a proxy in our collateral liquidity analysis. Using the formulation of illiquidity given in Equation (1), we can back out the value of λ that produces an equivalent price impact under an $\alpha = 1$ shock. The price of the collateral used for IM and the capital buffer will decrease as more of the collateral is liquidated. Appendix A describes in detail how the equilibrium algorithm is updated to account for illiquidity.

Figure C.1 plots the size of systemic loss, L, and deadweight loss, D, under different assumptions of collateral quality through the parameter λ. By comparing these different scenarios we can get a sense of the increase in systemic losses from lower collateral liquidity and derive an estimate of

[21] Information is from the Federal Reserve 2015 CCAR Severely Adverse Scenario spreadsheet (https://www.federalreserve.gov/supervisionreg/ccar-2015.htm) on advanced country corporate loans.

Table C.1. Advanced Economy CCAR Shock on Corporate Loans By Quality

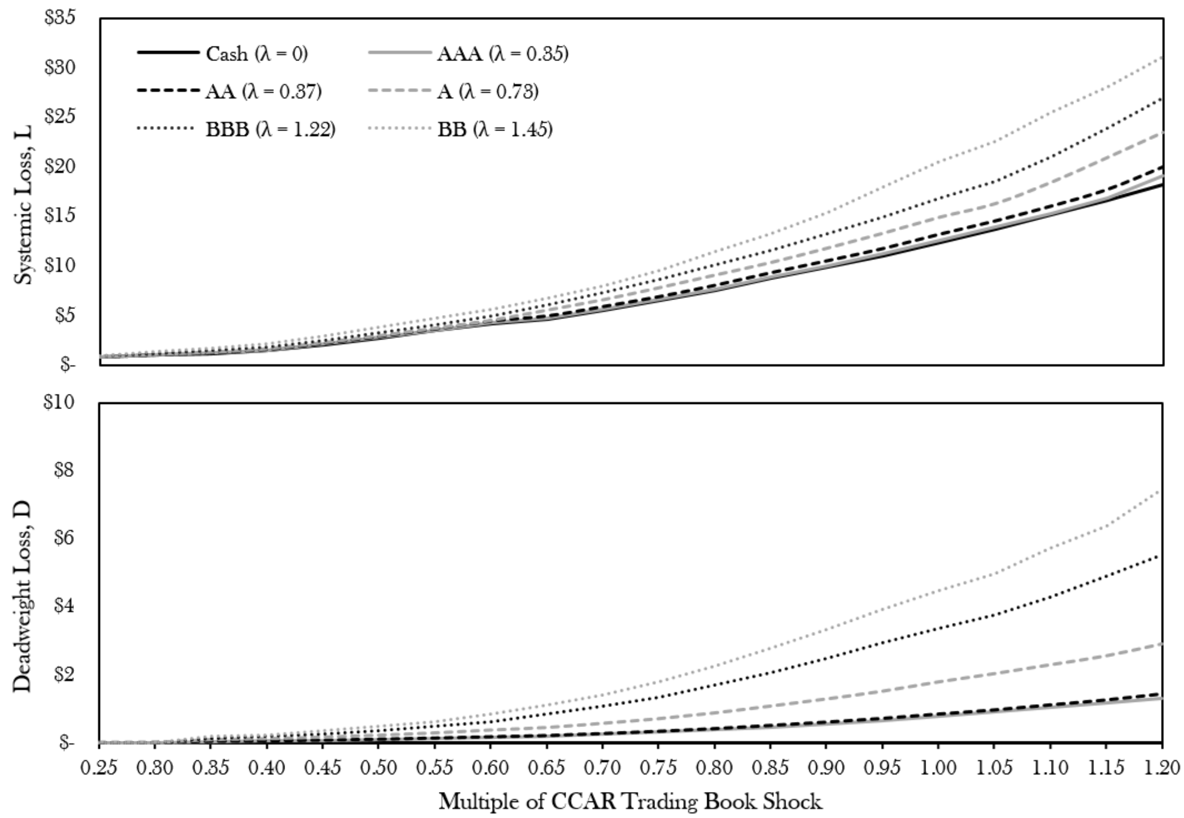
Loan Quality	**Value Loss**
AAA	-6.2%
AA	-6.7%
A	-13.4%
BBB	-22.6%
BB	-26.9%

Note: The table presents the decrease in value of Corporate Loans based on the Federal Reserve 2015 Severely Adverse CCAR Shock for advanced economies.
Source: Federal Reserve CCAR Severely Adverse Market Shock.

the impact of fire sales on losses. We find that as the average collateral quality decreases, the impact on systemic and deadweight losses sharply increases. At $\alpha = 1.2$, when collateral quality is set to BBB the systemic losses are 40 percent greater than when compared to AAA collateral. The consequences are even more dramatic when comparing deadweight losses, as BBB collateral produces three times the level of deadweight losses. Overall, the comparative results highlight how collateral illiquidity can greatly amplify the losses from stress and furthermore speaks to the importance of eligible collateral standards and haircuts.[22] Importantly, CCPs for the most part require highly liquid collateral in practice and high haircuts on lower grade collateral, which both help to mitigate these losses.

[22] See the CFTC's Margin Rule.

Figure C.1. Collateral and Financial System Losses



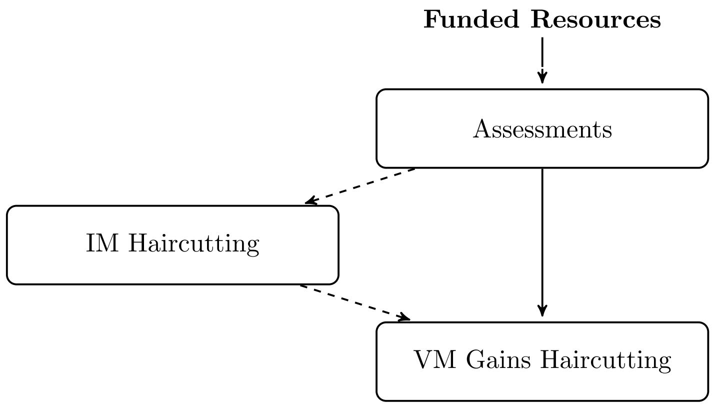
Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. Each line represents a different level of collateral liquidity, λ, which accounts for the fire-sale like effects that could occur as firms sell collateral to fulfill payments. At low λ values, i.e. high liquidity, and low shock multiples additional loss consequences are minimal. However, the consequence of high λ values, i.e. low liquidity, or large shock multipliers are significant, creating greater systemic losses.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

Appendix D End-of-Waterfall Mechanisms

The final stages of the default waterfall are based on mechanisms that are not as widely implemented, and their exact rules can vary significantly across CCPs. In addition, their placement at the end of the waterfall means they are used only in rare situations where all previous levels have been used. There is thus limited historical precedent for these recovery mechanisms. These factors mean that they are not well understood in practice, and therefore research and testing on their effectiveness is crucial.

The final stages include assessments, initial margin haircutting, and variation margin gains haircutting, depicted in Figure D.1. Assessments are the most commonly employed of these three mechanisms. At the time of the loss, if the CCP has already used up the previous stages of the

Figure D.1. Stages of the End-of-Waterfall



Note: The chart depicts the series of end-of-waterfall mechanisms in the waterfall that will be accessed if the funded resources are insufficient to cover total default losses in the event of a clearing member or client default. The solid arrows depict the most common set of waterfall resource contingencies, with the dashed arrows showing alternative contingency paths.
Source: Authors' creation.

default waterfall, it will have the ability to assess further funds from its clearing members. There is usually a cap on the amount that can be assessed from each clearing member. This cap varies across CCPs. For instance, ICE Clear Credit imposes a cap of 300 percent of guarantee fund contributions, while CME Group imposes a cap of 275 percent of guarantee fund contributions for a single clearing member default and 550 percent of guarantee fund contributions for multiple member defaults. Because assessments are not prefunded, they are less burdensome to clearing members than IM or guarantee fund contributions, which must be given in advance. However, being unfunded also means that assessments may not be available from clearing members that are under stress. Thus assessments may not provide the CCP sufficient resiliency if many clearing members are under stress simultaneously and unable to pay the assessed amounts.

Initial margin haircutting (IMH) allows the CCP to take the IM of clearing members that did not default. This opens up a very large pool of funds to the CCP, as IM values are usually quite large. However, IMH has the downside of potentially distorting clearing member incentives and causing clearing members to enter contracts that require less IM. It can also conflict with regulatory capital requirements for bank holding companies. Basel regulations place higher risk weights on non-bankruptcy remote capital held with the CCP. Utilizing IMH could thus raise clearing costs for clearing members under the current regulatory system. Finally, in some jurisdictions such as the U.S., IM may also be held at a third party that the CCP does not have access to in a crisis. Because of these issues, IMH tends to be an unpopular mechanism (ISDA (2013)).[23] Though IMH

[23]Though legally segregated operationally commingled is meant to prevent IMH in event of default of a member

is an option for some CCP jurisdictions, no CCP currently implements it in their default waterfalls (as suggested by the layout of Figure 1).

Variation margin gains haircutting (VMGH) allows the CCP to continue receiving VM payments that are owed to it but to reduce the VM payments that it owes. In theory, VMGH could allow the CCP to withstand an unlimited loss, as the CCP could reduce its VM payments to zero. By reducing the payments that the CCP owes, VMGH spreads losses to other clearing members and thus is a form of risk sharing. Also, placing losses on agents that are gaining in the market may reduce overall stress compared with placing losses on already-losing clearing members. However, as VMGH takes an equal pro rata approach to loss distribution, it can be harmful to firms that are hedged, and therefore generate contagion losses outside of the derivatives market.

D.1 Member Assessments

Each of the m clearing members is assessed according to the riskiness of its portfolio. This assessment is usually done via a similar method as in determining guarantee fund contributions. The difference is that assessments are not prefunded and must be collected at the time of the shock from the remaining capital of the clearing members. The total amount that can be assessed thus depends on the capital that each clearing member has available. We assume that assessments have a lower priority than a firm's VM payment obligations. Firms that are under stress cannot be assessed, and firms that are not under stress can contribute only up to their capital remaining after VM payments.

Given the payments that it receives, a clearing member k will have an amount of capital left over, θ_k, of

$$\theta_k \equiv \Bigg[b_k - \Bigg[\sum_{i \neq k} \bar{p}_{ki} + \sum_{i \in C_k} (\bar{q}_{0ki} + \bar{q}_{ik0}) - \sum_{i \neq k} ((p_{ik} + z_{ik}) \wedge \bar{p}_{ik}) - \sum_{i \in C_k} ((q^c_{ik0} + z_{ik0}) \wedge \bar{q}_{ik0}) + q_{0ki}) \Bigg]^+ \Bigg]^+ . \quad \text{(C.1)}$$

The CCP typically also has an upper cap on the amount it can assess each clearing member. Similarly to the rules of existing CCPs, we assume that the assessment amount of a clearing member is capped at β times the guarantee fund contribution. Therefore $\min(\beta\gamma_k, \theta_k)$ is the most that can

in the U.S., under the bankruptcy code even individually segregated client funds can be treated as if they were held commingled in a single account (Ruffini (2015)).

be raised from clearing member k.

For a given set of payment vectors, $\alpha_0 \equiv \min(\sum_{k\in M} \min(\beta\gamma_k, \theta_k), \dot{s}_0)$ is the *assessment ability* of the CCP. If this amount covers the remaining payment obligations of the CCP, the CCP will have zero remaining stress after assessments. If this amount is not sufficient, then assessments will not fully cover the CCP's payment obligations, and the CCP will have to implement VMGH, IMH, or move into resolution procedures. We note that α_0 is an endogenous value that depends on the stress of the CCP and its members given a specified set of payment vectors.

Given an assessment amount α_0, the CCP's remaining stress going on to the next stage is

$$\ddot{s}_0 \equiv \dot{s}_0 - \alpha_0. \tag{C.2}$$

Note that α_0 is endogenous and depends on the equilibrium stress of clearing members. It thus needs to be determined by a fixed point between the value of the stress of each firm s_k. However, this calculation will be affected by VMGH and IMH. We discuss the calculation of the final stress in the next two sections.

D.2 Initial Margin Haircuts

If the CCP is allowed to implement IMH, it will take unused IM from its contracts with its clearing members and clients in order to cover its stress. It will do a pro rata haircut of the IM still available among all its contracts,

$$z_0^r \equiv \sum_{k\in M} [z_{k0} - (\bar{p}_{k0} - p_{k0})]^+ + \sum_{k\in M} \sum_{i\in C_k} [z_{ik0} - (\bar{q}_{ik0} - q_{ik0}^c)]^+, \tag{C.3}$$

up to a maximum of its stress after assessments $\ddot{s}_0$. Note that p_{i0} and p_{ik0} are determined endogenously, so the remaining pool of IM, z_0^r, is determined endogenously as well.

If IMH is able to satisfy all of the CCP's stress $\ddot{s}_0$, then the CCP will make all of its payments. In this case $p_{0k} = \bar{p}_{0k}$ and $q_{0ki} = \bar{q}_{0ki}$. On the other hand, if the pool of IM is not enough to cover the CCP's stress then the CCP will either move onto VMGH if available, or it will default and enter resolution procedures. We do not explicitly model the resolution procedures. If using VMGH, define the stress as

$$\ddot{s}_0^z \equiv [\ddot{s}_0 - z_0^r]^+. \tag{C.4}$$

D.3 Variation Margin Gains Haircuts

The final stage in the waterfall is VMGH. Here we assume that the CCP has an assessment power as denoted in the previous section. In the case of VMGH, the CCP will prorate outgoing VM payments to all firms to cover its shortfall $\ddot{s}_0$, or $\ddot{s}_0^z$ if IMH was used. This occurs via a similar mechanism as to how firms prorate their VM payments to others in the model.

We define the combined payment obligations for the CCP to derive how the prorating occurs. We let $\bar{p}_0^g = \sum_{i \neq 0} \bar{p}_{ki} + \sum_{k \in M} \bar{q}_{0ki}$.

We use these combined payment obligations to derive the CCP's relative payment liability to different firms. If IMH is present, then replace $\ddot{s}_0$ by $\ddot{s}_0^z$ in the following equations.

$$a_{0k} = \bar{p}_{0k} / \bar{p}_0^g, \tag{C.5}$$

$$a_{0ki} = \bar{q}_{0ki} / \bar{p}_0^g. \tag{C.6}$$

$$p_{0k} = \bar{p}_{0k} - a_{0k}\ddot{s}_0 \quad \forall k \in M, \tag{C.7}$$

$$q_{0ki} = \bar{q}_{0ki} - a_{0ki}\ddot{s}_0 \quad \forall k \in M. \tag{C.8}$$

We note that under VMGH the model functions similarly as the Eisenberg-Noe model since the CCP prorates its outgoing payments proportionally like any other node.

D.4 Calculating Loss at End-of-Waterfall

Clearing members may suffer additional losses from the CCP's use of clearing member contributions to the guarantee fund, $\hat{\gamma}_k$, assessments on the firm by the CCP, $\hat{\alpha}_k$, and IM losses in cases where IMH is used, $\hat{z}_{k0}$. The total default waterfall losses are given by

$$\text{Default Waterfall Loss} = \hat{\gamma}_k + \hat{\alpha}_k + \hat{z}_{k0}. \tag{C.9}$$

In calculating these losses, we consider only the usage of funds to cover obligations of other clearing members. Guarantee funds that are used to cover a clearing member's own default are not counted in losses, but guarantee funds used to cover another clearing member's default are counted in losses.

Recall that the total guarantee fund contributions of clearing member k are denoted γ_k. We denote the amount of guarantee fund used to cover a clearing member's *own obligations* $\dot{\gamma}_k$. This guarantee fund is taken from clearing member k when clearing member k is short payments to the CCP and owes more than its IM z_{k0} can cover. The value of $\dot{\gamma}_k$ is given by

$$\dot{\gamma}_k \equiv \min\left(\left[\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0} - \left(p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}\right)\right]^+, \gamma_k\right), \quad \forall k \in M. \tag{C.10}$$

Suppose clearing member k does not use all of its own contributions γ_k and losses to the CCP from other firms go beyond the resources of the CCP's capital contribution b_0. Then clearing member k will have part, if not all, of its guarantee fund contribution used to cover the obligations of other clearing members. Guarantee fund contributions will be taken pro rata the remaining contributions of each clearing member to cover the CCP's shortfall.

Let the amount that the CCP needs to cover if it uses the previous waterfall layers be given by g_0:

$$g_0 \equiv \left[\sum_{k\in M}\left(\bar{p}_{0k} + \sum_{i\in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0}) - \dot{\gamma}_k\right) - b_0\right]^+. \tag{C.11}$$

The guarantee fund losses for clearing member k, $\hat{\gamma}_k$, are given by the following:

$$\hat{\gamma}_k \equiv \min\left(\frac{\gamma_k - \dot{\gamma}_k}{\sum_{j\in M} \gamma_j - \dot{\gamma}_j} g_0, \gamma_k - \dot{\gamma}_k\right), \quad \forall k \in M. \tag{C.12}$$

If this amount is still not enough to cover the CCP's losses, then the end-of-waterfall mechanisms will be used. Each clearing member will be assessed pro rata its total amount available for assessments, $\min(\beta\gamma_k, \theta_k)$. The assessment losses for clearing member k are given by

$$\hat{\alpha}_k \equiv \frac{\min(\beta\gamma_k, \theta_k)}{\sum_{j\in M} \min(\beta\gamma_j, \theta_j)} \min\left(\dot{s}_0, \sum_{j\in M} \min(\beta\gamma_j, \theta_j)\right), \quad \forall k \in M. \tag{C.13}$$

If IMH is an option for the CCP and is necessary, it will be performed pro rata across the clearing member contributions to the IMH pool. Recall that the total pool of IM that the CCP has available is

$$z^r_0 = \sum_{k\in M} [z_{k0} - (\bar{p}_{k0} - p_{k0})]^+ + \sum_{k\in M}\sum_{i\in C_k} [z_{ik0} - (\bar{q}_{ik0} - q^c_{ik0})]^+. \tag{C.14}$$

The total IM left for a clearing member k including its client clearing transactions is

$$\dot{z}_{k0} \equiv [z_{k0} - (\bar{p}_{k0} - p_{k0})]^+, \quad \dot{z}^0_{k0} \equiv \sum_{i\in C_k} [z_{ik0} - (\bar{q}_{ik0} - q^c_{ik0})]^+, \quad \forall k \in M. \tag{C.15}$$

The total amount of IMH that the CCP needs depends on the level of its stress $\ddot{s}_0$. The amount of IMH losses for each clearing member k, $\hat{z}_{k0}$, is given by

$$\hat{z}_{k0} \equiv \frac{\dot{z}_{k0} + \dot{z}_{k0}^0}{z_0^r} \min(\ddot{s}_0, z_0^r), \quad \forall k \in M. \tag{C.16}$$

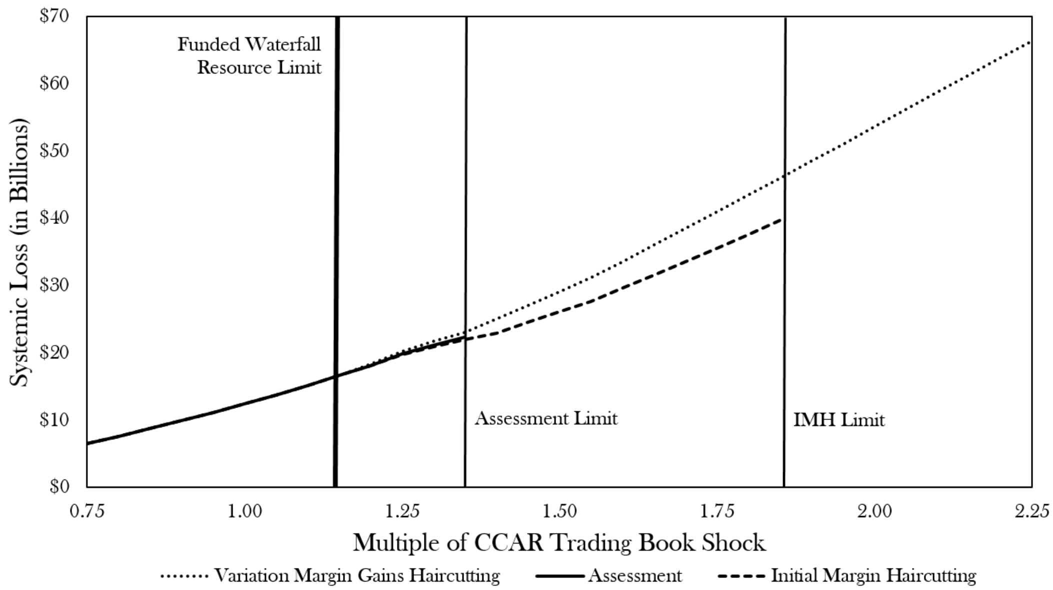
D.5 Empirical Results

If losses are large enough to exceed the total available funded waterfall resources, the CCP must use an end-of-waterfall mechanism to obtain additional resources and/or decrease its payments. As discussed previously, the major end-of-waterfall mechanisms are assessments, variation margin gains haircutting (VMGH), and initial margin haircutting (IMH). Our survey of global CCPs found that assessments and VMGH are both common in CCP waterfall structures today. IMH, on the other hand, is not part of any current CCP mechanisms; however, it is a widely discussed mechanism and its use may come into place in the future.

For simplicity, we isolate each mechanism in our evaluations of their impact on resilience and clearing member incentives, and we assume that the CCP cannot combine them. For each mechanism, we vary the size of the CCAR shock by α, and we calculate the total systemic loss L. This allows us to find the level of α at which the CCP runs out of resources to cover payments. Note that under VMGH the CCP never technically defaults because it can always reduce its payments to zero. However, once the CCP uses VMGH it will start transmitting its stress, $\ddot{s}$, to other institutions. On the other hand, assessments and IMH will not add stress to the network as long as the CCP has enough unfunded resources to fulfill its ponlayment obligations. With these mechanisms, the CCP transmits stress only when it runs out of clearing member funds to assess or initial margin to haircut.

Figure D.2 shows the systemic losses of the three different mechanisms. The vertical lines on the plot highlight important thresholds. Going from left to right, the first line indicates where the CCP's funded waterfall resources hit their limit, at $\alpha = 1.18$. After this threshold, the three mechanisms are activated by the CCP, which results in differences in the systemic losses. The second line, at $\alpha = 1.36$, indicates where the assessment mechanism reaches its limit in providing enhanced resiliency to the CCP, due to clearing members running out of resources to assess. The final line, at $\alpha = 1.87$, indicates where the IMH mechanism reaches its limit. The much greater alpha limit for IMH highlights how much larger the pool of IM funds is relative to assessments for the CCP. Additionally, it is important to note that both of these mechanisms provide lower total systemic losses than VMGH.

Figure D.2. Total Market Losses and CCP Default Threshold by End-of-Waterfall Mechanism



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. Each line represents the amount of loss suffered under the three end-of-waterfall mechanisms, assuming each is implemented independently of the other. The vertical lines represent the limit at which a mechanism no longer has resource to draw upon. The assessment mechanism has limited additional resilience in our example, whereas initial margin haircuts (IMH) allows the CCP to maintain full payments under a 50 percent larger stress. Variation margin gains haircuts provide the most resilience, though they create the largest systemic losses.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

The caveat to both assessments and IMH is that they have limits to the protection they offer. They require additional resources to cover payments, but under times of crisis, such additional resources may be limited. In contrast VMGH, the worst performing of the mechanisms by the measure of total systemic losses, actually provides the greatest degree of resiliency in allowing the CCP to continue operating even with limited funds. This suggests why the mechanism is typically used by CCPs as a final stage for the waterfall, and why it has been accepted for use in the United States.

Let us now consider how each of these mechanisms distribute losses across different types of firms. CCP members would be asked to contribute additional resources if assessments and/or IMH were to be used, so they must thus be properly incentivized to do so. To evaluate this concern, we estimate additional systemic losses beyond the funded waterfall resources, and we determine the percentage of losses split among the various firm types. We compute the collective losses in equilibrium using Equations (25) - (29) for each group, and we present the relative percentage of loss suffered by each firm type at $\alpha = 1.35$, where all three mechanisms are active, in Table D.1.

Table D.1. End-of-Waterfall Mechanism Losses at $\alpha = 1.35$

	Assessments		IMH		VMGH	
Systemic Loss	4.2		4.2		4.9	
by firm type	Amount($b)	Percent	Amount($b)	Percent	Amount($b)	Percent
Members	3.1	73.1%	3.0	71.4%	3.2	65.3%
Clients	0.7	17.4%	0.8	19.0%	1.3	26.5%
Bilateral	0.4	9.4%	0.4	9.5%	0.4	8.2%

Note: The table shows the collective systemic losses suffered by each firm type (in $ billions) under each of the end-of-waterfall mechanisms, assessments, initial margin haircuts (IMH) and variation margin gains haircuts (VMGH).
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

Table D.1 shows that clearing members are better off under IMH on an absolute loss basis, as this creates only $3.0 billion in additional losses. IMH creates the least amount of systemic losses as well. Though assessments create an equal amount of systemic losses for this α, the mechanism distributes the losses more heavily onto the clearing members.

VMGH reduces the percentage loss suffered by clearing members, as they don't need to contribute any additional resources, and clients and bilateral firms bear more of the burden. However, it also leads to higher total systemic losses across all firms, including the clearing members themselves. VMGH creates additional spillover losses throughout the network, as clearing members that do not receive payments may default on their counterparties. These spillover losses can eventually splash over onto other clearing members, resulting in even more losses. In equilibrium, the costs to clearing members as a whole could thus be made lower by contributing additional resources directly to the CCP. While clearing members may not prefer to be on the hook for contributing additional resources during a crisis, we find that contributing such resources would actually be more beneficial to them than VMGH. This nonintuitive result suggests that clearing members could inadvertently increase systemic losses for the market, and their own losses, if they were to face VMGH rather than the other two mechanisms.