OFFICE OF FINANCIAL RESEARCH

STAFF DISCUSSION PAPER

No. 2015-03 | July 1, 2015

The Effects of Housing Adjustment Costs on Consumption Dynamics

Benjamin Kay
Office of Financial Research
benjamin.kay@treasury.gov

The Effects of Housing Adjustment Costs on Consumption Dynamics

Benjamin Kay
U.S. Treasury's Office of Financial Research
benjamin.kay@treasury.gov

First Draft: 5/8/2011 Current Draft: 6/30/2015

Abstract

Building on Flavin and Nakagawa (2008), this paper models household optimal consumption and portfolio selection when consumption services are generated by both non-durable consumption and by holding a durable good housing. Housing is illiquid in that a non-convex adjustment cost must be paid when it is sold. It is shown that optimal consumption of housing is not a constant fraction of wealth but instead depends on the ratio of wealth to housing and the price of housing. Households adjust housing infrequently, waiting for large wealth changes before adjustment. As in models without this adjustment cost, households adjust non-durable consumption each period. Unlike in frictionless models, non-durable consumption is not a constant fraction of wealth. For particular parameters of the utility function and asset markets drawn from the literature, model simulations match aggregate consumption dynamics better than alternative frictionless models, even those with homes as assets. The simulations also predict differing responses of households with different fractions of their wealth in housing.

The views and opinions expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the official policy or position of the Department of Treasury or of any other agency of the U.S. Government. I thank the macroeconomics seminar participants at UCSD, the Boston Federal Reserve, Federal Deposit Insurance Corporation, and Federal Reserve Board for their helpful comments and suggestions. Special thanks go to Jim Hamilton, Irina Telyukova, Harry Markowitz, Giacomo Rondina, Davide Debortoli, and Marjorie Flavin.

I Introduction

The Great Recession and subsequent slow recovery have highlighted the serious macroeconomic consequences of problems in housing markets. Housing related industries like construction, furniture, real estate sales, and home improvement have suffered worse than the economy as a whole.[1] National home price declines of 30 percent left an estimated fifteen million households with negative home equity and the mortgage delinquency rate five times its historical average.[2], [3] Surprisingly, amidst this loss of wealth and employment, non-durable consumption (hereafter just consumption) was essentially unchanged.

It is surprising in part because home ownership is at the center of household assets and liabilities. In the last of the Federal Reserve Board's *Survey of Consumer Finances* (SCF) before the crisis (2007), 69 percent of households owned their own home.[4] Among homeowners, housing is typically their largest asset. The median percentage of net worth accounted for by their primary residence is 84%. Debts on the home also are typically the household's largest liability. The median percentage of household debt secured by the primary residence is 90 percent. Falling home prices and fixed liabilities suggest that home-owning households should have seen large declines in their net worth since 2006, even if homeowners did not invest in the financial markets. In many models of the household's basic decision over consumption and investment it is optimal to chose them proportionally to wealth (e.g., Merton [1969] and Constantinides [1986]) and therefore, a large wealth effect on consumption might be expected.

It is also surprising that changes in home values have had little effect on consumption because housing services are a large part of household spending. According to the Bureau of Economic Analysis's 2011 *National Income and Product Accounts* (NIPA), Housing and Utility Services is 18 percent of personal expenditures (second only to Non-durable Consumption's 23 percent). This aggregate hides even more exceptional cross-sectional importance. The U.S. Department of Housing and Urban Development estimates that 11 percent of households spend 50 percent or more of their annual income on housing. As long as housing is a normal good (and Hanushek and Quigley [1980] provides evidence that it is), then the substitution effect will exacerbate the wealth effect to reduce consumption.

[1]Leamer [2007] details the outsized role of housing in most post W.W. II recessions. Goodman and Mance [2011] document the 2007-2009 fall in employment in the construction industry was more severe than other post-war recessions. The U.S. Bureau of Economic *Analysis of Value Added by (NAICS) Industry* documents that furniture and related products economic value added fell by a third from 2006 to 2009. The California Department of Real Estate's Licensee/Examinee Statistics for Fiscal Year 2010/2011 shows a 20 percent decline in the number of licensed real estate agents from 2007 to 2010.

[2]How to Stop the Drop in Home ValuesMartin Feldstein, p. A29 10/12/11, The New York Times

[3]Board of Governors of the Federal Reserve System, *FRED Economic Data*, Delinquency Rate On Single-Family Residential Mortgages 2010 compared to 1990-2006 average.

[4]Of all other financial and real assets only transaction accounts (e.g., checking accounts) are more common.

Households will substitute out of consumption goods and into the now cheaper housing.

This paper resolves much of this anomaly by introducing a non-convex transaction cost for changing holdings of housing into the household's basic problem of allocating consumption optimally over time in a world with uncertainty in asset returns. To adjust housing, households must pay an adjustment cost equal to a fixed fraction of their home's value. This represents the costs to the household of selling a home and moving their possessions. Two other key features of housing are replicated. First, housing acts as a risky asset on the household balance sheet. Second, housing is in the utility function. Housing provides a flow of services when combined with non-durable consumption. [5] Because marginal utility (U_H) and cross-partial utility (U_{HC}) are positive, this captures that people prefer larger bedrooms and that it is easier to cook a large meal in large kitchen.

These features generate different predictions of consumption, investment, and value functions than models without housing or with housing but lacking housing frictions. Households make infrequent and large housing adjustments but frequent and mostly small consumption changes. Because of this, the marginal utility of consumption depends on current housing (relative to wealth). In this specification, when households experience negative wealth shocks they reduce consumption by much less than their reduction in wealth unless the shock induces them to move (in which case they reduce it more than the wealth shock). This friction also causes households to have preferences over risk that depend upon current housing (relative to wealth). Households that have just adjusted their housing are relatively more risk averse. Households near the adjustment bounds (determined by where households have so much or little housing they move) are relatively risk tolerant. This changes the curvature of the value function to be less curved (in an Arrow-Pratt relative risk aversion sense) than in models without the friction near the adjustment bounds but more curved for those who have just moved.

This paper contributes to two principle fields in macroeconomic literature. The first studies the use of housing in macroeconomic models. This is the first paper in that literature to contain housing, housing frictions, non-durable consumption, Constant Elasticity of Substitution (CES) Utility, and estimates of consumption and investment policy functions. The second studies the excess smoothness of aggregate consumption with respect to fluctuations in observable wealth. While housing has

[5] This need not be solely the utility value of living in a bigger home. It might also be a reduced form representation of how housing services complement home production and leisure.

been used to study this problem before, this paper is the first to investigate the effect of housing on consumption dynamics when frictions and non-durable goods are also modeled.

Henderson and Ioannides [1983] provides an early theoretical presentation integrating housing and non-durable consumption. H. [1989] considers housing as an investment asset but uses a rental services model to abstract away from how the stock of housing enters household utility. Berkovec and Fullerton [1992] integrates these asset and utility approaches, injecting housing into the consumption and investment decisions. This approach was widely adopted. For example, Lustig and Nieuwerburgh [2005] and Piazzesi et al. [2007] are respectively partial equilibrium and general equilibrium models employing this approach. This is a budget share theory of housing importance because it justifies an emphasis on housing because of its large share of consumption and investment.

There is an alternative perspective, acknowledging that the budget share theory is important but focusing on the significant frictions in trade of housing. Topel and Rosen [1988] models housing as an asset facing convex adjustment costs. Grossman and Laroque [1990a] introduces a more realistic non-convex adjustment cost for durable consumption, the only consumption in the model. Flavin and Nakagawa [2008] extends Grossman and Laroque with non-durable consumption and housing price dynamics. This allows for richer adjustment behavior by households and for asset prices to be consistent with the consumption-beta model of T. [1979] and Lucas [1978]. Flavin and Nakagawa cannot solve for the consumption and investment policy functions and so do not address individual and aggregate consumption dynamics. This paper solves for the policy functions of a related model and therefore can address questions about aggregate consumption and investment that they could not.

Mehra and Prescott [1985] identified the anomalous relationship between volatile asset markets and smooth consumption data. This paper builds upon the literature of possible explanations.[6] By integrating housing services into the utility function, it alters household preferences. Other important preference modification solutions include Epstein and Zin [1989] and Constantinides [1990]. Mehra [2007] and Dynan [2000] provide empirical evidence that these alternative preference modifications cannot resolve the anomaly with realistic preference parameters.

Including incomplete markets or missing assets in the household portfolio can resolve the anomaly.

[6] See Mehra [2007] for a survey of the enormous literature of possible complete and partial solutions.

Mehra and Prescott [1985] speculates that omitted human capital may explain the anomaly.[7] Guiso et al. [1996] finds that a combination of income risk, health risk, and borrowing constraints on labor income can explain one quarter of the anomaly. Appendix C considers a simple extension to the model that integrates human capital. This paper confirms that housing is an important asset to include in the household portfolio, especially when incorporated with realistic frictions.

Alternative modeling of asset returns can also resolve the anomaly. Rietz [1988] and Barro [2006] both make use of low probability but severe and (in expectation) permanent wealth shocks to depress household equity holdings and consumption response to asset returns. However, on realistic disaster magnitudes only part of the anomaly is resolved. Though this paper does not address disasters explicitly, it does model asset returns at a yearly frequency which allows for more extreme movements in the wealth portfolio between portfolio adjustments. In a single year, the model allows for -30 percent stock market returns and -14 percent housing market returns. Mehra et al. [2011] sensibly points out that borrowing and lending rates are not the same, and much of what looks like an equity premium is in fact the costs of financial intermediation. The household's optimal investment policy makes all households (weakly) net-borrowers. Therefore the model uses borrowing rates based on mortgage rates rather than the more commonly used lending rates based on risk-free bonds.

The rest of the paper is organized as follows. Section II describes the model and lays out a transformation that reduces the dimensionality of the problem. Section III describes the computational techniques used to solve the problem. Section IV discusses the parameter values that describe household preferences, adjustment costs, and the parametric assumptions about asset returns. Section V shows the solution to the value and policy functions of the model laid out in Section II. It also examines the aggregate consumption dynamics predicted by the model and contrasts them with alternative frictionless models and NIPA measured true consumption. Section VI concludes.

II Model

The model is a simplified version of Flavin and Nakagawa [2008]. The principal simplification is to abstract from multiple housing markets to a single risky housing market. An additional difference

[7] This is analogous to the Roll critique from Roll [1977]. If the observed portfolio is not the market portfolio then observed portfolio dynamics need not predict consumption dynamics.

is that Flavin and Nakagawa model the househod decision in continuous time while this paper does so in discrete time. Adapting the problem to discrete time also requires changes to housing and risky asset return processes.

II.1 Budget Constraint and the Evolution of the Wealth Equation

Let B_t , X_t, and $P_t \cdot H_t$ respectively denote the amounts (in units of non-durable consumption) of risk-free, risky, and housing assets chosen by the consumer at time t.[8] Since this is the exhaustive set of assets in the model, household wealth W_t is defined as follows:

$$W_t \equiv R_f \cdot B_{t-1} + R_{m,t} \cdot X_{t-1} + P_t \cdot H_{t-1}$$

where R_f is the gross return of a risk-free asset between $t-1$ and t, $R_{m,t}$ is the realized gross return of a risky asset between $t-1$ and t, and P_t is the price of square feet of housing in units of consumption at time t. The household then allocates this stock of wealth among consumption and savings to satisfy the budget constraint[9]:

$$W_t = C_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1} \cdot P_t + B_t + X_t + H_t \cdot P_t$$

We also can rewrite this in the form of Assets - Savings = Consumption:

[8]Instead of a single risky asset, without loss of generality this could be a mean-variance efficient portfolio of risky assets uncorrelated with housing returns.

[9]This may seem like an odd budget constraint. The following derivation may provide some insight. A budget constraint shows Existing Assets + Income = Consumption + Savings. In the terms of this model, existing assets are the holdings from the last period are $B_{t-1} + X_{t-1} + P_{t-1}H_{t-1}$. Income (think dividend, capital gains, and interest income in this paper) is the returns on those assets $r_f \cdot B_{t-1} + r_{m,t} \cdot X_{t-1} + (P_t - P_{t-1}) H_{t-1}$. These resources must equal the two expenditures categories. The first is consumption of non-durable goods and services C_t and housing adjustment costs $\mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1}P_t$. All other wealth is allocated to investments in bonds, risky assets, and housing totaling $B_t + X_t + H_t \cdot P_t$. Defining r_f and $r_{m,t}$ and the net return analogs to R_f and $R_{m,t}$ respectively we get the following budget constraint:

$$(B_{t-1} + X_{t-1} + P_{t-1}H_{t-1}) + (r_f \cdot B_{t-1} + r_{m,t} \cdot X_{t-1} + (P_t - P_{t-1}) H_{t-1})$$

$$= \left(C_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1}P_t\right) + (B_t + X_t + H_t \cdot P_t)$$

Combining terms, the left-hand side is just W_t as defined above

$$\begin{array}{rcl} R_f \cdot B_{t-1} + R_{m,t} \cdot X_{t-1} + P_t H_{t-1} & = & C_t + B_t + X_t + \left(H_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1}\right) \cdot P_t \\ W_t & = & \end{array}$$

$$(R_f \cdot B_{t-1} + R_{m,t} \cdot X_{t-1} + P_t H_{t-1}) - (B_t + X_t + H_t \cdot P_t) = C_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1} \cdot P_t$$

Using the definition of state variable W_t to substitute out the bond control variable (B_t) we combine the definition of W_{t+1} and the budget constraint:

$$W_{t+1} = R_f \cdot B_t + R_{m,t+1} \cdot X_t + P_{t+1} H_t$$

$$= R_f \cdot \big(W_t - C_t - X_t - \big(H_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot H_{t-1}\big) \cdot P_t\big) + R_{m,t+1} \cdot X_t + P_{t+1} \cdot H_t$$

. This can be simlified by distributing and collecting terms:

$$W_{t+1} = R_f \cdot (W_t - C_t) + (R_{m,t+1} - R_f) \cdot X_t + \left(P_{t+1} - R_f \cdot P_t \cdot \left(1 + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot \frac{H_{t-1}}{H_t}\right)\right) H_t$$

This is the equation of motion for household wealth which depends on the state variables W_t, H_{t-1}, and P_t, the control variables X_t, H_t, and C_t, and the random variables P_{t+1}, R_f, and $R_{m,t+1}$.

II.2 Felicity and Value Functions

The household felicity function is in constant elasticity of substitution (CES) form, taking as arguments non-durable consumption and units of housing:

$$U(C_t, H_t) = \frac{(C_t^\alpha + \gamma \cdot H_t^\alpha)^{\frac{1-\rho}{\alpha}}}{1-\rho}$$

The constant rate of substitution between non-durable and durable consumption is controlled by α. The parameter γ converts the units of housing as measured by P_t in the budget constraint into the units of housing consumed by the household (see Appendix B for a full discussion). In frictionless models, the parameter $1-\rho$ controls the elasticity of inter-temporal substitution (EIS) and the curvature of the both the value function and the felicity function. In this model, ρ controls only the constant curvature of the felicity function, the EIS and value function curvature depend on the state variables as well.

Let $V(W_t, H_{t-1}, P_t)$ be the supremum of the expected utility that the consumer can achieve from initial conditions (W_t, H_{t-1}, P_t). Then $V(W_t, H_{t-1}, P_t)$ satisfies the following Bellman equation:

$$V(W_t, H_{t-1}, P_t) = \sup_{C_t, X_t, H_t} \left[\frac{(C_t^\alpha + \gamma H_t^\alpha)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \mathbb{E}_t \left[V(W_{t+1}, H_t, P_{t+1}) \right] \right]$$

Define Φ as follows:

$$\Phi \equiv -\ln(\beta) - (1-\rho) \cdot (R_f - 1) - \frac{(\mathbb{E}[R_{m,t+1} - 1])^2}{2 \cdot Var(R_{m,t+1})} \cdot \frac{1-\rho}{\rho}$$

Grossman and Laroque 1990 show that if $\Phi > 0$ then the value function in this problem is homogeneous in H_{t-1} and W_t of degree $1-\rho$. For the parameters used in this paper, this condition holds and $H_{t-1}^{1-\rho} \cdot V\left(\frac{W_t}{H_{t-1}}, 1, P_t\right) = V(W_t, H_{t-1}, P_t)$. Therefore, we can rewrite the Bellman as follows:

$$V(W_t, H_{t-1}, P_t) = H_{t-1}^{1-\rho} \cdot V\left(\frac{W_t}{H_{t-1}}, 1, P_t\right)$$

$$= \sup_{C_t, X_t, H_t} \left[\frac{\left(\left(\frac{C_t}{H_{t-1}}\right)^\alpha + \gamma \cdot \left(\frac{H_t}{H_{t-1}}\right)^\alpha\right)^{\frac{1-\rho}{\alpha}} \cdot H_{t-1}^{1-\rho}}{1-\rho} + \beta \cdot H_t^{1-\rho} \cdot \mathbb{E}_t \left[V\left(\frac{W_{t+1}}{H_t}, 1, P_{t+1}\right) \right] \right]$$

$$\Rightarrow V\left(\frac{W_t}{H_{t-1}}, 1, P_t\right) = \sup_{C_t, X_t, H_t} \left[\frac{\left(\left(\frac{C_t}{H_{t-1}}\right)^\alpha + \gamma \cdot \left(\frac{H_t}{H_{t-1}}\right)^\alpha\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot \mathbb{E}_t \left[V\left(\frac{W_{t+1}}{H_t}, 1, P_{t+1}\right) \right] \right]$$

Now define housing intensive variables that scale the state and control variables by H_{t-1}:

$$\begin{array}{cccc} W_t & H_t & X_t & C_t \\ \downarrow & \downarrow & \downarrow & \downarrow \\ y_t \equiv \frac{W_t}{H_{t-1}} - \lambda \cdot P_t & h_t \equiv \frac{H_t}{H_{t-1}} & x_t \equiv \frac{X_t}{X_{t-1}} & c_t \equiv \frac{C_t}{H_{t-1}} \end{array}$$

Appendix A shows that the value function can be rewritten as:

$$G(y_t, P_t) = \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^{\alpha} + \gamma \cdot h_t^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t \left[g\left(y_{t+1}, P_{t+1}\right)\right] \right]$$

$$= \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^{\alpha} + \gamma \cdot h_t^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t \left[G\left(R_f \cdot \frac{y_t - c_t + \mathbf{1}_{\{h_t=1\}} \cdot \lambda \cdot P_t}{h_t} \right.\right.\right.$$

$$\left.\left.\left. + \left(R_{m,t+1} - R_f\right) \cdot \frac{x_t}{h_t} + P_{t+1} \cdot (1-\lambda) - R_f \cdot P_t \ , \ P_{t+1} \right) \right] \right]$$

, where $G(y_t, P_t) \equiv V(y_t + \lambda \cdot P_t, 1, P_t)$. This transformed problem has two states: y_t and P_t, three controls: c_t, x_t, and h_t, and two random processes: P_{t+1} and $R_{m,t+1}$ which eliminates one state variable from the original problem. This transformed problem is solved computationally in Section V. Though the transformed variables have slightly less intuitive interpretations, the reduced dimensionality substantially eases solving the problem computationally and the results can be transformed back to present intuitive policy functions and other results.

III Computational Modeling

This problem of two states (y_t, P_t) and three controls (c_t, x_{t+1}, h_{t+1}) does not have a closed-form solution for the policy and value functions. However, it can be solved with computational techniques. The general approach is value function iteration with a discretized state and adaptive grid policy space. Judicious use of Howard's improvement step speeds up the convergence of the value function. After reaching convergence with discrete policy choices, a more accurate value function is generated by allowing policy choices to be continuous. The global optimization method Pattern Search is employed to find the optimal policies and calculate the value function.[10] Final iteration tolerances are within machine precision.

This paper accounts for stochastic returns with discrete approximations to historical returns. For stock market returns, the method of Tauchen [1986] is employed. Specifically, this paper uses six states to approximate the returns of the stock market. The housing price process is described with a 15 state transition matrix. Conditional on today's price P_t, no more than five future prices P_{t+1}

[10] Implemented in Matlab's Global Optimization Toolkit and detailed in Kolda et al. [2003].

have positive probability. The stock market return and housing price processes are depicted in Tables 1 and 2, respectively. In keeping with the findings of Flavin and Nakagawa [2008] that found the correlation between housing and stock returns was effectively zero, this paper assumes that housing and risky asset returns are independent.

The distribution of annual stock market real returns is calibrated from Fama-French market return data (deflated with the CPI, pre-tax, and net of dividends) to match the mean and variance of returns from 1950-2010. The distribution of housing returns is calibrated on the Case-Shiller 10-city Index (deflated with the CPI and pre-tax, hereafter CS10) to match the mean, variance, and skew of the historical returns from 1987-2010. The range of housing states (depicted in table 2) allows for housing price states above peak real prices in 2006 and below trough prices in 1995. Figure 1 graphs the historical prices of the CS10 against the model implied price process. In general, the fit is good. The largest gap is less than $3 \ \frac{\$}{Ft^2}$ which is small relative to regional and inter-temporal variations in prices per square foot.

Individual households face idiosyncratic home price risk as well. Unfortunately, the literature finds disparate estimates of the idiosyncratic home price risk. Bourassa et al. [2009] found the standard deviation of individual home prices is 1.2-2.6 times that of the standard deviation of the whole market in New Zealand. Goetzmann [1993] found a range of 1.5 to 3 for four U.S. cities. On the higher end, Englund et al. [2002] found this ratio to be 5.7 in Sweden. This is analogous (but potentially not identical given differing time series properties) to the true variance of household prices being $\sigma^2_{House} \in \sigma^2_{Case-Shiller} \cdot [1.2, 5.7]$ in this model.

Because the cross-sectional and time-series nature of idiosyncratic component of house prices is not well understood, this paper uses aggregate house price dynamics only. Sensitivity testing of the simulation shows that greater price risk lowers portfolio holdings of the risky and housing assets, thereby further decreasing sensitivity of non-durable consumption to home price movements. This is consistent with Heaton and Lucas [2000] that finds that households facing idiosyncratic risks decrease their holdings of risky assets. Either way, treating this greater risk as an increased aggregate housing price risk or an idiosyncratic price risk would make household wealth less sensitive to asset price movements. Through this mechanism, the simulation results of consumption dynamics would be further dampened with respect to the frictionless models. Therefore, the assumption that households

Table 1: Tauchen Method Applied to Stock Returns from 1950-2009

Real Stock Market Return	Probability of Outcome
-29.8%	4.82%
-14.3%	15.5%
1.11%	29.7%
16.6%	29.7%
32.0%	15.5%
47.5%	4.82%
Predicted / Actual (1950-2010) Real Mean Return	8.8% / 8.8%
Predicted / Actual (1950-2010) Return Standard Deviation	18.6% / 18.6%

Source: Stock market returns from Fama-French U.S. Research Returns Data (1950-2010). Inflation from the Bureau of Labor Statistics' Consumer Price Index - All Urban Consumers series.

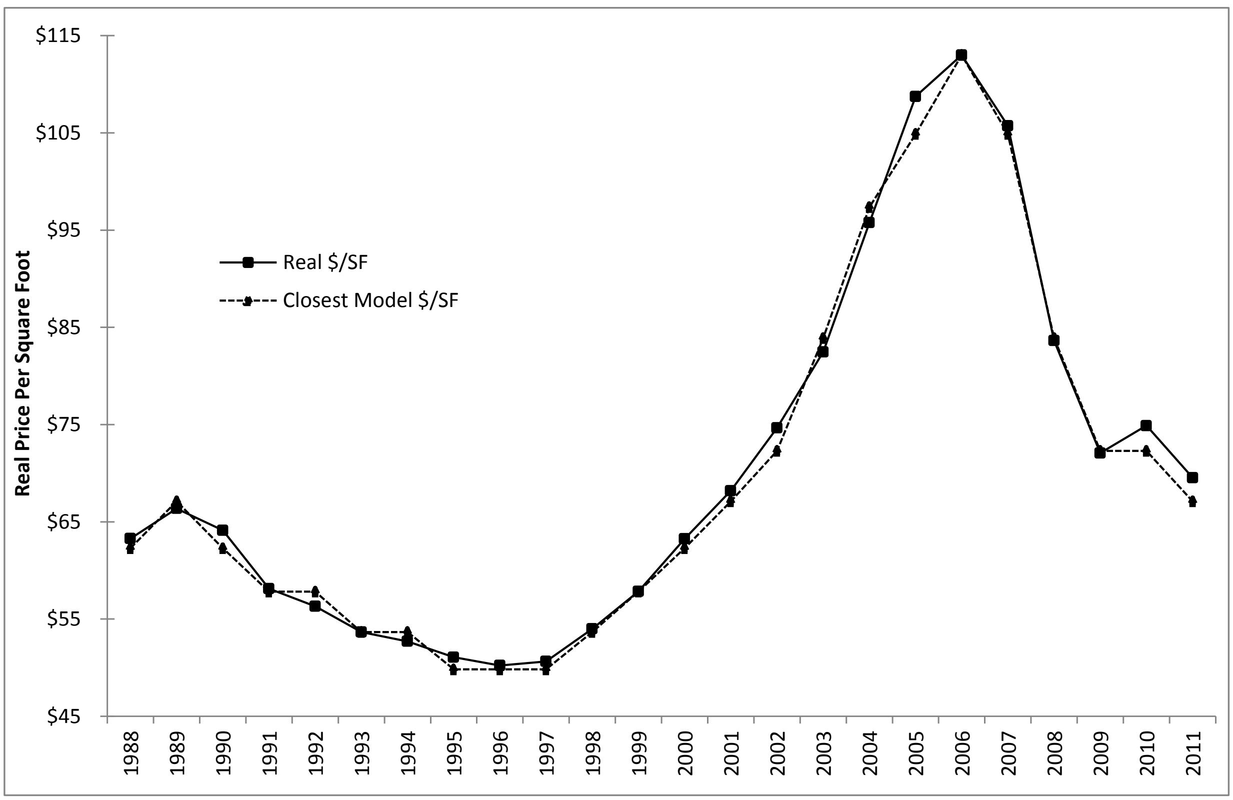
Table 2: A 15 State Markov Model of Home Price Dynamics

$P_t \backslash P_{t+1}$	\$46	\$50	\$54	\$58	\$62	\$67	\$72	\$78	\$84	\$90	\$97	\$105	\$113	\$122	131
\$46	.44	.51	.05												
\$50	.26	.18	.51	.05											
\$54	.21	.05	.18	.51	.05										
\$58		.21	.05	.18	.51	.05									
\$62			.21	.05	.18	.51	.05								
\$67				.21	.05	.18	.51	.05							
\$72					.21	.05	.18	.51	.05						
\$78						.21	.05	.18	.51	.05					
\$84							.21	.05	.18	.51	.05				
\$90								.21	.05	.18	.51	.05			
\$97									.21	.05	.18	.51	.05		
\$105										.21	.05	.18	.51	..05	
\$113											.21	.05	.18	.51	.05
\$122												.21	.05	.18	.56
\$131													.21	.05	.74

$R_{H,t+1}$	True	Model ($P_t = 113$)
Mean	1.015	1.015
Variance	.00875	.0087
Skew	-0.6510	-0.6510

Source: The Standard & Poor's Case–Shiller 10-city Home Price Index from 1987-2010. Inflation from the Bureau of Labor Statistics' Consumer Price Index - All Urban Consumers series.

Figure 1: Model Implied Price Process Approximates The Real Price History



face no idiosyncratic price risk is a conservative one that reduces the model fit.

IV Simulation Parameters

The simulation uses preference parameter estimates from Flavin and Nakagawa [2008].

$$\gamma = 1 \qquad \rho = 1.8 \qquad \beta = .98 \qquad \alpha = -6.7$$

A rho parameter of 1.8 is well within laboratory experiments of relative risk aversion over lotteries (and consistent the findings of Szpiro [1986] from insurance data). Though the correct inter-temporal discount rate to use in representative agent modeling is still much debated, $\beta = .98$ seems reasonable for a simulation at annual frequencies.[11]Appendix B shows that the purpose of γ is to primarily convert between the units of of housing determining utility (which may be in square feet, square yards, or hectares, or what have you) and the price per square foot P_t. That Flavin and Nakagawa [2008] estimate γ as one implies that the utility function takes square feet and not other area measures as an input. Setting $\alpha = -6.7$ implies a low substitutability between durable and non-durable consumption. For example, if $\alpha = 1$ then durable and non-durable goods would be perfect substitutes and no adjustment in the level of durable goods would be needed. Taking the $\lim \alpha \to 0$ gives Cobb-Douglas utility in the two goods, and so this substantially lower substitutability than that.

The simulation calibrates the friction and asset return dynamics from several empirical sources.

$$P_{2006} = 113 \qquad R_f = 1.042 \qquad \lambda = .05$$

Real stock market return data calculated from Bureau of Labor Statistics' *Consumer Price Index - All Urban Consumers* series and Fama-French *U.S. Research Returns Data* (1950-2010). While home quality and land prices (with associated amenities) vary a great deal, US Census Bureau data estimates the cost of new construction at the peak of the housing boom at $113 a square foot.[12] An annual home

[11]See for example Discounting and Intergenerational Equity (1999) by Portney and Weyant for justifications of using everything between a discount rate of 0 and stock market returns. Trachtenberg [2011] argues that the proper discount rate for a social planner is negative because of rising willingness to pay for safety, environmental protection, and medical care.

[12]According to US Census data (using reports 2011 reports *Median and Average Sales Price of Houses Sold by Region* and *Median and Average Square Feet of Floor Area in New Single-Family Houses Completed by Location*), the median new

price return series is calculated from the CS10. These data motivate the return distributions in tables 1 and 2. While transaction costs associated will selling a home vary significantly (from very low for retirees selling a home "for sale by owner" on Realtor.com for a local move to very high for a busy professionals selling a problem home and moving across country), the most accurate and relevant estimate 5 percent from Haurin and Gill [2002], motivating the value of lambda used in this paper. This may be a low estimate because it is calculated from military families who plausibly have lower than typical moving costs. Ommeren and Leuvensteijn [2005] estimate the equivalent of λ as 6 - 22 percent in several European countries. Sensitivity analysis confirms that a larger value of λ dampens the response of consumption to wealth shocks. This is consistent with Grossman and Laroque [1990a] which finds higher housing transaction costs reduces the fraction of wealth held in risky assets.

V Simulation Results

In the overview of the model in Section II, the model was transformed into various intensive variables (e.g., $c_t = \frac{C_t}{H_t}$) to eliminate a state variable from the optimization process. This transformed problem is the one solved computationally. Once solved, the policy functions can be rewritten in more intuitive quantities. Now policies are normalized to be fraction of total wealth W_t (e.g., $\frac{C_t}{W_t} = \frac{c_t}{y_t + P_t \cdot \lambda}$). These new policy functions describe what fraction of wealth goes to what purpose at each level of y_t. There is also a renormalization of y_t so that policies are instead a function of $\frac{W_t}{H_{t-1}}$. Where y_t is a measure of wealth net of transaction costs relative to the quantity of housing, $\frac{W_t}{H_{t-1}}$ is a measure of pre-adjustment cost wealth relative to the quantity of housing. While y_t is easier to manipulate analytically it can be confusing when comparing policy functions across different values of P_t and λ. Two households with the same H_{t-1} have three ways to have different y_t (different W_t, λ, or P_t) but only one way to have a different ratio of wealth to housing (different wealth). This also simplifies comparison with figure one of Grossman and Laroque [1990b] which plots the equivalent of $\frac{W_t}{H_{t-1}} - 1$ against $\frac{X_t}{W_t}$. Figures 2, 3, and 4 show the model's policy functions for housing, consumption, and risky assets. In an analogous frictionless model, all three of these plots would be horizontal lines with levels

home price at the peak of boom (2007 Q2 in their data)was $257,400 and a median size of 2,277 square feet which implies a construction cost of approximately $113 a square foot inclusive of land costs. New homes are often built with higher ceilings, on bigger lots, to higher standards of finish, and in more expensive areas, so this may be a significant overstatement of the square footage price of existing existing homes. However, if newer homes provide more services for a given square footage, they may still be effectively the same price for units of service flow.

determined by expected returns, preferences, and P_t because expenditures are constant fractions of wealth conditional on P_t. The housing adjustment cost alters the policy functions considerably from the frictionless case. Each policy function is considered in turn.

V.1 Housing Policy

Figure 2 shows the housing policy function for various values of P_t. Within the inaction region on housing (the S-s bounds), the wealth share invested in housing is mechanical:

$$\frac{P_t H_t}{W_t} = \frac{P_t H_{t-1}}{W_t} = \frac{P_t}{\frac{W_t}{H_{t-1}}}$$

Holding everything else constant, doubling $\frac{W_t}{H_{t-1}}$ halves the budget share of housing. Outside of the S-s bounds, households select a new home. Within the range of house prices studied in this paper, households adjust into homes costing approximately 60-65 percent of household wealth. The strong complementarity between durable and non-durable consumption insures that households want to consume them in fixed proportions if possible. Therefore, after adjustment the price impacts the quantity of housing but not the budget share of housing.

The price of housing influences the the S-s bounds. Notice that the S-s bounds both widen and shift to the right as house prices increase. A higher house price widens the bounds because the cost of adjustment $\lambda \cdot H_{t-1} \cdot P_t$ is also higher. When the adjustment cost is larger households partially compensate by adjusting the policy to pay the adjustment cost less frequently. For the intuition, consider that in the extreme, if housing were free transaction costs would be zero and all households could adjust. The rightward shift is a product of the upper S-s bound increasing more than the lower one as P_t increases. This happens because of the income effect from the pattern of adjustment. A household adjusting down is cashing out a too valuable home and using the proceeds to buy a smaller one. A higher home price means this sale raises more money. In contrast, a household adjusting up is a net buyer of housing. A higher price means they pay more for a given change in housing units. Therefore, household trading up needs a larger $\frac{W_t}{H_{t-1}}$ before adjusting is optimal than they do when house prices are lower. This unequal income effect combined with the equal transaction costs effect moves the upper S-s more than the lower one.

V.2 Consumption Policy

Figure 3 shows the consumption policy function. Notice that a greater fraction of wealth is spent on non-durable consumption (C_t/W_t) when $\frac{W_t}{H_t}$ is relatively small. The intuition is as follows. When a household has relatively too much house for their current level of wealth they would like to cut both non-durable and durable consumption. However, the loses from paying the transaction cost outweighs the gains from adjusting durable consumption. Recall that durable and non-durable consumption are complements in the model. Under this complementarity, the high level of unchanged durable consumption raises the marginal utility of non-durable consumption compared with what it would be if the household had adjusted the durable good. Therefore, the optimal share of wealth to spend on non-durable consumption is relatively higher. Conversely, if the household has relatively too little house for their current wealth then this complementarity depresses the marginal utility of non-durable consumption. This reduces the optimal share of wealth to spend on non-durable consumption.

This may seem counterintuitive. How can a household with too much housing afford to spend more of their wealth on consumption? The answer is in two parts. First, when comparing two households (within the S-s bounds) facings the same economy and current housing H_{t-1}, the one with the higher $\frac{W_t}{H_{t-1}}$ consumes more units of non-durable consumption (C_t). It is only the share of wealth consumed that is higher for the household with the smaller $\frac{W_t}{H_{t-1}}$. The higher fraction of wealth consumed is not enough to compensate for the lower wealth. Second, they do not plan on being in that position forever. Eventually when they follow the optimal policy they will either exit the S-s bound and adjust their level of housing or experience enough positive wealth shocks that they move back into a region where they consume a smaller fraction of their wealth each period.

Varying the price of housing does not alter any of this basic logic. However, it does change the domain and range of the policy function. The range is governed by the transaction cost and income effects from the housing policy function discussion. The range is controlled by wealth and substitution effects. The income effect is that lower housing prices mean households can afford more of everything. The substitution effect is that lower housing prices raise the relative cost of non-durable consumption, making households purchase relatively more housing. Because of the strong complementarity of durable and non-durable consumption the wealth effect is stronger than the substitution effect and households consume a higher fraction of their wealth when house prices are lower.

Figure 2: Housing policy function

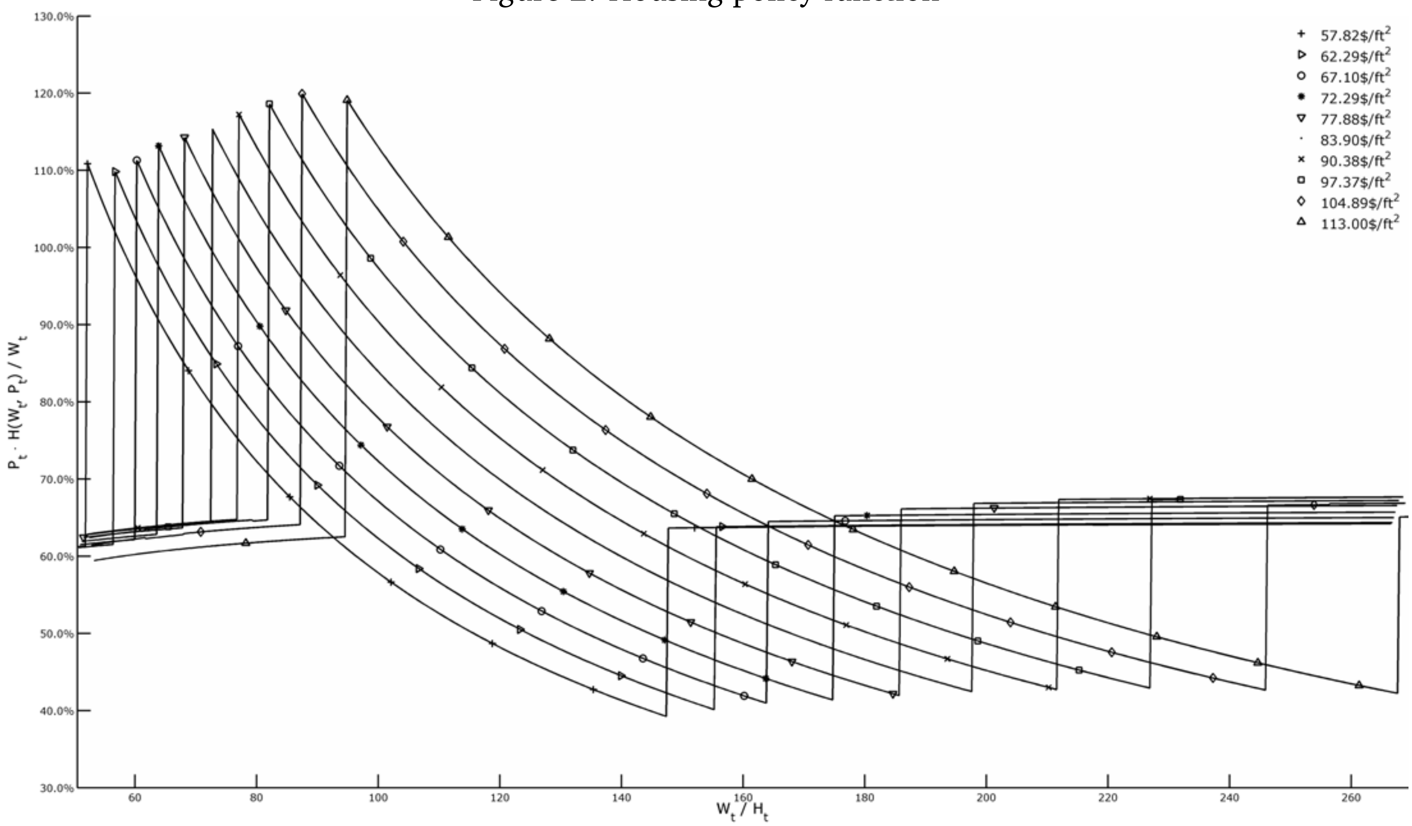
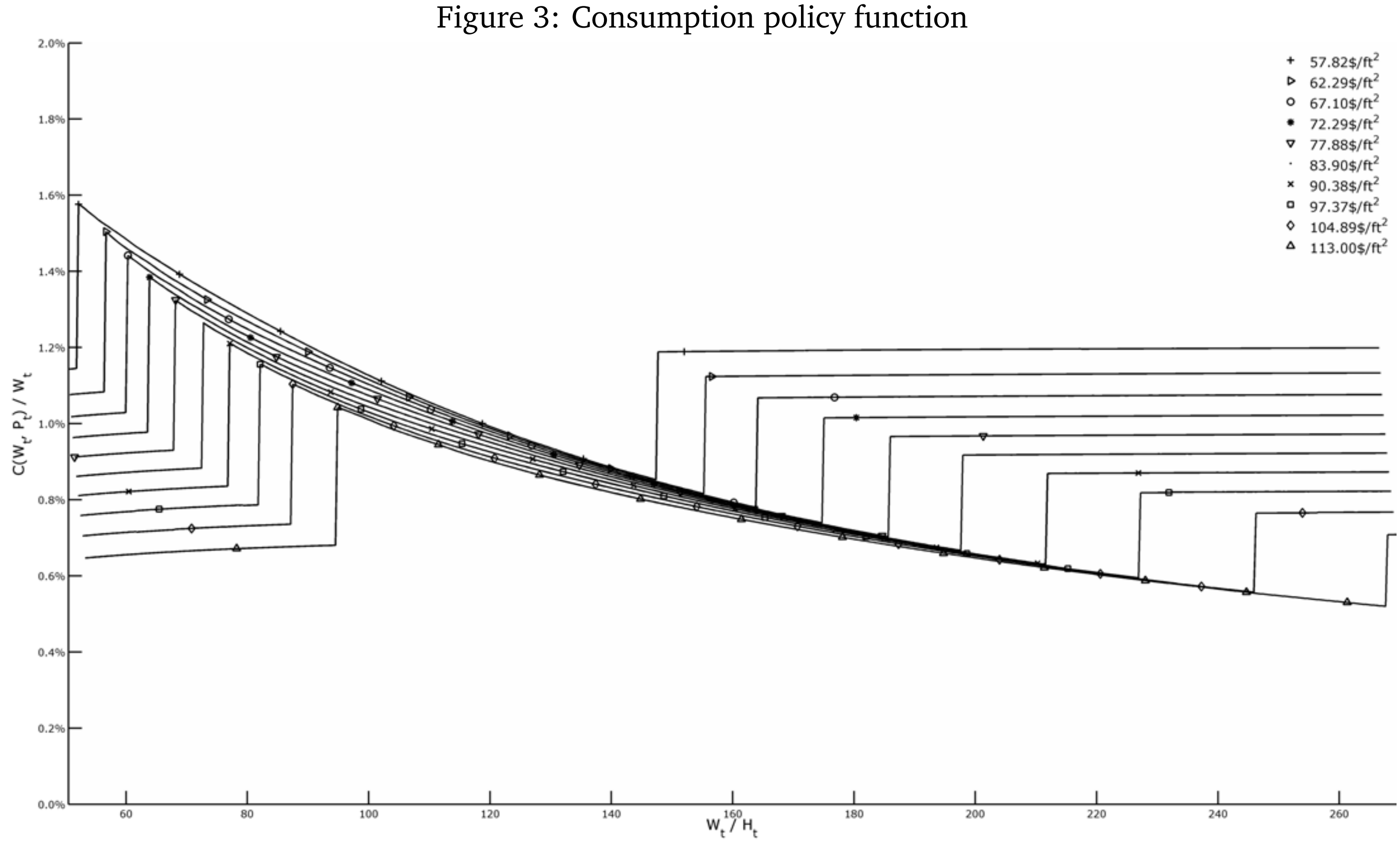


Figure 3: Consumption policy function



V.3 Risky Asset Policy

Figure 4 shows the risky asset policy function. Notice the general 'U' shape of the risky asset's share of wealth with respect to $\frac{W_t}{H_{t-1}}$. This is caused by household risk preferences that depend on $\frac{W_t}{H_{t-1}}$. One way to see this in figure 5. In the frictionless setting, there is a constant curvature of the value function. With the frictions, the curvature of the value function depends on $\frac{W_t}{H_{t-1}}$. When $\frac{W_t}{H_{t-1}}$ is near the S-s bounds there is less curvature. When $\frac{W_t}{H_{t-1}}$ is near the return point (the value of $\frac{W_t}{H_{t-1}}$ chosen when adjusting) there is more curvature. Those households with high curvature are more risk adverse (in an Arrow-Pratt sense) than those with low curvature.

Alternatively, consider a household near the upper S-s bound. If they chance a risky investment and it pays off they can afford to adjust their housing position. This makes them better off two ways, they can afford more non-durable consumption and the ratio of wealth to housing is also more optimal. On the other hand, if the investment does poorly then the ratio of wealth to housing falls towards the return point, becoming more optimal. This partly offsets the of wealth. By improving the ratio both the good and bad outcomes are better than the straight wealth effects suggest. This lowers the required certainty equivalent and makes the household less risk averse (again in an Arrow-Pratt sense). On the other hand, consider a household at the return point. Positive and negative returns on the risky investment both move the household away from the efficient wealth to housing ratio. Now the efficiency effect is reversed, raising the certainty equivalent because good and bad outcomes are both worse than if there were only a wealth effect.

Notice that this discussion does not depend on P_t. This ensures that the portfolio holdings are are the same near these points. However, because of the transaction cost and income effects these points shift right and spread apart as the price of housing increases. This widens the "U" shape of the policy function but leaves the levels at the three anchor points unchanged.

V.4 The Value Function

These three optimal policy functions imply the household's value function. In the frictionless case the value function inherits the constant relative risk aversion form for the felicity function. In this model with frictions and complementarity the overall level of the value function is lower because there is an inefficiency induced by not consuming the durable and non-durable consumption in optimal (under no frictions) proportions and a wealth effect that households are poorer because they have to actually pay the transaction cost. Grossman and Laroque [1990a] prove that in their model with housing and a non-convex adjustment cost (but without house price dynamics or non-durable consumption) that outside of the S-s bounds (the adjustment region) the value function takes a form $M \cdot y_t^{1-\rho}$ (where M is a constant) and that value function has the same curvature as in the frictionless case. This result is also found in this paper's simulations: in the adjustment region the value function takes the form $M(P_t) \cdot y_t^{1-\rho}$ and the curvature of the value function is the same as the frictionless case and controlled by ρ. Within the S-s bounds there is a hump where the value function is greater than $M(P_t) \cdot y_t^{1-\rho}$. Figure 5 shows this hump, the difference $G(y_t, P_t) - M(P_t) \cdot y_t^{1-\rho}$.[13]

The hump occurs because adjusting housing is costly. It is optimal to adjust only when the wealth effect of paying the transaction cost is offset by the efficiency gains of altering the consumption bundle. However, households can choose not to adjust housing. The right but not the obligation to adjust housing is an option held by the household. The value of that option to the household (measured in utility) is the difference $G(y_t, P_t) - M(P_t) \cdot y_t^{1-\rho}$. This hump represents the option value of not adjusting. Outside of the S-s bounds the household is worse off not adjusting and the right to not adjust this period has no value.

The option value for a given value of y_t is sensitive to P_t. Again, this is because of a wealth and efficiency effect. When the price of housing is higher so is the adjustment cost and therefore the value of not paying that cost is higher (wealth). Unfortunately for the household, when P_t is higher they expect to spend longer in their new home and farther away from the optimal ratio of housing to wealth then they would at lower prices. Overall, as home prices increase the inefficiency effect dominates and the option value declines.

V.5 Implications for Consumption Dynamics

To study the model's replication of empirical macroeconomic consumption dynamics, it is useful to study the model's predicted consumption changes to historical changes in home prices and stock market returns. Figure 7 highlights the basic model results. It shows agents' consumption and wealth

[13]This is analogous to G&L's figure 2.

Figure 4: Risky asset policy function

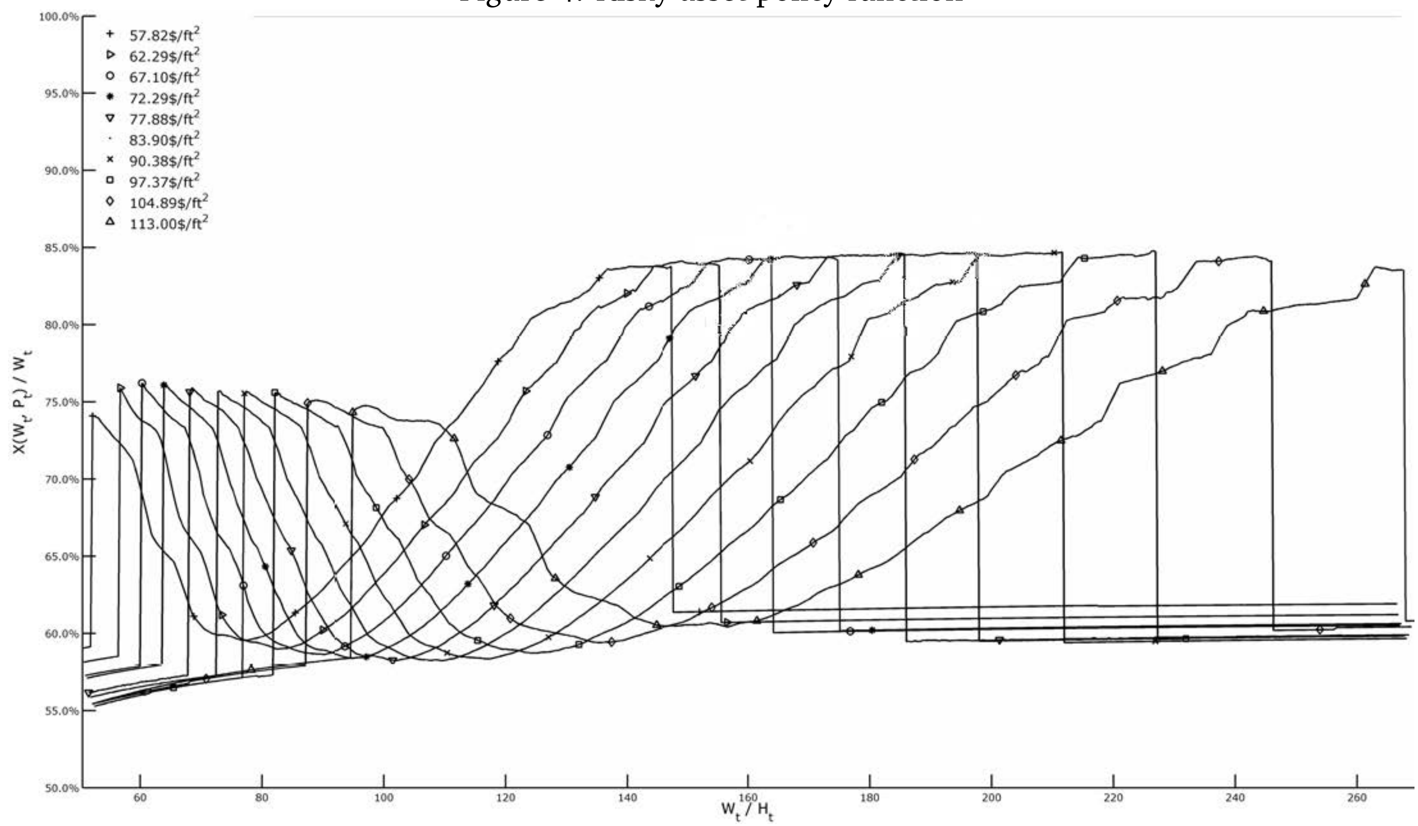


Figure 5: The value of the option to delay adjustment depends on P_t

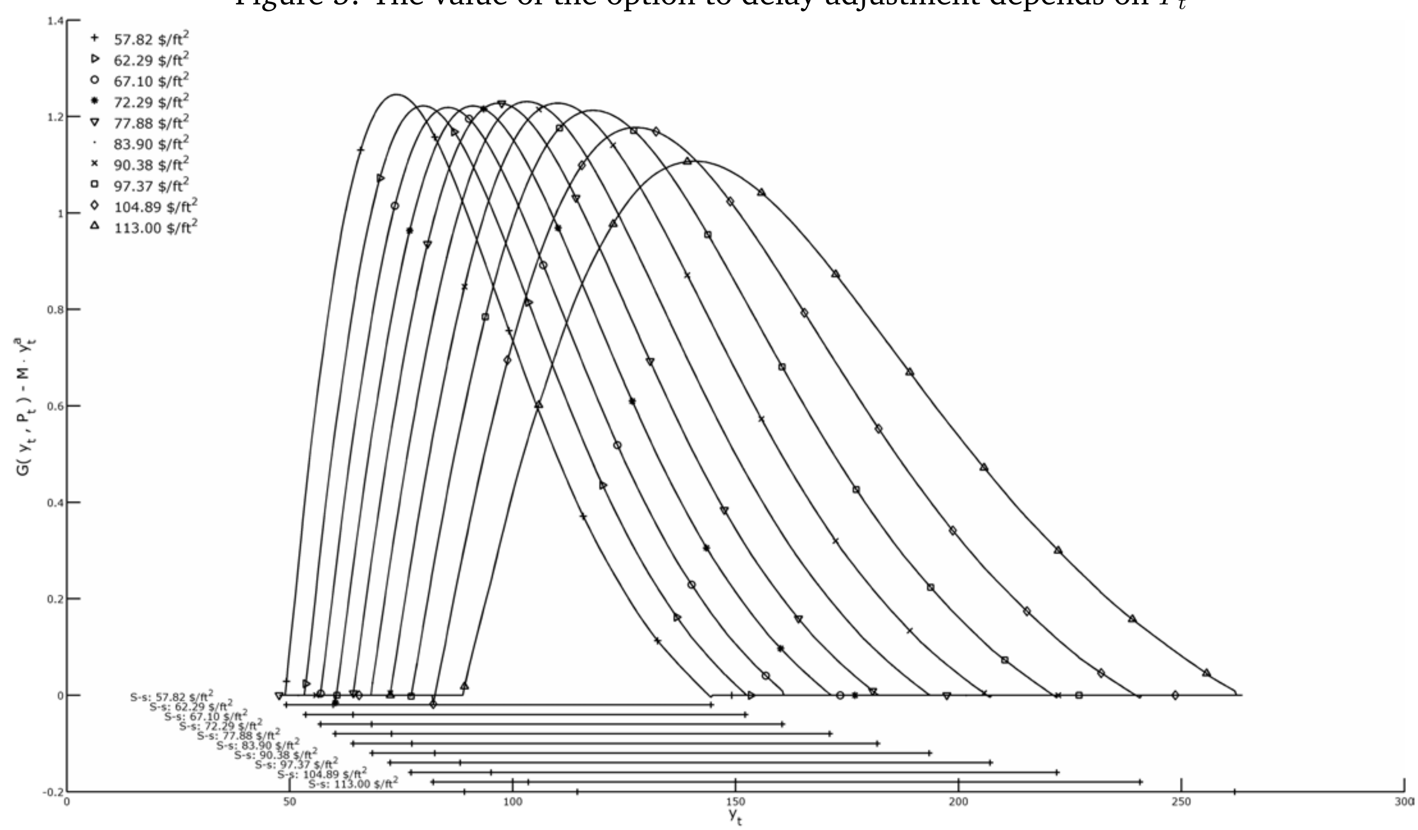
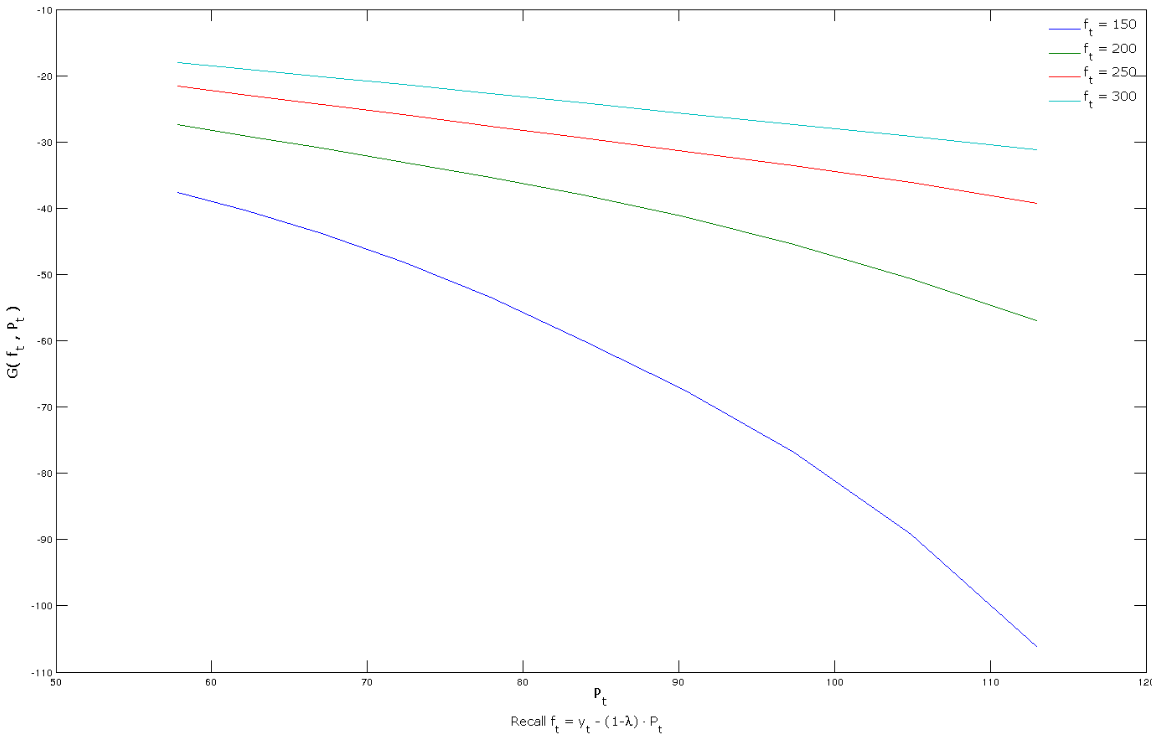


Figure 6: Home Price Increases Make the Household Worse Off Holding Fixed Housing and Financial Assets



Define $F_t \equiv W_t - \lambda P_t H_{t-1} - P_t \cdot H_{t-1}$ as household resources not held in housing. Define $f_t \equiv \frac{F_t}{H_{t-1}}$ as the housing intensive version. Holding these resources constant and varying P_t shows that higher housing prices make the household worse off. The wealth effect of the more valuable home and the substitution effect of substituting into non-durable consumption moving out of housing veThis is caused by the transaction cost and complementarity between housing and consumption.

responses from 2006 to 2008 when the US stock market fell 39 percent and national home prices fell 31%. Relatively house rich agents (y_t large pre-crash) lowered their non-durable consumption by approximately 10 percent . Progressively more house poor (relatively too much housing) agents decreased their consumption by larger amounts but less than 20 percent. A subset of agents (those with smaller y_t) move into smaller homes and have vastly lower consumption with declines of 45 percent or more. The other line in figure 7 is the changes in wealth for each value of pre-crash y_t. In a frictionless model (like Lucas [1978]) where agents make the same investment decisions consumption is a constant fraction of wealth and so this would also be the consumption response. Overall, this model delivers much smaller (and more realistic) consumption adjustment for most agents. Some agents adjust more in the model then they would in a frictionless setting. However, real households forced to downsize due to diminished wealth or job prospects are likely to make atypically large non-durable consumption adjustments. Therefore, general response shape is realistic, even if the precise magnitudes are potentially too large.

Figures 8 and 9 respectively examine the path of consumption predicted by the model and over the historical asset returns in the years 1995-2010 and 2005-2010. They also show the response of two alternative frictionless models. The first, "Frictionless Stock Market Model" assumes that all household wealth is held in the market portfolio of stocks. The second, "Frictionless Stock And Housing Portfolio Model Consumption" assumes that households hold their assets in a mixture of housing (45%) and risky assets (55%) consistent with portfolio composition in the 2004 Survey of Consumer Finance. Again, since in both models consumption can be costlessly adjusted, percent changes in wealth are percent changes in consumption.

Within the model, agent consumption changes depend on y_t. Therefore, this requires some assumption about the initial distributions of y_t in the economy before the the shocks are introduced. Two methods are employed. First, the joint distribution of of P_t and risky asset returns implies a steady state distribution of $y_t|P_t$. Alternatively, a uniform distribution of y_t values between the S-s bounds is used. As a further refinement these distributions can start at different times. Figure 8 starts the simulation in 1987. However, since NIPA data on true non-durable and service consumption starts only in 1995, only those results starting in 1995 are shown. Figure 9 starts the simulation just before the crash in 2005. Tables 3 and 4 provide the data used to create these plots, as well as the sum of

Figure 7: Changes in consumption depend on y_t

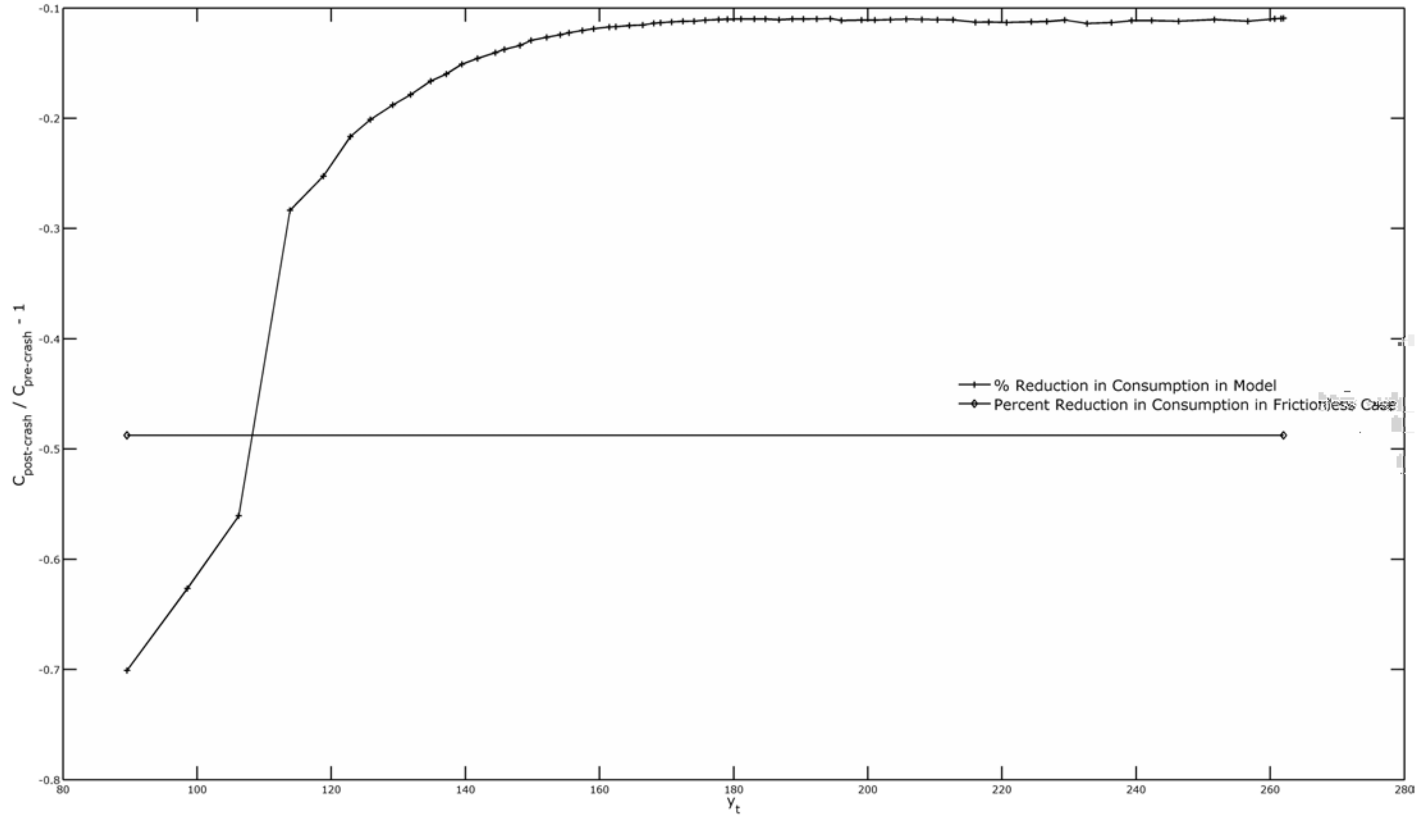
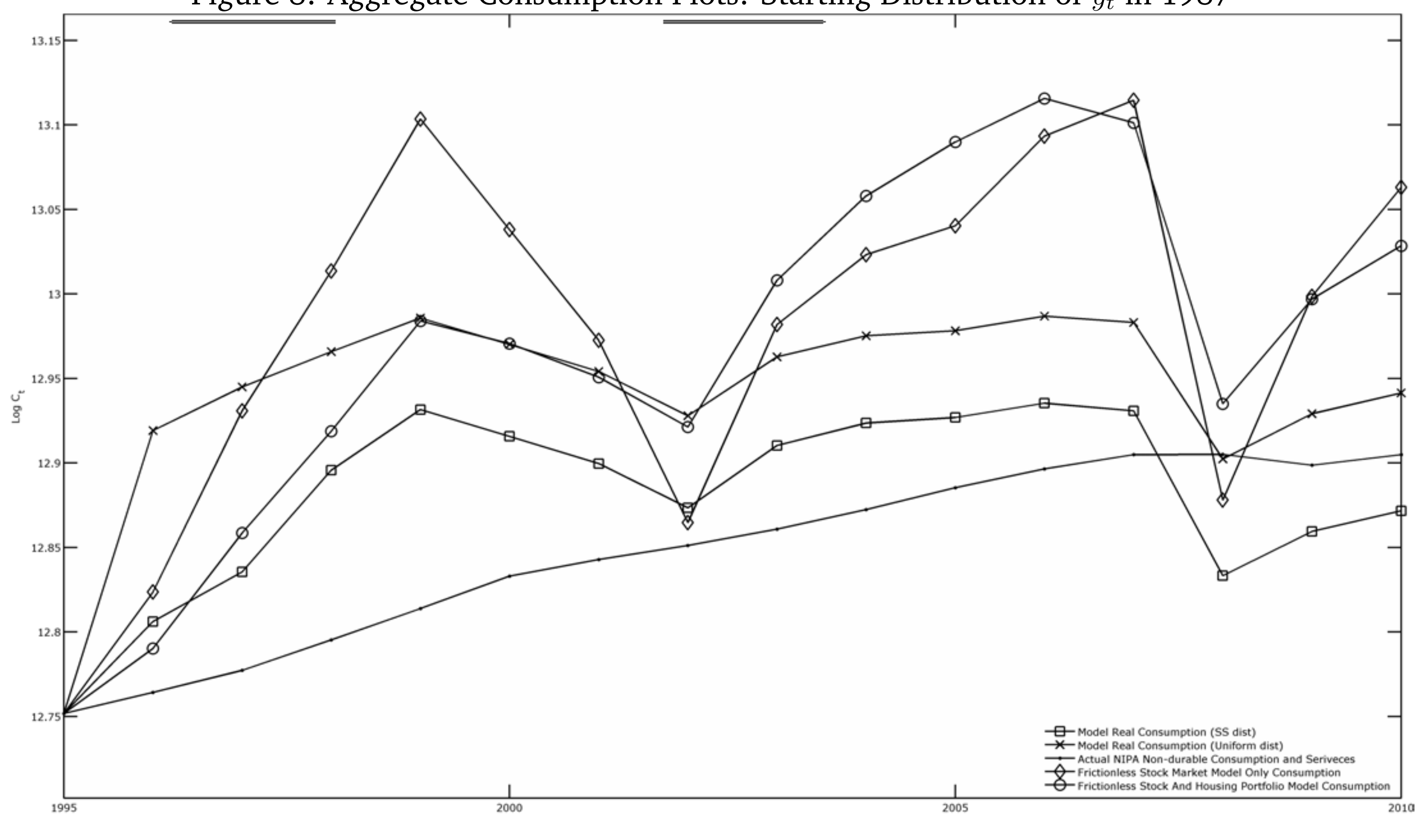


Figure 8: Aggregate Consumption Plots: Starting Distribution of y_t in 1987



squared error from true consumption process reported in the NIPA.

Both y_t distributions have much lower mean squared error with respect to the observed series than the two frictionless models. When starting in 1987, the steady state distribution has much smaller tracking error. When starting in 2005, the uniform distribution has slightly lower tracking. In both simulations, both calibration roughly have a third of the error of the frictionless models. The model predictions are particularly good in period of the Great Recession, with much smaller consumption responses.

In practice, many households walk away from mortgage obligations rather than suffer too much non-durable consumption reduction from continuing to pay a mortgage with a balance greater than the home's value. Therefore model prediction of exits on the low end should be a proxy for delinquent mortgages, perhaps with some delay. There are health and labor force reasons why people get into delinquency, so for many years national mortgage delinquencies were low and steady in America. The model predicts that depending on starting period and distribution of y_t that between two and ten percent of households would hit the lower S-s bound and adjust into a smaller home in 2008 as a result of the great recession after more than a decade of no households hitting this lower bound. This is roughly contemporaneous with the massive increase in mortgage delinquencies from historical levels.

It is possible that another omitted asset, human capital, could instead deliver these results. Appendix B provides a back of the envelope estimate of how large these human capital effects might be. These human capital effects alone are not large enough to deliver realistic consumption dynamics. However, when combined with the housing model, human capital gives even more accurate replication of aggregate consumption dynamics.

VI Conclusion

Models without non-convex transaction costs on adjusting housing holdings are more tractable. Therefore, we would prefer them if they gave the same quantitative and qualitative predictions. However, these frictions deliver substantially different consumption and investment policy functions. These functions have implications on the co-movement of aggregate consumption dynamics and asset prices that look much more like the actual co-movement of these series than that of models without this key

Table 3: Goodness of Fit: Starting Distribution of y_t in 1987*

Year	Model Real Consumption (SS dist)	Model Real Consumption (Uniform dist)	Actual NIPA Non-durable Consumption and Services	Frictionless Stock Market Model Only Consumption	Frictionless Stock And Housing Portfolio Model Consumption
1995**	12.752	12.752	12.752	12.752	12.752
1996	12.806	12.919	12.764	12.824	12.79
1997	12.836	12.945	12.777	12.931	12.859
1998	12.896	12.966	12.795	13.014	12.919
1999	12.932	12.986	12.814	13.104	12.984
2000	12.916	12.97	12.833	13.038	12.971
2001	12.9	12.954	12.843	12.973	12.951
2002	12.873	12.928	12.851	12.865	12.921
2003	12.91	12.963	12.861	12.982	13.008
2004	12.924	12.975	12.872	13.023	13.058
2005	12.927	12.978	12.885	13.04	13.09
2006	12.935	12.987	12.897	13.093	13.116
2007	12.931	12.983	12.905	13.115	13.101
2008	12.833	12.902	12.905	12.878	12.935
2009	12.859	12.929	12.899	12.999	12.997
2010	12.872	12.941	12.905	13.063	13.028
SSE	188.4	358.2	.	1379.9	525.8

* - Case-Shiller data starts in 1987

** - NIPA non-durable consumption data starts in 1995

Table 4: Goodness of Fit: Starting Distribution of y_t in 2005

Year	Model Real Consumption (SS dist)	Model Real Consumption (Uniform dist)	Actual NIPA Non-durable Consumption and Services	Frictionless Stock Market Model Only Consumption	Frictionless Stock And Housing Portfolio Model Consumption
2005	12.885	12.885	12.885	12.885	12.885
2006	12.927	12.9	12.897	12.938	12.926
2007	12.948	12.905	12.905	12.96	12.94
2008	12.862	12.823	12.905	12.723	12.802
2009	12.886	12.847	12.899	12.844	12.865
2010	12.898	12.858	12.905	12.908	12.894
SSE	94.7	77.9	.	774.7	252.2

friction.

The model also makes household level predictions. Households are predicted to smoothly reduce consumption in response to small wealth shocks and discontinuously to large wealth shocks that induce them to adjust their housing. They are also predicted to make infrequent large adjustment to the house holdings. These predictions are consistent with the microeconomic evidence. Households that have recently adjusted their housing are predicted to hold less of the risky asset while those considering moving for financial reasons (y_t near the S-s bounds) should be more risk tolerant. This could be why Banks et al. [2002] and Flavin and Yamashita [2002] find that young households who on average have moved more recently hold less of their wealth in stocks.

There are other omitted assets on the household balance sheet. Particularly, human capital (or alternatively labor income) is not treated and it is large, illiquid, difficult to borrow against, and considerably less variable (especially in aggregate) than housing or stock markets. Households also have access to bankruptcy and social insurance. There are other macroeconomic dynamics buffeting the household beyond asset returns. All should affect household consumption and investment and therefore it would be a surprise if adding housing alone would perfectly match aggregate dynamics. Though the resulting dynamics are still too volatile relative to observed non-durable consumption, this paper shows that a serious treatment of housing goes far towards generating realistic household behavior and aggregate dynamics. A natural extension to this work is adding labor income or human capital to the model. The calculations in Appendix B suggest that between housing and human capital, most of the consumption dynamics can be captured.

VII Bibliography

References

James Banks, Richard Blundell, and James P. Smith. Wealth portfolios in the uk and the us. Technical report, National Bureau of Economic Research, August 2002. URL `http://www.nber.org/papers/w9128`. Working Paper.

Robert J. Barro. Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of*

Economics, 121(3):823–866, August 01 2006.

James Berkovec and Don Fullerton. A general equilibrium model of housing, taxes, and portfolio choice. *Journal of Political Economy*, 100(2):pp. 390–429, Apr. 1992. URL http://www.jstor.org/stable/2138612.

Steven C. Bourassa, Donald R. Haurin, Jessica L. Haurin, Martin Edward Ralph Hoesli, and Jian Sun. House price changes and idiosyncratic risk: The impact of property characteristics. *Real Estate Economics*, 37:259–278, 2009. URL http://dx.doi.org/10.1111/j.1540-6229.2009.00242.x.

George M. Constantinides. Capital market equilibrium with transaction costs. *Journal of Political Economy*, 94(4):pp. 842–862, Aug. 1986. URL http://www.jstor.org/stable/1833205.

George M. Constantinides. Habit formation: A resolution of the equity premium puzzle. *The Journal of Political Economy*, 98(3):519–543, Jun. 1990. URL http://www.jstor.org/stable/2937698.

Karen E. Dynan. Habit formation in consumer preferences: Evidence from panel data. *The American Economic Review*, 90(3):pp. 391–406, Jun. 2000. URL http://www.jstor.org/stable/117335.

Peter Englund, Min Hwang, and John M. Quigley. Hedging housing risk*. *The Journal of Real Estate Finance and Economics*, 24(1):167–200, 01/01 2002. URL http://dx.doi.org/10.1023/A:1013942607458. M3: 10.1023/A:1013942607458.

Larry G. Epstein and Stanley E. Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969, Jul. 1989. URL http://www.jstor.org/stable/1913778.

Marjorie Flavin and Shinobu Nakagawa. A model of housing in the presence of adjustment costs: A structural interpretation of habit persistence. *American Economic Review*, 98(1):474–495, March 2008. URL http://ideas.repec.org/a/aea/aecrev/v98y2008i1p474-95.html.

Marjorie Flavin and Takashi Yamashita. Owner-occupied housing and the composition of the household portfolio. *The American Economic Review*, 92(1):345–362, Mar. 2002. URL http://www.jstor.org/stable/3083338.

Giuseppe Folloni and Giorgio Vittadini. Human capital measurement: A survey. *Journal of Economic Surveys*, 24(2):248–279, 2010.

William Nelson Goetzmann. The single family home in the investment portfolio. *The Journal of Real Estate Finance and Economics*, 6(3):201–222, 05/01 1993. URL http://dx.doi.org/10.1007/BF01096958. M3: 10.1007/BF01096958.

Douglas Gollin. Getting income shares right. *Journal of Political Economy*, 110(2):pp. 458–474, April 2002. URL http://www.jstor.org/stable/10.1086/338747.

Christopher Goodman and Steven Mance. Employment loss and the 2007-09 recession: An overview. *Monthly Labor Review*, 1:1–6, 2011.

Sanford J. Grossman and Guy Laroque. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica*, 58(1):25–51, Jan. 1990a. URL http://www.jstor.org/stable/2938333.

Sanford J. Grossman and Guy Laroque. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica*, 58(1):25–51, Jan. 1990b. URL http://www.jstor.org/stable/2938333.

Luigi Guiso, Tullio Jappelli, and Daniele Terlizzese. Income risk, borrowing constraints, and portfolio choice. *The American Economic Review*, 86(1):pp. 158–172, Mar. 1996. URL http://www.jstor.org/stable/2118260.

Goulder Lawrence H. Tax policy, housing prices, and housing investment. *Regional Science and Urban Economics*, 19(2):281–304, 5 1989.

Eric A. Hanushek and John M. Quigley. What is the price elasticity of housing demand? *The review of economics and statistics*, 62(3):pp. 449–454, Aug. 1980. URL http://www.jstor.org/stable/1927113.

Donald R. Haurin and H. Leroy Gill. The impact of transaction costs and the expected length of stay on homeownership. *Journal of Urban Economics*, 51(3):563–584, 5 2002.

John Heaton and Deborah Lucas. Portfolio choice in the presence of background risk. *The Economic Journal*, 110(460):1–26, 2000.

J. V. Henderson and Y. M. Ioannides. A model of housing tenure choice. *The American Economic Review*, 73(1):pp. 98–113, Mar. 1983. URL `http://www.jstor.org/stable/1803929`.

Dale Jorgenson and Barbara M. Fraumeni. *The Measurement of Saving, Investment, and Wealth*, chapter The accumulation of human and nonhuman capital, 1948-84, pages 227 – 286. University of Chicago Press, 1989. URL `http://www.nber.org/chapters/c8121.pdf`.

Nicholas Kaldor. *Capital Accumulation and Economic Growth*. The Theory of Capital. St. Martin's Press, 1961.

Tamara G. Kolda, Robert Michael Lewis, and Virginia Torczon. Optimization by direct search: New perspectives on some classical and modern methods. *SIAM Review*, 45(3):pp. 385–482, Sep. 2003. URL `http://www.jstor.org/stable/25054427`.

Edward Leamer. Housing is the business cycle. *Proceedings, Federal Reserve Bank of Kansas City*, pages 149–233, 2007. URL `http://www.kansascityfed.com/publicat/sympos/2007/PDF/Leamer_0415.pdf`.

Robert E. Lucas. Asset prices in an exchange economy. *Econometrica*, 46(6):1429–1445, Nov. 1978. URL `http://www.jstor.org/stable/1913837`.

Hanno N. Lustig and Stijn G. Van Nieuwerburgh. Housing collateral, consumption insurance, and risk premia: An empirical perspective. *The Journal of Finance*, 60(3):1167–1219, Jun. 2005. URL `http://www.jstor.org/stable/3694924`.

Rajnish Mehra. *Handbook of the equity risk premium*. Elsevier, 2007. ISBN 0444508996.

Rajnish Mehra and Edward C. Prescott. The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2):145–161, 3 1985.

Rajnish Mehra, Facundo Piguillem, and Edward C. Prescott. Costly financial intermediation in neoclassical growth theory. *Quantitative Economics*, 2(1):1–36, 2011.

Robert C. Merton. Lifetime portfolio selection under uncertainty: The continuous-time case. *The review of economics and statistics*, 51(3):247–257, Aug. 1969. URL `http://www.jstor.org/stable/1926560`.

Jos Van Ommeren and Michiel Van Leuvensteijn. New evidence of the effect of transaction costs on residential mobility*. *Journal of Regional Science*, 45(4):681–702, 2005.

Monika Piazzesi, Martin Schneider, and Selale Tuzel. Housing, consumption and asset pricing. *Journal of Financial Economics*, 83(3):531–569, 3 2007.

Thomas A. Rietz. The equity risk premium a solution. *Journal of Monetary Economics*, 22(1):117–131, 7 1988.

Richard Roll. A critique of the asset pricing theory's tests part i: On past and potential testability of the theory. *Journal of Financial Economics*, 4(2):129–176, 3 1977. URL `http://www.sciencedirect.com/science/article/pii/0304405X77900095`.

George G. Szpiro. Measuring risk aversion: An alternative approach. *The review of economics and statistics*, 68(1):156–159, Feb. 1986. URL `http://www.jstor.org/stable/1924939`.

Breeden Douglas T. An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics*, 7(3):265–296, 9 1979.

George Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181, 1986.

Robert Topel and Sherwin Rosen. Housing investment in the united states. *Journal of Political Economy*, 96(4):pp. 718–740, Aug. 1988. URL `http://www.jstor.org/stable/1830471`.

Ben Trachtenberg. Health inflation, wealth inflation, and the discounting of human life. *Oregon Law Review, Vol.89, p.1313, 2011*, 89:1313, 2011. Accepted Paper Series.

A Transforming the Problem with Housing Intensive State and Control Variables

This appendix exploits the homogeneity of the value function and defines alternative housing intensive state and control variables to reduce the dimensionality of the value and policy functions. Define the intensive variables as follows:

$$\begin{array}{cccc} W_t & H_t & X_t & C_t \\ \downarrow & \downarrow & \downarrow & \downarrow \\ y_t \equiv \frac{W_t}{H_{t-1}} - \lambda \cdot P_t & h_t \equiv \frac{H_t}{H_{t-1}} & x_t \equiv \frac{X_t}{X_{t-1}} & c_t \equiv \frac{C_t}{H_{t-1}} \end{array}$$

Take the value function from section II.2:

$$V\left(\frac{W_t}{H_{t-1}}, 1, P_t\right) = \sup_{C_t, X_t, H_t} \left[\frac{\left(\left(\frac{C_t}{H_{t-1}}\right)^{\alpha} + \gamma \cdot \left(\frac{H_t}{H_{t-1}}\right)^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot \mathbb{E}_t \left[V\left(\frac{W_{t+1}}{H_t}, 1, P_{t+1}\right)\right]\right]$$

and simplify the value function by substituting with the intensive variables

$$V\left(y_t + \lambda \cdot P_t, 1, P_t\right) = \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^{\alpha} + \gamma \cdot h_t^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t \left[V\left(y_{t+1} + \lambda \cdot P_{t+1}, 1, P_{t+1}\right)\right]\right]$$

Simplify the problem further by defining $G\left(y_t, P_t\right) \equiv V\left(y_t + \lambda \cdot P_t, 1, P_t\right)$ and make that substitution as follows:

$$G\left(y_t, P_t\right) = \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^{\alpha} + \gamma \cdot h_t^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t \left[g\left(y_{t+1}, P_{t+1}\right)\right]\right]$$

Recall the equation of motion for wealth

$$W_{t+1} = R_f \cdot \left(W_t - C_t\right) + \left(R_{m,t+1} - R_f\right) \cdot X_t + \left(P_{t+1} - R_f \cdot P_t \cdot \left(1 + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot \frac{H_{t-1}}{H_t}\right)\right) H_t$$

and the definition of y_t:

$$y_{t+1} = \frac{W_{t+1}}{H_t} - \lambda \cdot P_{t+1}$$

Combine these two equations to substitute out W_{t+1} and create an equation of motion for y_t:

$$y_{t+1} = \frac{R_f \cdot (W_t - C_t) + (R_{m,t+1} - R_f) \cdot X_t + \left(P_{t+1} - R_f \cdot P_t \cdot \left(1 + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot \frac{H_{t-1}}{H_t}\right)\right) H_t}{H_{t-1}} \cdot \frac{H_{t-1}}{H_t} - \lambda \cdot P_{t+1}$$

Replace (C_t, X_t, H_t) with their intensive forms (c_t, x_t, h_t):

$$y_{t+1} = \frac{R_f \cdot \left(\frac{W_t}{H_{t-1}} - c_t\right) + (R_{m,t+1} - R_f) \cdot x_t}{h_t} + P_{t+1} - R_f \cdot P_t \cdot \left(1 + \frac{\mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda}{h_t}\right) - \lambda \cdot P_{t+1}$$

Add and subtract $\lambda \cdot P_t$ from the first parentheses:

$$y_{t+1} = R_f \cdot \frac{\left(\frac{W_t}{H_{t-1}} - c_t + \mathbf{1}_{\{H_t \neq H_{t-1}\}} \cdot \lambda \cdot P_t + \lambda \cdot P_t - \lambda \cdot P_t\right)}{h_t} + (R_{m,t+1} - R_f) \cdot \frac{x_t}{h_t} + P_{t+1} - R_f \cdot P_t - \lambda \cdot P_{t+1}$$

Replace $\frac{W_t}{H_{t-1}} - \lambda \cdot P_t$ with y_t:

$$y_{t+1} = R_f \cdot \frac{\left(y_t - c_t + \left(\mathbf{1}_{\{H_t \neq H_{t-1}\}} - 1\right) \cdot \lambda \cdot P_t\right)}{h_t} + (R_{m,t+1} - R_f) \cdot \frac{x_t}{h_t} + P_{t+1} \cdot (1 - \lambda) - R_f \cdot P_t$$

Simplify by replacing $\mathbf{1}_{\{H_t \neq H_{t-1}\}} - 1$ with $\mathbf{1}_{\{h_t = 1\}}$:

$$y_{t+1} = R_f \cdot \frac{y_t - c_t + \mathbf{1}_{\{h_t=1\}} \cdot \lambda \cdot P_t}{h_t} + (R_{m,t+1} - R_f) \cdot \frac{x_t}{h_t} + P_{t+1} \cdot (1 - \lambda) - R_f \cdot P_t$$

$$\Rightarrow y_{t+1} = R_f \cdot \frac{y_t - c_t + \mathbf{1}_{\{h_t=1\}} \cdot \lambda \cdot P_t}{h_t} + (R_{m,t+1} - R_f) \cdot \frac{x_t}{h_t} + P_{t+1} \cdot (1 - \lambda) - R_f \cdot P_t$$

Substitute the equation for y_{t+1} into the transformed Bellman:

$$G\left(y_t, P_t\right) = \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^\alpha + \gamma \cdot h_t^\alpha\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t\left[g\left(y_{t+1}, P_{t+1}\right)\right]\right]$$

$$= \sup_{c_t, x_t, h_t} \left[\frac{\left(c_t^\alpha + \gamma \cdot h_t^\alpha\right)^{\frac{1-\rho}{\alpha}}}{1-\rho} + \beta \cdot h_t^{1-\rho} \cdot \mathbb{E}_t\left[G\left(R_f \cdot \frac{y_t - c_t + \mathbf{1}_{\{h_t=1\}} \cdot \lambda \cdot P_t}{h_t}\right.\right.\right.$$

$$\left.\left.\left. + (R_{m,t+1} - R_f) \cdot \frac{x_t}{h_t} + P_{t+1} \cdot (1 - \lambda) - R_f \cdot P_t\ ,\ P_{t+1}\right)\right]\right]$$

B The Role of γ as Unit Converter

Assume square feet is the true relationship with non-durable consumption

$$\frac{\left(C^{\alpha}+\gamma_{f^2}H_{f^2}^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho}$$

. This can be rewritten as

$$\frac{\left(C^{\alpha}+\left(\gamma_{f^2}^{\frac{1}{\alpha}}H_{f^2}\right)^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho}$$

. Alternatively we could have a specification in square yards (a year is three feet or 0.9144 meters):

$$\frac{\left(C^{\alpha}+\left(\gamma_{y^2}^{\frac{1}{\alpha}}H_{y^2}\right)^{\alpha}\right)^{\frac{1-\rho}{\alpha}}}{1-\rho}$$

. This suggests that if $\gamma_{f^2}^{\frac{1}{\alpha}}H_{f^2}=\gamma_{y^2}^{\frac{1}{\alpha}}H_{y^2}$then the felicity will be unchanged. This is desirable because the units of measurement alone should not determine any household decisions. By the definition of the units we know that

$$9\cdot P_{Hf^2}=P_{Hy^2}$$

$$9\cdot H_{f^2}=H_{y^2}$$

$$\gamma_{f^2}^{\frac{1}{\alpha}}H_{f^2}=\gamma_{y^2}^{\frac{1}{\alpha}}\cdot 9\cdot H_{f^2}\Rightarrow\gamma_{f^2}=\gamma_{y^2}\cdot 9^{\alpha}$$

. Is this definition of gamma sensible? One way is to check is if the resulting intratemporal Euler is the correct one when everything is substituted in for the old values.

The intratemporal Euler of the square foot model is

$$\frac{1}{\gamma_{f^2}} \cdot \left(\frac{C}{H_{f^2}}\right)^{\alpha-1} = \frac{P_c}{P_{Hf^2}}$$

. The intratemporal Euler of the square yard model is

$$\frac{1}{\gamma_{y^2}} \cdot \left(\frac{C}{H_{y^2}}\right)^{\alpha-1} = \frac{P_c}{P_{Hy^2}}$$

. Substitute in $\gamma_{y^2} \cdot 9^{\alpha}$ for γ_{f^2}, $\frac{H_{y2}}{9}$ for H_{f^2} and $\frac{P_{Hy2}}{9}$ for P_{Hf^2}:

$$\frac{1}{\gamma_{y^2} \cdot 9^{\alpha}} \cdot \left(\frac{9 \cdot C}{H_{y^2}}\right)^{\alpha-1} = \frac{9 \cdot P_c}{P_{Hy^2}}$$

. Simplify

$$\Leftrightarrow \frac{9^{\alpha-1}}{9^{\alpha-1}} \cdot \frac{1}{\gamma_{y^2}} \cdot \left(\frac{C}{H_{y^2}}\right)^{\alpha-1} = \frac{P_c}{P_{Hy^2}} \Leftrightarrow \frac{1}{\gamma_{y^2}} \cdot \left(\frac{C}{H_{y^2}}\right)^{\alpha-1} = \frac{P_c}{P_{Hy^2}}$$

So yes, it gives the expected results. We can safely work in price per square foot and then rely on gamma to convert them into the proper utility units. But since Flavin and Nakagawa (2008) finds that gamma is essentially 1 (1.015 page 491), this means that the true units are extremely close to square feet. We can work backwards from gamma estimates to get the units of housing that go into the utility function.

$$\left(\frac{\gamma_{f^2}}{\gamma_{?^2}}\right) = \left(\frac{?^2}{f^2}\right)^{\alpha} \Rightarrow \left(\frac{\gamma_{f^2}}{\gamma_{?^2}}\right)^{\frac{1}{\alpha}} = \left(\frac{1.015}{1}\right)^{\frac{1}{-6.7}} = 0.9978 = \frac{?^2}{f^2}$$

. Therefore the model fits the utility unit of housing such that gamma is 1 at .9978 the size of a square foot. There is minimal loss of precision to measure housing services in square-feet instead.

C Estimating the Effect of Adding Human Capital

There is a literature exploring the effect of human capital on portfolio choice but nothing with a serious treatment of housing . Guiso et al. [1996] considers the effects of uninsurable income risk and borrowing constraints on the household portfolio but explicitly excludes primary residence from consideration. Heaton and Lucas [2000] examines the roll of uninsurable background risks of which labor income is their canonical example. They also ignore housing in their quantitative model. Realistic integration of human capital into the setting of this paper is beyond the scope of the paper. Instead, this section treats human capital as a large but frictionlessly adjusted asset in the household portfolio alongside stocks, bonds, and housing.

Denote human capital holdings as M_t and the return on human capital as $R_{H,t}$. Wealth $\tilde{W}_t$ is defined as

$$\tilde{W}_t \equiv R_f \cdot B_{t-1} + R_{m,t} \cdot X_{t-1} + P_t H_{t-1} + R_{H,t} \cdot M_{t-1}$$

where all other terms are as defined in the body of the paper. We can rewrite this as a total return on wealth equation in terms of the shares of wealth invested in each asset:

$$R_{\tilde{W}_t} \equiv \frac{\tilde{W}_t}{\tilde{W}_{t-1}} = R_f \cdot \frac{B_{t-1}}{\tilde{W}_{t-1}} + R_{m,t} \cdot \frac{X_{t-1}}{\tilde{W}_{t-1}} + P_t \frac{H_{t-1}}{\tilde{W}_{t-1}} + R_{H,t} \cdot \frac{M_{t-1}}{\tilde{W}_{t-1}}$$

In a frictionless Lucas tree economy, all asset holdings are fixed fractions of wealth determined by the expected value and covariances of their returns and household preferences. Denote these fractions respectively as $\theta_B \equiv \frac{B_t}{\tilde{W}_t}$, $\theta_X \equiv \frac{X_t}{\tilde{W}_t}$,$\theta_H \equiv \frac{P_t \cdot H_t}{\tilde{W}_t}$, and $\theta_M \equiv \frac{M_t}{\tilde{W}_t}$. Then the total return on wealth equation simplifies to become:

$$R_{\tilde{W}_t} = R_f \cdot \theta_B + R_{m,t} \cdot \theta_X + \frac{P_t}{P_{t-1}} \cdot \theta_H + R_{H,t} \cdot \theta_M$$

In this setting consumption C_t is also a constant fraction of wealth $\theta_C \equiv \frac{C_t}{\tilde{W}_t} = 1 - \theta_B - \theta_X - \theta_H - \theta_M$. This implies that the evolution of consumption is as follows:

$$C_t = \theta_C \cdot \tilde{W}_t = \theta_C \cdot R_{\tilde{W}_t} \cdot \tilde{W}_{t-1} = R_{\tilde{W}_t} \cdot C_{t-1} \Rightarrow \frac{C_t}{C_{t-1}} = R_{\tilde{W}_t}$$

Define the fraction of wealth saved:

$$\theta_s \equiv \theta_B + \theta_X + \theta_H + \theta_M$$

Define θ_1 as the fraction of wealth invested in housing and financial assets :

$$\theta_1 \equiv \theta_B + \theta_X + \theta_H = \theta_s - \theta_M$$

Define θ_2 as the fraction of wealth invested in housing and financial assets that is invested in housing:

$$\theta_2 \equiv \frac{\theta_H}{\theta_1}$$

As in the paper's body for the frictionless case, θ_B is assumed to be zero (through a zero net supply argument). This implies:

$$R_{\tilde{W}_t} = R_{m,t} \cdot \theta_X + \frac{P_t}{P_{t-1}} \cdot \theta_H + R_{H,t} \cdot \theta_M$$

$$= \left(R_{m,t} \cdot \frac{\theta_X}{\theta_1} + \frac{P_t}{P_{t-1}} \cdot \frac{\theta_H}{\theta_1} \right) \cdot \theta_1 + R_{H,t} \cdot \theta_M$$

Also define θ_2 as the fraction of financial wealth (wealth invested but not invested in human capital) that is invested in housing:

$$\theta_2 \equiv \frac{H_t}{B_t + X_t + H_t} = \frac{\theta_H}{\theta_B + \theta_X + \theta_H}$$

$\theta_2 = \frac{\theta_H}{\theta_H + \theta_X}$ and $1 - \theta_2 = \frac{\theta_X}{\theta_H + \theta_X}$. Then the evolution of consumption equation can be rewritten as follows:

$$\frac{C_t}{C_{t-1}} = R_{\tilde{W}_t} = R_{m,t} \cdot \theta_X + \frac{P_t}{P_{t-1}} \cdot \theta_H + R_{H,t} \cdot \theta_M$$

$$= \left[\left(R_{m,t} \cdot \frac{\theta_X}{\theta_1} + \frac{P_t}{P_{t-1}} \cdot \frac{\theta_H}{\theta_1} \right) \cdot \theta_1 + R_{H,t} \cdot \theta_M \right]$$

$$= \left(R_{m,t} \cdot (1 - \theta_2) + \frac{P_t}{P_{t-1}} \cdot \theta_2 \right) \cdot (\theta_s - \theta_M) + R_{H,t} \cdot \theta_M$$

The 2004 Survey of Consumer Finance Survey establishes $\theta_2 \approx .45$. To estimate the consumption process predicted by this setup requires an estimate of θ_M. Estimating individual and aggregate human capital is a complex problem that is the focus of ongoing research (see Folloni and Vittadini [2010] for a review of the history and modern attempts at measurement). Jorgenson and Fraumeni [1989] estimates the stock of America's human capital using school enrollment and demographic data. The paper finds human capital to be be about 92 percent of the total capital stock. However, this includes the value of non-market income (especially leisure) and so the numbers are not directly comparable to θ_M because that measure should only count resources convertible into durable and non-durable consumption. They estimate that the value of labor income only is about 18 percent of the total. This implies $\theta_M = .67$.

An alternative is to use assumptions about competition and factor shares to generate θ_M. Kaldor [1961] documented the stable shares of income going to capital and labor. Gollin [2002] confirms that "estimated labor shares that are essentially flat across countries and over time" and finds that two-thirds remains a good estimate for the United States. In a simple model with competitive factor markets, the marginal returns of a dollar of human capital will be equalized with the marginal returns of non-human capital (machines, intellectual property, brands, and so on). Under the further assumption of constant returns to scale of production, this will equalize the returns of not just marginal capital but all capital. If all capital earns the same rate of return and human capital gets twice as large a share, then the human capital stock must be twice as large as as the non-human capital stock. This implies that θ_M is roughly two-thirds and $\theta_s - \theta_M$ is roughly one-third and matches almost perfectly estimates of Jorgenson and Fraumeni [1989].

Lacking updated time series data on the return to human capital, it is approximated with a constant growth of two percent per year. Under the assumptions, the effects on consumption variance in the frictionless models are exact. In the housing and frictions model, the human capital cannot be added simply as another asset as it can in the frictionless models. Instead, the paper assumes that $\theta \cdot W_t$ is invested in human capital but the same consumption policy holds as without human capital. Table 5 shows the results of this simplified treatment of human capital alongside the paper's preferred calibration without them. Introducing a large and risk-less asset to the household balance sheet delivers the intuitive result that wealth is less volatile and therefore so is consumption.

Figure 9: Aggregate Consumption Plots: Starting Distribution of y_t in 2005

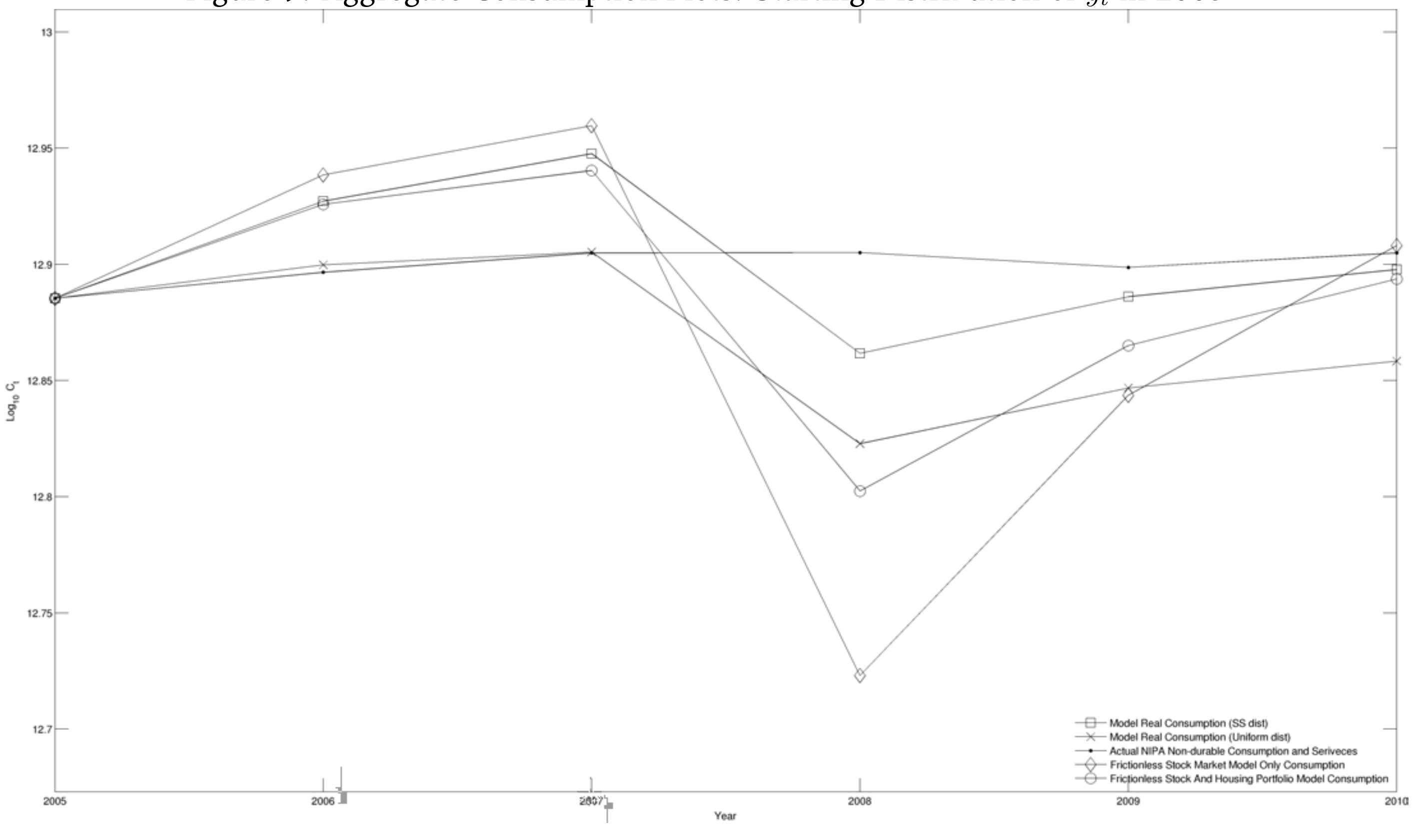


Table 5: Estimate of the Effect of Adding Human Capital to Model

	Std. Dev. $\Delta \ln C_t$		
	$\theta_M = 0$	$\theta_M = \frac{2}{3}$	
Model (historical shocks 95-10)	No Human Capital	With Human Capital	$\frac{M}{X+H}$ Required to Match
NIPA Non-durable Consumption	0.7%	0.7%	NA
Frictions & Housing model	3.9%	1.3%	4.6
Housing and Stock, no frictions	6.3%	2.1%	8.1
Stock Only, no frictions	10.0%	3.3%	13.3